This paper presents the modeling and control of two techniques for the induction machine "IM": a chattering-free Sliding Mode Controller based on Integrated Control (SMIC) and a PI based control technique. Input/output linearization will be used to ensure a decoupled control of the different outputs as well as the stability of the system. The proposed approach is designed for improving the system stability, behaviour and performance. The SMIC yields more accuracy and reduces the chattering resulting from the high frequency control switching. The controller performance is demonstrated via result of simulation in which the proposed controller effectiveness is compared to conventional PI.

Keywords: PI controller, input/output linearization, sliding mode integrated, electromagnetic torque, and rotor flux control.

1. Introduction

The induction machine "IM" control is quite difficult, in other words, compared to the DC machine, the qualities of the IM, which is considered as a high performance electric actuator, can not be used without resorting to advanced and significantly complex control strategies. These are related, on the one hand, to certain parameters and/or variables which may be unattainable, and on the other hand to the nonlinear multi-variable dynamic behavior of this kind of machine [1-2].

However, conventional and modern control theories allow an accurate control of undisturbed linear processes with known parameters. This explains why the PI controller, for example, is widely used in control applications considering its simplicity and efficiency.

The main disadvantage of the PI controller is the difficulty to handle system uncertainty due to the parameter variations and external disturbances [3-4].

Variable structure and associated sliding mode control "SMC" [5-6-7], is a robust control scheme which is widely used to control non-linear systems towards the model uncertainties, disturbances as well as changes of the manipulated load [8-9]. The main feature of SMC is robustness against parameters variations and external disturbances [10-11].

To apply the sliding mode control to a nonlinear system, we will use the input/output linearization method. The obtained system after this transformation is a linear decoupled system with m inputs [12].

The disadvantage associated with the SMC is the emergence of the "chattering" phenomenon.

Sliding mode integrated control "SMIC" is a control technique that overcomes this drawback [13]. It is to define a sliding surface which is integrated according to system states in a way that it is attractive. The idea is to ensure that the dynamic behavior of the equivalent reduced system (when the sliding condition is verified) becomes insensitive to...
modeling errors and other disturbances. The results obtained by simulation on Matlab represent a comparative study between the SMIC and PI controller.

2. Notation

The notation used throughout the paper is stated below.

\[
\begin{align*}
\phi_{dr}, \phi_{qr} & \quad \text{rotor flux components} \\
V_{sd}, V_{sq} & \quad \text{stator voltage components} \\
I_{sd}, I_{sq} & \quad \text{stator current components} \\
\sigma & \quad \text{leakage factor} \\
R_s, R_r & \quad \text{stator and rotor resistances} \\
L_s, L_r & \quad \text{stator and rotor inductances} \\
M_{sr} & \quad \text{mutual inductance} \\
C_{em} & \quad \text{the electromagnetic torque} \\
C_r & \quad \text{load torque} \\
J & \quad \text{moment of inertia of the IM} \\
\Omega_m & \quad \text{mechanical speed} \\
\omega_s & \quad \text{stator pulsation} \\
\tau_r & \quad \text{rotor time constant} \\
\text{SMC} & \quad \text{Sliding Mode Control} \\
\text{SMIC} & \quad \text{Sliding Mode Integrated Control} \\
e_1, e_2 & \quad \text{torque and flow errors (for SMC)} \\
e_{1i}, e_{2i} & \quad \text{torque and flow errors (for SMIC)} \\
n_1, n_2 & \quad \text{order of system} \\
v_1, v_2 & \quad \text{torque and flow vectors control (for SMC)} \\
w_1, w_2 & \quad \text{torque and flow vectors control (for SMIC)} \\
S_1, S_2 & \quad \text{torque and flow sliding surfaces (for SMC)} \\
S_{1i}, S_{2i} & \quad \text{torque and flow sliding surfaces (for SMIC)} 
\end{align*}
\]

3. IM modelling

Prior to the IM equating, some assumptions are considered [1] [14][15]:
- The gap is constant.
- The Hysteresis, the saturation and the eddy currents are neglected.
- The magneto-motive forces generated by the stator and rotor phases have a sinusoidal distribution.
3.1. Mathematical model for the IM

**Electrical equations**

\[ V_{ds} = R_S I_{ds} + \frac{d\phi_{ds}}{dt} - w_S \phi_{qS} \]
\[ V_{qs} = R_S I_{qs} + \frac{d\phi_{qs}}{dt} + w_S \phi_{dS} \]  
\[ 0 = R_r I_{dr} + \frac{d\phi_{dr}}{dt} - w_S \phi_{qr} \]
\[ 0 = R_r I_{qr} + \frac{d\phi_{qr}}{dt} + w_S \phi_{dr} \]  
\[ \text{With} \]
\[ \phi_{ds} = L_S I_{ds} + L_m I_{dr} \]
\[ \phi_{qs} = L_S I_{qs} + L_m I_{qr} \]
\[ \phi_{dr} = L_m I_{ds} + L_r I_{dr} \]
\[ \phi_{qr} = L_m I_{qs} + L_r I_{qr} \]  

**Torque equation**

The equation is defined by:
\[ C_{em} = \frac{3}{2} p \frac{M_{Sr}}{L_r} \phi_r I_{qs} \]  

4. Synthesis of the IM controllers

The IM state equations are as follows:

\[ \frac{dl_{sd}}{dt} = -C_1 I_{sd} + w_S I_{sq} + C_2 \phi_{rd} + C_3 p \Omega_m \phi_{rq} + C_4 V_{sd} \]  
\[ \frac{dl_{sq}}{dt} = C_1 I_{sq} + w_S I_{sd} + C_2 \phi_{rq} - C_3 p \Omega_m \phi_{rd} + C_4 V_{sq} \]  
\[ \frac{d\phi_{rd}}{dt} = C_5 I_{sd} - C_6 \phi_{rd} + (w_S - p\Omega_m) \phi_{rq} \]  
\[ \frac{d\phi_{rq}}{dt} = C_5 I_{sq} - C_6 \phi_{rq} - (w_S - p\Omega_m) \phi_{rd} \]
\[
\frac{d\Omega_m}{dt} = C_y \left( \phi_{rd} I_{sq} - \phi_{rq} I_{sd} \right) - C_y \Omega_m - C_y C_r
\]  
(8)

While:
\[
\sigma = 1 - \frac{M^2_{Sr}}{L_s L_r}
\]

4.1. Control loop of the rotor flux

The decoupling allowed by the oriented flux and the relation (3) can give:
\[
\frac{d\phi_{rd}}{dt} = \frac{M_{Sr} R_r}{L_r} I_{sd} - \frac{R_r}{L_r} \phi_{rd}
\]  
(9)

From where the direct stator current is determined by:
\[
I_{sd} = \frac{1}{M_{Sr}} \left( \phi_{rd} + \frac{L_r}{R_r} \frac{d\phi_{rd}}{dt} \right)
\]  
(10)

Let \( T_r = \frac{R_r}{L_r} \) the rotor time constant and \( T_s = \frac{L_s}{R_s} \) the stator one. The relations (7) and (13) can lead to:
\[
V_{sd} = \frac{R_s}{M_{Sr}} \left( \phi_{rd} + (T_s + T_r) \frac{d\phi_{rd}}{dt} \right) + \sigma T_s T_r \frac{d^2 \phi_{rd}}{dt^2}
\]

\[- w_s \sigma L_s I_{sq} = V_{Sdf} + V_{Sdc} \]

\[
(11)
\]

To ensure the decoupling between the two axes, the term \( V_{Sdc} \) must be compensated:
\[
V_{Sdf} = \frac{R_s}{M_{Sr}} \left( \phi_{rd} + (T_s + T_r) \frac{d\phi_{rd}}{dt} \right) + \sigma T_s T_r \frac{d^2 \phi_{rd}}{dt^2}
\]

\[
V_{Sdc} = - w_s \sigma L_s I_{sq}
\]

(12)

The system transfer function is:
\[
G(p) = \frac{\phi_{rd}(p)}{V_{Sdf}(p)} = \frac{M_{Sr}}{R_s} \frac{1}{1 + (T_s + T_r) p + \sigma T_s T_r p^2}
\]

(13)

Are \( p_1 \) and \( p_2 \) the roots of denominator, such as \( p_2 >> p_1 \).

\[
p_1 = \frac{2 \sigma T_s T_r}{T_s + T_r + \Delta}; \quad p_2 = \frac{2 \sigma T_s T_r}{T_s + T_r - \Delta}
\]

To compensate for the dominant time-constant \( p_2 \). The regulator of flow is of type PI.
\[ F(p) = K_f \frac{R_s \left(1 + T_f p\right)}{M_{sr} p} \]

The transfer function in closed loop is:
\[ H_F(p) = \frac{\phi_{rd}(p)}{\phi_{rd_{-ref}}(p)} = \frac{1}{1 + \frac{2z}{w_n} p + \frac{1}{w_n^2} p^2} \]

with
\[ T_f = p_2 ; \quad w_n = \sqrt{\frac{k_f}{p_1}} ; \quad z = \frac{1}{2} \sqrt{\frac{k_f}{p_1}} \]

To ensure a fast establishment of flow leading to a just oscillating system, one chooses
\[ z = \frac{1}{\sqrt{2}}. \]

The error of flow is \( e = e_{pI} = \phi_{rd_{-ref}} - \phi_{rd} \).

The following figure represents the functional diagram of the loop of regulation of flow.

![Diagram](image)

**Fig. 1.** Control loop of the rotor flux.

4.2. Control loop of the electromagnetic torque

That is to say the response of flow is faster than the response of the couple then flow reaches its end value \( \dot{\phi}_{rd} = \dot{\phi}_{rd0} \), from where the couple is given by the following expression:
\[ C_{em} = \frac{3}{2} \frac{M_{sr}}{L_r} \phi_{rd0} I_{sq} \]

The equation of the tension \( V_{sq} \) becomes:
\[ V_{sq} = R_s I_{sq} + \sigma L_s \frac{dI_{sq}}{st} + \phi_{rd} w_s \frac{M_{sr}}{L_r} + \sigma L_w w_s I_{sd} \]  
(15)

Let

\[ V_{sq} = V_{sqc} + V_{sqt} \]  
(16)

The \( V_{sqc} \) component represents a decoupling term that we have to compensate.

\[ V_{sqc} = \phi_{rd} w_s \frac{M_{sr}}{L_r} + \sigma L_s w_s I_{sd} \]  
(17)

\[ V_{sqt} = R_s I_{sq} + \sigma L_s \frac{dI_{sq}}{dt} \]

The system transfer function becomes:

\[ G(p) = \frac{C_{em}(p)}{V_{sqt}(p)} = \frac{3M_{sr} P \phi_{rd0}}{2L_r R_s (1 + \sigma T_s p)} \]  
(18)

We choose a PI controller; its transfer function is given by:

\[ F(p) = \frac{2L_r R_s}{3M_{sr} P \phi_{dr0}} \cdot K_c \left(1 + \frac{T_c p}{p}\right) \]  
(19)

Let \( T_c = \sigma T_s \), the transfer function in closed loop will be:

\[ H_F(p) = \frac{C_{em}(p)}{C_{em\_ref}(p)} = \frac{1}{1 + \frac{1}{K_c} p} \]  
(20)

\[ H_F(p) = \frac{C_{em}(p)}{C_{em\_ref}(p)} = \frac{1}{1 + \frac{1}{K_c} p} \]  
(21)

The error of torque is \( e = e_{1\_p} = C_{em\_ref} - C_{em} \).

The following figure represents the functional diagram of the loop of regulation of the torque.

\[ \text{Fig. 2. Control loop of the electromagnetic torque.} \]
5. Sliding Mode Integrated Control "SMIC"

5.1. Sliding Mode Control "SMC"

The sliding mode takes place in grate application fields present of the fields and opens interesting prospects in the electric machines control domain.

It is to define a sliding surface according to the system states so that it is gravitational [16].

The main disadvantage of the SMC is emergence of the chattering phenomenon (oscillation or disturbance in the evolution of the controlled parameters). Chattering phenomenon appears because the variable structure supposes that the control can be commutated instantly.

5.2. Operating principle

To reduce the chattering effect, we suggest using the variable structure control law with sliding mode integrated control "SMIC"[13].

The SMIC is a second controller integrated into the sliding mode controller. It consists in bringing each considered point of the system "N_j" to a second integrated sliding surface (S_ij=0) so that it is attractive. The point "N_j" is brought to the intersection of two surfaces S and S_ij. The oscillation either side of the surface S=0 is removed until convergence to equilibrium point located on this surface S=0 [17].

Switching command must be with an infinite frequency and zero amplitude.

Generation of the control vector $\mathbf{W}_d$ is performed using the following rallying law:

$$S'_i = -Q_i \text{s}gn(S_i) - L_i \lambda_i(S_i) = 0$$

(22)

Where:

$$Q_i = \text{diag}[q_{i1}, q_{i2}, \ldots, q_{im}], \quad q_{ij} > 0$$

$$L_i = \text{diag}[l_{i1}, l_{i2}, \ldots, l_{im}], \quad l_{ij} > 0$$

$$\lambda_i(S_i) = \begin{bmatrix} \lambda_1(S_{i1}) & \lambda_2(S_{i2}) & \cdots & \lambda_m(S_{im}) \end{bmatrix}^T, \quad S_{ij}\lambda_j(S_{ij}) > 0, \quad \lambda_j(0) = 0$$

Fig. 3. Principle diagram of the sliding mode control SMC and sliding mode integrated control SMIC.
5.3. Input/Output Linearization

The Input/Output linearization technique is an open loop transformation of a nonlinear dynamical system to a decoupled linear system with \( m \) inputs (in this case \( m=2 \)) having all its poles at the origin.

The main idea of this approach is to allow expressing the output variables of the system in terms of the input variables through a nonlinear state feedback.

\[
S'_i = -Q_i \text{sgn}(S_i) - L_i \hat{\lambda}(S_i) = 0
\]

\[
y = h(X)
\]

Where:

- \( f \) and \( h \) are applications respectively of \( R^n \) in \( R^n \) and \( R^m \).

\[
f(x) = \begin{bmatrix} f_1(x) & f_2(x) & f_3(x) & f_4(x) \end{bmatrix}^T = AX
\]

\[
f_1(X) = -\frac{R_r}{L_r} \phi_{dr} + M_{Sr} \frac{R_r}{L_r} i_{ds}
\]

\[
f_2(X) = \left(w_s - P\Omega_m\right) \phi_{dr} + M_{Sr} \frac{R_r}{L_r} i_{qs}
\]

\[
f_3(X) = \frac{M_{Sr} R_r}{\sigma L_S L_r} \phi_{dr} - \frac{1}{\sigma L_S} \left( R_S + \frac{M_{Sr}^2 R_r}{L_r} \right) i_{ds} + w_s i_{qs}
\]

\[
f_4(X) = -\frac{M_{Sr}}{\sigma L_S L_r} P\Omega_m \phi_{dr} + w_s i_{ds} + \frac{1}{\sigma L_S} \left( R_S + \frac{M_{Sr}^2 R_r}{L_r} \right) i_{qs}
\]

- \( X \) is the vector of state of dimension \( n=4 \).

\[
X = \begin{bmatrix} \phi_{dr} & \phi_{qr} & i_{ds} & i_{qs} \end{bmatrix}^T
\]

- \( u \) is the vector of entry of dimension \( m=2 \).

\[
u = \begin{bmatrix} v_{ds} & v_{qs} \end{bmatrix}^T
\]

- \( D(x) \) is a matrix of dimension \( nxm \) whose columns are fields of vectors \( d_i(x) \).

\[
D(x) = \begin{bmatrix} d_1(x) & d_2(x) \end{bmatrix}
\]

\[
d_1 = \begin{bmatrix} 0 & 0 & \frac{1}{\sigma L_S} & 0 \end{bmatrix}^T
\]

\[
d_2 = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{\sigma L_S} \end{bmatrix}^T
\]

- \( y = h(x) \) is the vector of exit of the system of dimension \( m \)
The elements of the vector of exit \( h(x) = \left[ h_{11}(x) \quad h_{12}(x) \right]^T \) are respectively the electromagnetic torque \( h_{11}(x) \) and rotor flow \( h_{12}(x) \).

\[
h(X) = \left[ h_{11}(X) \quad h_{12}(X) \right]^T = \left[ C_{em}^\prime \quad \phi_r \right]^T
\]

To obtain the system Input/Output linearization, it is necessary to derivate the output variables as many times as necessary using the Lie derivative and the relative degree [13] [14].

Fig. 4. Obtained system, after applying the Lie derivative.

5.4. Hyper surface commutation

The general form suggested in [18] [19] [20] to determine a further sliding surface is:

\[
S_i(X) = \left( \frac{d}{dt} + \alpha \right)^n \left( \int e_i \right)
\]

The two components are defined by:

\[
S_{i1} = K_{i1}\left(S_{i1} - C_{em}^\prime \right) + K_{i2}\int\left(S_{i1} - C_{em}^\prime \right) \tag{29}
\]

\[
S_{i2} = K_{i3}\left(S_{i2} - \phi_r \right) + K_{i4}\int\left(S_{i2} - \phi_r \right) \tag{30}
\]

\( e_{i1} \) and \( e_{i2} \) : the torque errors and the flux errors :

\[
e_{i1} = S_1 - C_{em}^\prime \quad \text{and} \quad e_{i2} = S_2 - \phi_{dr}^\prime
\]

The existence condition of the sliding mode is defined by the Lyapunov equation [21] [22] [23]:

\[
S_i S_i^\prime \leq 0 \tag{32}
\]

5.5. Equivalent control computing

The existence of an equivalent command is a needful condition for the existence of a sliding mode integrated on the switching surface \( S_i=0 \).

By defining the function \( \lambda(S_i) = S_i \), the relationships (24) and (30) allow to deduce the
equivalent command $W_i$. The components $W_1$ and $W_2$ of the linear system control vector $W_i$ are defined by:

\[
W_1 = K_{i1}.K_{i3}[-K_{i2}.e_{i1} + q_{i1}.\text{sgn}(S_{i1}) + l_{i1}.S_{i1}]
\]

\[
W_2 = K_{i1}.K_{i3}[-K_{i4}.e_{i2} + q_{i2}.\text{sgn}(S_{i2}) + l_{i2}.S_{i2}]
\]

(33) (34)

Where :
- $e_{i1}$ ; $S_{i1}$ ; $w_1$ : $y_{i-ref} = C_{em-ref}$ and $y_i = C_{em}$ for the torque control.
- $e_{i2}$ ; $S_{i2}$ ; $w_2$ ; $y_i = \phi_{r-ref}$ ; $y_i = \phi_r$ for the flux control.

Fig. 5. General diagram of the sliding mode integrated control.

6. Simulation results

Comparison between the two control techniques for the IM is first illustrated by Figure 6 to Figure 15. Performance of both control techniques is evaluated by:

- Stability in steady state.
- Response quickness.
- Relative weakness of the static error.

The simulation is performed with unloading startup, at $t=60s$ rotation is reversed, then a load torque $C_r=20Nm$ is introduced at $t=100s$.

Fig. 6 represents the evolution of the real and the reference electromagnetic torque of the IM in the presence of radial force $C_r=20N$ à $t=100s$.

It is noted that the electromagnetic torque does not admit oscillations and reaches steady operation with a response time $T_{rPI}=2.95s$ and $T_{rSMC}=0.09s$. The machine answers successfully the inversion of its rotation.

Fig. 7 shows the impact of the applied controls on the flux response along the two axes ($d$, $q$):
- Along the axis ($d$): control by integrated sliding mode is less sensitive than PI controller to rotation direction reversing as well as load torque variation.
- Along the axis ($q$): the flux is negligible relatively to control.

Changes in the IM flux demonstrates the robustness of the control slide, it follows exactly the desired set point without overshooting and without static error even when the load torque changes or rotation is reversed.

The evolution of the direct rotor flux is not a static error with short response time;

Fig. 8 is a representation of the IM speed evolution with both commands techniques.

The IM speed response is similar to that of a first order system without overshooting. The IM reaches steady state in almost 4.19s with a PI controller and 1.16s with the integrated sliding mode controller.

Velocity evolution with load introduction ($t=100s$) proves the robustness of the integrated sliding mode controller compared to the PI one.

Fig. 9 and 10 show the errors $e_i/e_{i1}$ and $e_i/e_{i2}$ between the real IM torque (respectively the IM flux) and the reference one. The response time error with the SMIC is significantly
A. Ltifi et al: A Sliding Mode Integrated Control Technique and the PI regulator for IM

lower than that with a PI controller. It is found that:
- Observing the error, we note the stability of the system in steady state without overshooting and with a short response time.
- Control by SMIC and PI are insensitive to load changes (from zero torque to 20Nm).
- System instability at startup.

Fig. 11 and 12 show the sliding surface evolution according to time. Both surfaces $S_{i1}$ and $S_{i2}$ quickly pass to zero. The "chattering" amplitude oscillation level of both surfaces $S_{i1}$ and $S_{i2}$ are negligible compared to $S_1$ and $S_2$ surfaces.

Fig. 13 and 14 represent the evolution of torque (respectively flux) derived error according to torque error (respectively flux). The impact of SMIC on the control is shown in two control phases. The first phase represents the system response speed when joining the sliding surface ($t_{ei1}=1.43s$ and $t_{ei2}=0.13s$). The second phase ensures, along this surface, the sliding and the maintenance trapped in the decision border when reaching the phase plan origin. The SMIC effectiveness is revealed in the second phase by the negligible "chattering" oscillation amplitude level.

Fig. 15 shows the $V_{ds}$ and $V_{qs}$ voltage that should be applied to IM to control the sliding mode and drive the PI controller.

Voltages reach steady state with a response time:
- $t_{rSMC} = 0.093s$ ; $t_{rPI} = 3.53s$ for $V_{qs}$ voltage
- $t_{rSMC} = 0.6s$ ; $t_{rPI} = 6.7s$  for $V_{ds}$ voltage

We mention that the PI controller is more sensitive to startup than sliding mode one.

![Fig. 6. Evolution of the electromagnetic torque with sliding mode integrated controller and PI controller.](image)
Fig. 7. Rotor flux response.

Fig. 8. Evolution of the IM speed.

Fig. 9. Variation of the error \((\epsilon_{i1} ; \epsilon_{1_{PI}})\) between the actual torque and the reference.
Fig. 10. Variation of the error ($e_{i2} ; e_{2\_PI}$) between the real flux and the reference flux.

Fig. 11. Evolution of the sliding surface ($S_{i1}$) over time.
Fig. 12. Evolution of the sliding surface (Si2) over time.

Fig. 13. Torque error derivative according to the torque error.

Fig. 14. Flux error derivative according to the flux error.
7. Conclusion

A comparative study of two IM control techniques has been made in this paper: PI control and sliding mode integrated control.

The disadvantage of the sliding mode control is the "chattering" phenomenon of. To overcome this drawback we developed a variable structure control technique based on an integrated sliding mode allowing obtaining a sliding mode on a surface of commutation without the "chattering" phenomenon.

SMIC is a technique that allows to improve the system stability and to reach the required behavior, by adding a second integrated sliding Mode Controller using a technique of input/output linearization which allows decoupling of the system that it has to be controlled and thus to facilitate the controller design.

Simulation results proved the robustness of this technique with overcomes the load disturbances. In addition, convergence towards reference is ensured with complete “chattering” phenomenon elimination. Results revealed the supremacy of the SMIC versus the PI controller particularly as regard to speed response and robustness towards load disturbance.

References


