Development of compact thermal model with two exchange surfaces

The design of electronic power converters includes many aspects such as magnetism, material choice, geometry and heat transfer. Their simulations require complex and iterative methods. The compact thermal model (CTM) can then be used to limit the computation time in the design stage. In the model adopted in this paper, the system is represented by two exchange surfaces. An impedance matrix is used to link the temperature of the surfaces to the heat fluxes. This matrix should be independent of the boundary conditions applied at each exchange surface. The computation of the impedance matrix elements versus the frequency is explained. The model is validated using finite element simulations.

Keywords: Thermal approach; thermal impedance matrix; boundary condition independence; simulation.

1. Introduction

Passive components used in power electronics (i.e. capacitors, inductors and transformers) are characterized by impedances measured at the operating frequency [1]-[2]-[3]. Putting more components in a smaller package has complicated the thermal process [4]. So, the thermal design has become of high priority in order to maintain system performance, reliability and low-cost products [5]. The compact thermal model (CTM) is a simple way to save time consuming at the simulation stage [6]-[7]. To develop these models for simple geometry, analytical approaches [8]-[9] can be done. For complicated device, numerical approach as the finite element methods must be performed [10]-[11]. To minimize the computation time, reduced models can be used. The nodal method has proved to be an effective method in many applications [12]-[13]. It is a flexible method to use particularity in the case of model reduction. This method is based on a thermal – electrical analogy. It can be adapted to different areas of physics and sensitivity studies. Kevin J. Kerns et al [14]-[15] introduces the PACT method which is a mathematical process that can be used to reduce a RC network by preserving its poles. J. Pailhes et al [16] presents a quadrupole technique for heat transfer modeling in multilayer spreader with heat sources. The idea is to use hyperbolic functions and numerical problems according to the argument value that depends on geometrical and thermophysical properties as well as characteristics times. The use of this formulation in the equivalent impedance networks allow to run efficiently the thermal behaviour of multilayered spreader. The reduced model proposed in this paper is dedicated to the modeling of magnetic components by using a matrix
description approach. The properties of this matrix are described in the Schmidt method using the mathematical tools related to positive definite matrix [17]-[18].

The paper is divided into three parts. First, the methodology of the thermal matrix description is exposed in the case of two exchange surfaces. Simulations are required to identify the thermal impedance matrix as function of frequency and to generate the reduced model. Thermal characterization methods use a confidence factor to help us to detect the error of simulation step [19]. Second, the boundary condition independence is demonstrated. It allows to ensure that the variation of the boundary conditions does not affect the actual model. Finally, taking into account all the boundary conditions applied in this model, a validation process of the thermal model is performed using FEM simulations.

2. Theoretical aspects

The model discussed in this paper is based on a thermal compact model (CTM) using the notion of thermal impedance matrix. This thermal impedance matrix expresses by a relationships between the temperatures and heat flux densities in a spreader. In this work, the thermal impedance is studied in the case of two-dimensional systems.

2.1. Thermal impedance matrix with two exchange surfaces

For homogenous spreader with two-directional conductions, the quadrupole method is obtained from the heat equation and Fourier's law solving by the finite element method.

\[
\rho C_p \frac{\partial T(x, y, t)}{\partial t} = \lambda \left[ \nabla T(x, y, t) \right] + P(x, y, t)
\]  \hspace{1cm} (1)

where \( T \) is the temperature field, \( \lambda \) is the thermal conductivity, \( \rho C_p \) is the volumetric heat capacity and \( P \) is the volume density of the heat sources.

The sample model test case is shown in Fig. 1.

![Fig. 1](image)

**Fig. 1.** Description of the problem geometry.

Fig. 1 shows an example spreader heat transfer path with 20 mm x 4 mm grid cells on the copper layer. The spreader is divided into 30432 nodes. This model contains two exchange surfaces. Using FEM allows to calculate an approximate solution of a problem in a discrete space representation using a geometrical mesh [20]. Thermal material behaviour is supposed linear isotropic and its properties are given in Table 1.
Table 1: Material thermal properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal conductivity (Wm⁻¹K⁻¹)</th>
<th>Specific heat capacity (Jkg⁻¹K⁻¹)</th>
<th>Density (kgm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>385</td>
<td>385</td>
<td>8930</td>
</tr>
</tbody>
</table>

The CTM is created by the extraction of temperatures and heat flux densities when an appropriate set of boundary conditions is imposed on the external surface. These boundary conditions are defined by:

- A Dirichlet condition that can impose a sinusoidal temperature distribution (BC1)

\[
T(x,t) = T_0(x) + T_0(x) \sin(\omega t)
\]  \hspace{1cm} (2)

where \( T_0 \) is the initial temperature and \( \omega \) the angular frequency.

- Exchange thermal condition that can impose a convective condition (BC2)

\[
\varphi(x,t) = -h \left[ T(x,t) - T_{ref} \right]
\]  \hspace{1cm} (3)

where \( h \) is the heat transfer coefficient and \( T_{ref} \) the reference temperature.

- Exchange thermal condition without heat source (BC3)

\[
\varphi(x,t) = -\lambda \frac{\partial T(x,t)}{\partial n} = 0
\]  \hspace{1cm} (4)

with \( n \) the normal to the exchange surface.

The impedance matrix can be found in the modeling of magnetic components [21] or in the formalism of the thermal quadrupoles [22]. In the thermal study, the thermal impedance matrix is defined in harmonic domain by:

\[
\begin{bmatrix}
\theta_1(\omega) \\
\theta_2(\omega)
\end{bmatrix} = j\omega
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
\phi_1(\omega)|_1 \\
\phi_2(\omega)|_2
\end{bmatrix}
\]  \hspace{1cm} (5)

where \( \theta \) and \( \phi \) are the Fourier transforms respectively of the temperature and heat flux densities, \( l_1 \) and \( l_2 \) are respectively the widths of the surfaces (S₁) and (S₂), and \( Z \) represents the thermal impedance matrix.

This thermal impedance is a complex values which may be represent by its modulus \( |Z| \) and its argument \( \Phi \). Both temperature and heat flux density are computed on each mesh node by using Flux 2D software [23]. The thermal impedance matrix must be square, symmetric and strictly definite positive, this propriety is demonstrated by Onsager [24]-[25] requires that the coefficients \( Z_{12} \) and \( Z_{21} \) are equal. In order to identify thermal impedance matrix, we impose one of these boundary conditions at each exchange surface. The aim of the compact thermal model is to find the matrix based on a preferably small number of detailed model simulations. By analogy with the magnetic characterization [26], 4-linked simulations are required to determine the impedance matrix.

The self-impedance \( Z_{11} \) and \( Z_{22} \) are identified using the first and second simulations, like shown in Table 2 [27].

\[
Z_{11}(\omega) = \frac{\theta_1(\omega)}{j\omega \phi_{l1}(\omega)|_1}
\]  \hspace{1cm} (6)
\[ Z_{22}(\omega) = \frac{\theta_{2b}(\omega)}{j\omega \phi_{2b}(\omega)|_2} \] (7)

To calculate \( Z_{1s} \) and \( Z_{2s} \) coefficients, we use the third and fourth simulations like shown in Table 2. The prefix “s” means that thermal impedance coefficient is in a short circuit [28]-[29].

\[ Z_{1s}(\omega) = \frac{\theta_{1c}(\omega)}{j\omega \phi_{1c}(\omega)|_1} \] (8)

\[ Z_{2s}(\omega) = \frac{\theta_{2d}(\omega)}{j\omega \phi_{2d}(\omega)|_2} \] (9)

Let “a”, “b”, “c” and “d” denotes the acquired simulations.

To identify the mutual impedance coefficients \( Z_{12} \) and \( Z_{21} \), we use 4 relations described below [30].

\[ Z_{1s}(\omega) = \frac{Z_{22}(\omega) - Z_{12}(\omega)}{Z_{11}(\omega)} \] (10)

\[ Z_{12}(\omega) = \sqrt{Z_{22}(\omega)(Z_{11}(\omega) - Z_{1s}(\omega))} \] (11)

Then, it is simple to deduce \( Z_{21} \) impedance from the simulations results:

\[ Z_{21}(\omega) = \sqrt{Z_{11}(\omega)(Z_{22}(\omega) - Z_{2s}(\omega))} \] (12)

Symmetry of impedance matrix related to two exchange surfaces results by this combination that links:

\[ \left| Z_{22}(\omega)Z_{1s}(\omega) \right| = \left| Z_{11}(\omega)Z_{2s}(\omega) \right| \] (13)

To check our simulations, the confidence factor is computed by:

\[ ind(\omega) = \frac{\left| Z_{22}(\omega)Z_{1s}(\omega) \right|}{\left| Z_{11}(\omega)Z_{2s}(\omega) \right|} \] (14)

The four simulations of thermal impedance are carrying out in frequency range.

Table 2: Set of boundary conditions

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>BC on (S₁)</th>
<th>BC on (S₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{11} )</td>
<td>(BC1)</td>
<td>(BC3)</td>
</tr>
<tr>
<td>( Z_{22} )</td>
<td>(BC3)</td>
<td>(BC1)</td>
</tr>
<tr>
<td>( Z_{1s} )</td>
<td>(BC1)</td>
<td>(BC2)</td>
</tr>
<tr>
<td>( Z_{2s} )</td>
<td>(BC2)</td>
<td>(BC1)</td>
</tr>
</tbody>
</table>


2.2. Numerical implementation

Both temperature and heat flux density are computed on each mesh node by using Flux 2D software. This section explains how to generate compact thermal models CTM in MATLAB by using Flux software.
First, designing a “full” model of an electronic power device:
A CTM should supply enough thermal information at the desired design. So the first step of the methodology consists to represent the geometry, to mesh it and to attribute the physical properties that characterize the studied device in the Flux software.

Second, to create a CTM is important to choose an appropriate set of boundary conditions. In this case, we applied boundary conditions directly on the model external surface. Three different boundary conditions sets were imposed on the model one by one. A powerful CTM must be independent from external boundary conditions.

Third, we generate a script file to automate the resolution. All local parameters can be extracted along two paths in the thermal device. This script python define a frequency array containing 40 frequency values ranging from 0.1 Hz to 1000 Hz (working on 4 decades and 10 points by decade) executed by Flux 2D. Each scenario has its own distribution on time and frequency \( f \). At a given frequency, the time interval is defined between 0 and \((8/f)\) with 160 steps. Then, we generate the following physical parameters: temperature and heat flux density which are distributed over each exchange surface. To minimize, the computation time, our study will be restricted on each frequency \( f \), and last period \([7/f, 8/f]\).

These scenarios address 4 simulations which vary with the nature of the boundary conditions applied on each exchange surface. In this case, each simulation has 40 scenarios, since each simulation possesses these 2 own exchange surfaces and its own frequency range.

Fourth, in steady state domain, temperature and heat flux density are extracted for each scenario. The results are saved as txt files. Using MATLAB software, these parameters (temperature and heat flux density) are calculated in complex notation on each mesh point.

The approach adopted in this paper is divided into three steps:
- We present the simulation using all boundary conditions previously described to determine the thermal impedance matrix.
- We demonstrate that model is boundary conditions independent.
- We validate the results and the repeatability of the global model using Flux software.

2.3. Results and discussion

The associated matrix for 2D spreader is constructed using the boundary conditions explained in the previous section (see Table 2). The identification of each coefficient of the impedance matrix takes a lot of time because many simulations of the model must be performed in transient domain. At given frequency (i.e. \( f = 0.1 \) Hz), the copper spreader is simulated by applying a Dirichlet condition (20 °C) at the first exchange surface, the second surface is thermally isolated like shown in Fig. 2-a.
Fig. 2. Temperature distribution at the first (a) and second (b) simulations.

As function with time, the variation of temperature is uniform on the surface \( S_1 \), however the variation of heat flux density is very important on this surface. To determine the coefficient \( Z_{22} \) of the impedance matrix, it is necessary to launch a new simulation by reversing the previously defined boundary conditions like shown in Fig. 2-b. To get a scalar value at each surface, we calculate the mean of temperature and heat flux density. We use this reduction process to identify the coefficients of the thermal impedance matrix.

Fig. 3 gives a comparison of extracted modulus and phases of the self-thermal impedances \( Z_{11} \) and \( Z_{22} \) as function of the frequency.

Fig. 3. Variation of the coefficients \( Z_{11} \) and \( Z_{22} \) versus frequency.

The modulus of the self-thermal impedance decreases with the increases of frequency however the phase is quite constant -130° at frequency range. To assess the mutual terms of the thermal impedance matrix, a third simulation is done. We apply new boundary conditions on the copper spreader. The first exchange surface is maintained at a prescribed temperature 20 °C and a heat sink condition is applied to the second exchange surface with high convection coefficient \( h = 10^{20} \text{ Wm}^{-1}\text{K}^{-1} \) and emissivity \( \varepsilon = 10^{20} \) like shown in Fig. 4-c. Temperature and heat flux density are extracted again on each exchange surfaces. The fourth simulation will be performed to prove that the thermal impedance matrix is square, symmetric and strictly positive definite like shown in Fig. 4-d.
Fig. 4. Temperature distribution at the third (c) and fourth (d) simulations.

The mutual thermal impedances $Z_{12}$ and $Z_{21}$ are complex number and are shown in log-log scale in Fig. 5.

Fig. 5. Variation of the coefficient $Z_{12}$ and $Z_{21}$ versus frequency.

The mutual impedance decreases with increasing frequency, however the phase is almost constant throughout the frequency. At large frequencies the two curves of mutual impedances become identical which confirm that the thermal impedance matrix is symmetric. As shown in Fig. 3 and Fig. 5, the thermal response is restricted to low frequencies range from a few kilohertz. This choice is suitable for low power applications because more we increase frequency, more simulation is costly and will be time-consuming [31]. The relative error on the confidence factor is shown in Fig. 6.
This factor is normally equal to 1, any significant deviation from this value must attract attention [32], from this value should alert on the quality of the simulations. A confidence factor is defined by the equation (14). This confidence factor gives useful information about the characterization of the material and helps to find simulations errors. Our results provide an acceptable confidence factor because the relative error is less than 0.35%. To conclude, four simulations are required to build the thermal impedance matrix in the case of two exchange surfaces. This matrix can then be used to create a CTM characterizing the thermal behaviour for different frequencies. Due to its low time consumption, this CTM could be used to simulate the thermal behavior in an electrical circuit.

3. Boundary condition independence (BCI)

3.1. Theoretical aspects

After focusing on the content of the thermal impedance matrix of our physical model, a comparative study is performed to show that our model is independent of the boundary conditions (BCI). This CTM should reflect the model behavior with acceptable precision under any set of boundary conditions [33]-[34]. Dirichlet conditions are applied on the model to impose different magnitudes of temperature on each exchange surface. The thermal impedance matrix \([Z] \) must present an intrinsic character compared to the temperature excitations.

3.2. Results and discussion

To validate the independence of the boundary conditions of our physical model, we use the same geometry described in the section (2.1), this simple geometry contains a conductive material with two exchange surfaces. The choice of the exchange surface is important because it describes the overall of thermal impedance matrix without considering the whole device. This model is simulated using the same boundary conditions. To show that physical model is BCI, we use the previous simulations by changing the magnitude of the temperature imposed respectively on surfaces \((S_1)\) and \((S_2)\). As described in the first simulation, the sinusoidal temperature is applied in different magnitudes on the first exchange surface, while the second exchange surface is isolated.
Fig. 7. The variation of the first coefficient $Z_{11}$ of the thermal matrix versus frequency.

Fig. 7 shows the variation of the first coefficient $Z_{11}$ of the impedance matrix versus frequency. The coefficient $Z_{11}$ is independent of the temperature excitations. Using the third simulation and to prove that the coefficient $Z_{12}$ is BCI, we also apply different magnitudes of temperature on the first exchange surface ($S_1$).

Fig. 8. The variation of the coefficient $Z_{12}$ of the impedance matrix versus frequency.

Fig. 8 shows the decreasing of $Z_{12}$ coefficient through frequency and tends towards zero value at high frequency. The thermal impedance coefficient $Z_{12}$ is independent of the temperature excitations. The symmetry of the thermal impedance matrix allows to prove that $Z_{21}$ is also independent of the temperature excitations. As described in the second simulation, the sinusoidal temperature is applied in different magnitudes on the second exchange surface, while the first one surface is isolated. We could obtain results similar to the Fig. 7 which prove that $Z_{22}$ is also BCI. To conclude, all thermal impedance matrix are BCI. It means that our CTM is independent of all external boundary conditions.

4. Model validation

4.1. Mathematical tools

In order to validate our model and to verify that this model is completely independent of the exterior mode(s) of heat transfer, the CTM obtained by FEM simulation is verified with a new simulation processes. This simulation is defined by using a Dirichlet condition on the first exchange surface and a convective condition on the second one. Let “e” denote this new simulation. Four physical parameters are extracted from this simulation; $\theta_{1e}$, $\theta_{2e}$, $\phi_{1e}$, $\phi_{2e}$. 
\[ \theta_{1e}(\omega) = j\omega(Z_{11}^{CTM} l_1 \phi_{1e}(\omega) + Z_{12}^{CTM} l_2 \theta_{2e}(\omega)) \]  
(15)

\[ \theta_{2e}(\omega) = j\omega(Z_{21}^{CTM} l_1 \phi_{1e}(\omega) + Z_{22}^{CTM} l_2 \theta_{2e}(\omega)) \]  
(16)

We suppose that the reference temperature is annulled on the second exchange surface. The convective condition applied at this surface is defined by:

\[ \phi_{2e}(\omega) = -h\theta_{2e}(\omega) \]  
(17)

By using a thermal impedance matrix \([Z]\), defined in the section (2.1) a mathematical combination will be developed.

\[ j\omega Z_{11}^{CTM} \phi_{1e}(\omega) l_1 - j\omega Z_{12}^{CTM} l_2 h \theta_{2e}(\omega) = \theta_{1e}(\omega) \]  
(18)

\[ j\omega Z_{21}^{CTM} \phi_{1e}(\omega) l_1 - (j\omega Z_{22}^{CTM} l_2 h + 1) \theta_{2e}(\omega) = 0 \]  
(19)

Using the equation (19) and (18):

\[ \theta_{2e}(\omega) = \frac{Z_{21}^{CTM} \theta_{1e}(\omega)}{(Z_{11}^{CTM} + l_2 Z_{11}^{CTM} Z_{22}^{CTM} h j\omega - l_2 Z_{12}^{CTM} Z_{21}^{CTM} h j\omega)} \]  
(20)

The computed heat flux density on the first exchange surface is defined by using the equations (20) and (18):

\[ \phi_{1e}(\omega) = \frac{(l_2 Z_{22}^{CTM} h j\omega + 1) \theta_{1e}(\omega)}{l_1 j\omega(Z_{11}^{CTM} + l_2 Z_{11}^{CTM} Z_{21}^{CTM} h j\omega - l_2 Z_{12}^{CTM} Z_{21}^{CTM} h j\omega)} \]  
(21)

By using the equations (17) and (20), we identify the heat flux density at the second exchange surface:

\[ \phi_{2e}(\omega) = \frac{-h Z_{21}^{CTM} \theta_{1e}(\omega)}{(Z_{11}^{CTM} + l_2 Z_{11}^{CTM} Z_{22}^{CTM} h j\omega - l_2 Z_{12}^{CTM} Z_{21}^{CTM} h j\omega)} \]  
(22)

The temperature on the first exchange surface \(\theta_{1e}\) is maintained at 20 °C, the other physical parameters are computed and compared with those obtained by FEM simulation in order to validate the proposed model.

4.2. Results and discussion

The forced convection condition (BC2) is applied on the second exchange surface. The heat transfer coefficient reaches 40000 Wm\(^{-2}\)K\(^{-1}\) like shown in Tokamaks model [35]-[36]. Both temperature and heat flux density of the CTM model are compared with the results of the FEM simulations.
Fig. 9. The variation of the temperature under the effect of forced heat transfer coefficient.

Fig. 10. The variation of the heat flux density $\phi_{2e}$ under the effect of forced heat transfer coefficient.

Fig. 9 and Fig. 10 show the decreasing temperature and heat flux density on the second exchange surface as function of frequency. The numerical simulation result is an agreement with the CTM curve. As can be seen, at upper frequency the numerical results deviate from the CTM model which confirms that the model is limited to low frequency.

Fig. 11. The variation of the heat flux density $\phi_{1e}$ under the effect of forced heat transfer coefficient.

Fig. 11 shows the increasing of the heat flux density as function of frequency. Under the effect of forced heat transfer coefficient, the heat flux density still higher at the first exchange surface.

It must be underlined that the computed results from CTM model follows the simulation results in the frequency interval from 0.1 Hz to 10 Hz using a forced convection conditions. On the high frequency side, the deviation of these results is taken into account. The computed results from CTM model are instantaneous and reduce the simulation time. This
could lead to choose a CTM results rather than a FEM simulations. The thermal model is now validated at low frequency and can be reused at every design step.

5. Conclusion

To speed up and ease the thermal characterization process, the engineers need small, accurate thermal model which can be reused at every design step. CTM are well-known techniques in power devices because it allows to save simulation time and to reduce the number of unknowns. They are also employed to characterize the thermal process and can be used in SPICE type electrical simulator for modeling the self and the mutual impedances and the global temperature dependence. We study our CTM approach based on the impedance matrix which links the temperature and heat flux density flowing through the exchange surfaces. In the case of two exchange surfaces, 4-linked simulations are necessary to determine the impedance matrix. This model has great advantage of good performance and lower cost. This CTM possesses many proprieties like symmetrical matrix which allows to be easily inserted into electrical software like PSPCE. CTM is BCI which ensures that the variation of the boundary conditions does not affect the actual model. We have validated our models with 2D FEM simulations by imposing a forced convection on the second exchange surface. Predicted temperatures and heat flux densities by the established CTM are compared to FEM simulation results. Important future work consists of extending our model to include more exchange surfaces whose the number imposes the size of the impedance matrix. This model could be implement in electrical devices to simultaneously simulate the magnetic behavior of power electronic converter and take into account thermal effects.

References