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J. Electrical Systems 12-3 (2016): 541-555

Regular paper

Robust Output Trajectory Tracking of Car-Like Robot Mobile

Journal of Electrical Systems

In this paper, we propose a robust output trajectory tracking based on the differential flatness and the integral sliding mode control of the car-like robot mobile. The trajectory planning and the dynamic linearization are based on the differential flatness property of the robot, whereas the integral sliding mode control is designed to solve the reaching phase problem with the elimination of matched uncertainties and minimization of unmatched one. The effectiveness of the proposed control scheme is demonstrated through simulation studies.

Keywords: Dynamic feedback linearizing, Flatness, Integral sliding mode control, Trajectory tracking, wheeled mobile robot.

Article history: Received 6 March 2016, Accepted 7 August 2016

1. Introduction

Wheeled mobile robots (WMRs) have attracted much research due to their theoretically interesting properties and its usefulness in many applications. These robots are a typical example of non-holonomic mechanisms where the constraints imposed on the motions are not integrable resulting from the assumption that there is no slipping of the wheels [1]. The main consequence of a non-holonomic constraint for the WMRs is the no corresponding of each path of the admissible configuration space to a feasible trajectory for the robot. Hence, feasible motion planning of WMRs received considerable attention in the past years. Control of WMRs has also been studied from several points of view, including set point stabilization, trajectory tracking and path following. Among nonlinear control techniques developed for WMRs, we mention: Lyapunov-based nonlinear controllers [2], controls based on backstepping approaches [3], model-based predictive controllers [4], flatness based controls [5], and discontinuous controls [6]. However, for WMRs, the basic limitations in the control design come from their kinematic dynamic as showed in [7].

The flatness property developed by Fliess et al., (1999) [8], as a new concept in automatic, is able to generate a feasible trajectory without integrating the dynamic of the system. Furthermore, the flatness provides a systematic design of linearized endogenous state feedback of the original system. Several real systems had proved to be flat and the flatness appears as a natural property of such systems like Induction Motors, non-holonomic mobile robot, voltage converter, conventional aircraft…etc. The application of the flatness to the control of WMRs is given in [9]–[11].

In the real implementation, it is desired to have an inherently robust control, which provides a fast convergence and good robustness properties with respect to the parameter variation and the external disturbances. The sliding mode control emerges as a robust approach and has been successfully applied to control problems in different real applications [6]. The sliding mode control has many advantages, among them, its finite

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time convergence to a stable manifold and insensitivity to disturbances and model uncertainties satisfying the matching condition. However, it has some disadvantages such as the chattering phenomena, the reaching phase and sensitivity to the unmatched perturbation. To enhance the robustness of the sliding mode control in the whole motion, it is interesting to eliminate the reaching phase and minimized the effect of the unmatched disturbances. This can be done by applying the integral sliding mode design proposed in [6], [12], [13]. The basic idea of this control is the inclusion of an integral term to the sliding manifold, which enables the system to start on the sliding manifold at the initial condition and eliminating the reaching phase. From the integral sliding manifold, we define two controllers [6]: The continuous controller that is a continuous time varying feedback designed to stabilize the nominal system and the discontinuous control used to minimize the disturbances.

The main objective of this work is the design of a robust controller for the trajectory tracking of the car-like robot subject to state dependent uncertainties. To attain this objective, we use the flatness and the sliding mode approaches. The flatness is used to generate a feasible trajectories and to transform the system to linear controllable perturbed by dynamic linearizing feedback. To enhance the robustness of the system, we add an integral sliding mode control to a classical continuous feedback.

The outline of this paper is as follows. In Section 2, kinematic model of car-like robot and problem formulation is presented. Then, the proposed output feedback design is derived in Section 3. In Section 4 stability analysis of the proposed controller is given. Then, simulation results are discussed in Section 5. Finally, Section 6 is devoted for the conclusion.

2. Kinematic model and problem formulation

The kinematic model of car-like mobile robot shown in Fig. 1 is described taking in consideration the non-holonomic constraints. A complete study of the kinematics model of wheeled mobile robots could be found in [13].

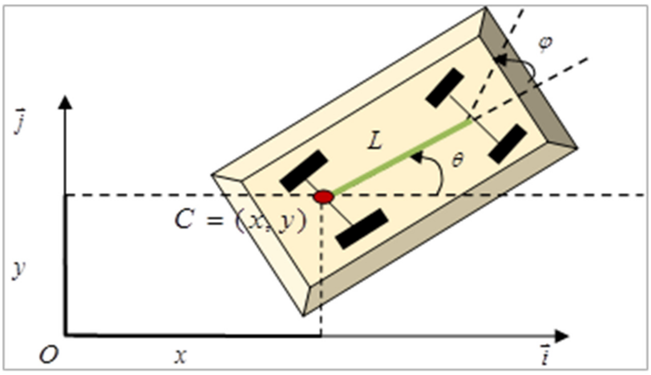

Fig. 1. Kinematic model of the car-like robot.

The midpoint of the rear axle of the mobile robot is noted C as shown in Fig. 1; the configuration of the car-like robot q is represented by the quadruplet

$$q = (x, y, \theta, \varphi) \in Q = IR^2 \times SO \tag{1}$$

Where (x, y) are the coordinates of point C , θ is the car orientation, φ is the orientation of the front wheels and L is the distance between the front and rear axles. The non-holonomic constraint, coming from allowing the wheels to roll and spin without slipping [13], is given by

$$\begin{gathered} \dot{x}\sin\theta - \dot{y}\cos\theta = 0 \\ \dot{x}\sin(\theta+\varphi) - \dot{y}\cos(\theta+\varphi) - L\cos\varphi\,\dot{\theta} = 0 \end{gathered} \tag{2}$$

Based on this constraint, the kinematic model of the car can be written as follows

$$\dot{q} = g(q)u \Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \frac{1}{L}\tan\varphi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{3}$$

Where u_1 and u_2 are the linear velocity of the wheels and its angular velocity around the vertical axis, respectively, they represent the control inputs [14].

The model of the car-like robot usually has external disturbance or unmodeled uncertainty. The kinematic model (3) can be modified by adding perturbations as follows,

$$\dot{q} = g(q)u + P(q,t) \Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \frac{1}{L}\tan\varphi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \tag{4}$$

Where $P(q,t)$ is an unknown vector that represents the modeling uncertainties and external disturbances (matched and unmatched). The following assumption is introduced.

Assumption 1: $P(q,t)$ is continuous and bounded for all $(q,t) \in IR^4 \times IR^+$; that is,

$$\left|P_i\right| < \rho_i, \quad i = 1,..,4 \tag{5}$$

Where ρ_i are a known positives constant.

Problem 1: solve the robust tracking of the output (x, y) to a reference trajectory (x_r, y_r) in the presence of the perturbation $P(q,t)$.

2.1. Trajectory planning

In practice, the reference trajectory is given by the initial condition at $t = t_0 : q_0 = (x_0, y_0, \theta_0, \varphi_0)$ and final condition at $t = t_f : q_f = (x_f, y_f, \theta_f, \varphi_f)$. Before computing the tracking control feedback, the designer should generate feasible reference trajectories that satisfy the nominal model (3) for a given interval of time $[t_0, t_f]$. The trajectory-planning problem is reformulated by the following problem.

Problem 2: plan a feasible state trajectory of (3) for a given initial condition at $t = t_0 : q_0 = (x_0, y_0, \theta_0, \varphi_0)$ and final condition at $t = t_f : q_f = (x_f, y_f, \theta_f, \varphi_f)$.

To solve the problem 2, we use the flatness property of the kinematic model of car like-robot given in (3). The flat output z is given by the pair of coordinates of the midpoint of the rear axle i.e. $z = (x, y)$ [5]. Indeed, from (3) we can algebraically describe all trajectories of the system in function of $z = (x, y)$ and finite number of their derivatives [14].

$$\theta = \operatorname{atan}\left(\frac{\dot y}{\dot x}\right)$$
$$\varphi = \operatorname{atan}\left(\frac{L\left(\ddot y \dot x - \dot y \ddot x\right)}{\left(\dot x^2 + \dot y^2\right)^{\frac{3}{2}}}\right) \tag{6}$$

$$u_1 = \pm\sqrt{\dot x^2 + \dot y^2}$$
$$u_2 = L\sqrt{\dot x^2 + \dot y^2}\,.\chi \tag{7}$$

with

$$\chi = \frac{\left[\dot y^2(\dot x\,\dddot y + 3\ddot x\ddot y) - \dot x^2(\dot y\,\dddot x + 3\ddot x\ddot y) + \dot x^3\dddot y - \dddot x\,\dot y^3 + 3\dot y\dot x(\ddot x^2 - \ddot y^2)\right]}{(\dot x^6 + \dot y^6 + \dot x^2(3\dot y^4 + L^2\ddot y^2) + \dot y^2(3\dot x^4 + L^2\ddot x^2) - 2L^2\dot y\ddot y\dot x\ddot x)}$$

The singularity in (6), which occur when $\dot x = 0$ and $u_1 = 0$ can be avoided by time parameterization the reference trajectory of the flat output $z = (x, y)$. To this end, we choose trajectory in the form

$$t \to z = (x(t), y(t)) = (\sigma(t), P(\sigma(t)))$$

From this choice, the states and the open loop control [11] can be written as

$$\theta = \operatorname{atan}\left(\frac{dp(\sigma(t))}{d\sigma}\right)$$
$$\varphi = \operatorname{atan}\left(L\chi(\sigma(t))\right) \tag{8}$$

$$u_1 = \sqrt{1 + \left(\frac{dp(\sigma(t))}{d\sigma}\right)^2}\,\dot\sigma(t)$$
$$u_2 = \left(L\frac{\frac{d}{d\sigma}(\chi(\sigma(t)))}{1 + L^2\chi^2(\sigma(t))}\right)\dot\sigma(t) \tag{9}$$

Where we have noted by

$$\chi(\sigma(t)) = \frac{\frac{d^2p(\sigma(t))}{d\sigma^2}}{\left(1 + \left(\frac{dp(\sigma(t))}{d\sigma}\right)^2\right)^{\frac{3}{2}}}$$

From these equations, we can remark that the singularity is avoided.

From (8) and (9), we conclude that to generate algebraically the state and input reference trajectories of (3), it suffices to generate the reference of the flat trajectories. Assume that the angles θ, φ belong to $\left]-\frac{\pi}{2}, +\frac{\pi}{2}\right[$ due to physical limitation and consider the initial conditions for the input $(u_1(t_0), u_2(t_0)) = (0,0)$.

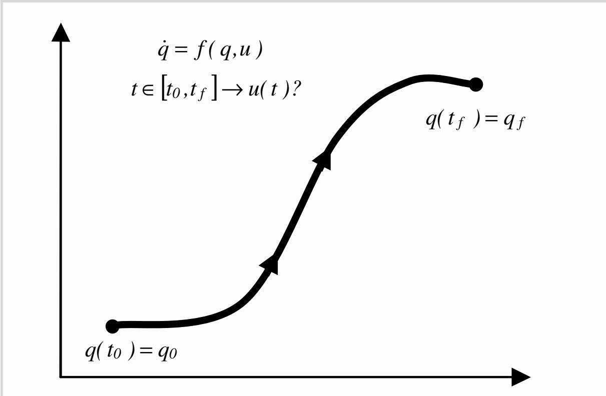

Fig. 2. Principle of trajectory planning.

According to (8) and (9), we need the initial and final conditions of the flat output and its derivative up to order two to compute the reference trajectory of the flat output. Hence, we consider a smooth curve $y_r = p(\sigma(t))$ connecting the initial and final points in (x, y)-plan. It is straightforward to find a polynomial of minimal degree five, which verifies the boundary conditions, we have

$$x_r = \sigma(t) = x_0 + (x_r - x_0)\left(\frac{t - t_0}{t_f - t_0}\right)^3 \left(10 - \left(\frac{t - t_0}{t_f - t_0}\right) + 6\left(\frac{t - t_0}{t_f - t_0}\right)^2\right) \tag{10}$$

$$y_r = P(\sigma) = y_0 + (y_r - y_0)\left(\frac{\sigma - \sigma_0}{x_r - x_0}\right)^3 \left(10 - \left(\frac{\sigma - \sigma_0}{x_r - x_0}\right) + 6\left(\frac{\sigma - \sigma_0}{x_r - x_0}\right)^2\right) \tag{11}$$

In the new time $\sigma(t)$, the dynamic became

$$\begin{bmatrix} \frac{dx}{d\sigma} \\ \frac{dy}{d\sigma} \\ \frac{d\theta}{d\sigma} \\ \frac{d\varphi}{d\sigma} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \frac{1}{L}\tan\varphi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} \tilde{P}_1 \\ \tilde{P}_2 \\ \tilde{P}_3 \\ \tilde{P}_4 \end{bmatrix} \tag{12}$$

Where $u_1 = w_1\dot{\sigma}, u_2 = w_2\dot{\sigma}$, $P_i = \tilde{P}_i\dot{\sigma}\quad i = 1,\ldots,2$ and $w = [w_1 , w_2]^T$ are the new controls.

Note that the flatness, the model (12) is linearizable by endogenous state feedback and consequently equivalent to the linear controllable perturbed system.

2.2. Control problem formulation

To resolve the tracking problem 1, we begin by computing the transformation that converts the system (12) to a linear perturbed system. We define the output error by $e = (e_x, e_y) = (x - x_r, y - y_r)$. By computing successively the derivatives of the output error up to the order three with respect to the new time $t \to \sigma(t)$; and defining the state transformation

$$
\begin{aligned}
\zeta_1 &= x - x_r, \\
\zeta_2 &= w_1 \cos\theta - \frac{dx_r}{d\sigma}, \\
\zeta_3 &= \frac{dw_1}{d\sigma}\cos\theta - \sin\theta \frac{\tan\varphi}{L} w_1^2 - \frac{d^2 x_r}{d\sigma^2} \\
\zeta_4 &= y - y_r, \\
\zeta_5 &= {}_1 \sin\theta - \frac{dy_r}{d\sigma}, \\
\zeta_6 &= \frac{dw_1}{d\sigma}\sin\theta + \cos\theta \frac{\tan\varphi}{L} w_1^2 - \frac{d^2 y_r}{d\sigma^2}
\end{aligned}
\tag{13}
$$

and noting $\dot{\zeta}_i = \frac{d\zeta_i}{d\sigma}$, the system (12) became in the new state

$$
\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 + D_{u1}(\zeta, \tilde{w}_1, \tilde{p}, \sigma) \\
\dot{\zeta}_2 &= \zeta_3 + D_{u2}(\zeta, \tilde{w}_1, \tilde{p}, \sigma) \\
\dot{\zeta}_3 &= \phi_1(\zeta, \tilde{w}_1) + \gamma_1(\zeta)\left[W + D_m(\zeta, \tilde{w}_1, \tilde{p}, \sigma)\right] + D_{u3}(\zeta, \tilde{w}_1, \tilde{p}, \sigma) \\
\dot{\zeta}_4 &= \zeta_5 + D_{u4}(\zeta, \tilde{w}_1, \tilde{p}, \sigma) \\
\dot{\zeta}_5 &= \zeta_6 + D_{u5}(\zeta, \tilde{w}_1, \tilde{p}, \sigma) \\
\dot{\zeta}_6 &= \phi_2(\zeta, \tilde{w}_1) + \gamma_2(\zeta)\left[W + D_m(\zeta, \tilde{w}_1, \tilde{p}, \sigma)\right] + D_{u6}(\zeta, \tilde{w}_1, \tilde{p}, \sigma)
\end{aligned}
\tag{14}
$$

Where $\zeta = [\zeta_1 \ \zeta_2 \ \zeta_3 \ \zeta_4 \ \zeta_5 \ \zeta_6]$, $\tilde{w}_1 = \left(w_1, \frac{dw_1}{d\sigma}\right)$ and $W = \left[\frac{d^2 w_1}{d\sigma^2}, w_2\right]^T$ are the new controls. The terms $\phi_i(\zeta, \tilde{w}_1)$ and $\gamma_i(\zeta)$ ($i = 1,2$) are given as

$$
\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} -3\sin\theta \frac{\tan\varphi}{L} w_1 \frac{dw_1}{d\sigma} - \cos\theta \left(\frac{\tan\varphi}{L}\right)^2 w_1^3 \\ 3\cos\theta \frac{\tan\varphi}{L} w_1 \frac{dw_1}{d\sigma} - \sin\theta \left(\frac{\tan\varphi}{L}\right)^2 w_1^3 \end{bmatrix}, \quad \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\frac{\sin\theta}{L\cos^2\varphi} w_1^2 \\ \sin\theta & \frac{\cos\theta}{L\cos^2\varphi} w_1^2 \end{bmatrix}
$$

The matched disturbances D_m and unmatched disturbances D_{uj} ($j = 1,...,6$) are given by

$$
D_m(\zeta, \tilde{w}_1, \tilde{P}, \sigma) = \begin{bmatrix} -\frac{\tan\varphi}{L} w_1^2 \tilde{P}_3 \\ L\cos^2\varphi \tilde{P}_4 \end{bmatrix}, \quad \begin{bmatrix} D_{u1} \\ D_{u2} \\ D_{u3} \\ D_{u4} \\ D_{u5} \\ D_{u6} \end{bmatrix} = \begin{bmatrix} \tilde{P}_1(q,\sigma) \\ -\sin\theta w_1 \tilde{P}_3 \\ -\sin\theta \frac{dw_1}{d\sigma} \tilde{P}_3 - \frac{d^3 x_r}{d\sigma^3} \\ \tilde{P}_3(q,\sigma) \\ \cos\theta w_1 \tilde{P}_3 \\ \cos\theta \frac{dw_1}{d\sigma} \tilde{P} - \frac{d^3 y_r}{d\sigma^3} \end{bmatrix}
$$

The input-output behavior of the system (14) expressed in term of the output error $e = (e_x, e_y) = (x - x_r, y - y_r)$ is

$$
\begin{aligned}
\frac{d^3 e_x}{d\sigma^3} &= \frac{d^2 w_1}{d\sigma^2}\cos\theta - 3\sin\theta \frac{\tan\varphi}{L} w_1 \frac{dw_1}{d\sigma} - \cos\theta \left(\frac{\tan}{L}\right)^2 w_1^3 - \frac{\sin\theta}{L\cos^2\varphi} w_1^2 w_2 + \psi_1\left(q, \tilde{w}_1, \tilde{p}, \sigma\right) \\
\frac{d^3 e_y}{d\sigma^3} &= \frac{d^2 w_1}{d\sigma^2}\sin\theta + 3\cos\theta \frac{\tan\varphi}{L} w_1 \frac{dw_1}{d\sigma} - \sin\theta \left(\frac{\tan}{L}\right)^2 w_1^3 + \frac{\cos\theta}{L\cos^2\varphi} w_1^2 w_2 + \psi_2\left(q, \tilde{w}_1, \tilde{p}, \sigma\right)
\end{aligned}
\tag{15}
$$

Where $\psi_1(q, \tilde{w}_1, \tilde{p}, \sigma)$ and $\psi_2(q, \tilde{w}_1, \tilde{p}, \sigma)$ are the disturbance terms given by

$$\psi_1(q,\tilde{w}_1,\tilde{p},\sigma) = -\sin\theta\left(2\frac{dw_1}{d\sigma}\tilde{P}_3 + w_1^2\tilde{P}_4\right) - \frac{\cos\theta\tan\varphi}{L}w_1\left(w_1+w_2\right)\tilde{P}_3 - \sin\theta w_1\dot{\tilde{P}}_3 - \cos\theta w_1\tilde{P}_3\tilde{P}_4 + \ddot{\tilde{P}}_1 - \frac{d^3x_r}{d\sigma^3}$$

$$\psi_2(q,\tilde{w}_1,\tilde{p},\sigma) = \cos\theta\left(2\frac{dw_1}{d\sigma}\tilde{P}_3 + w_1^2\tilde{P}_4\right) - \frac{\sin\theta\tan\varphi}{L}w_1\left(w_1+w_2\right)\tilde{P}_3 + \cos\theta w_1\dot{\tilde{P}}_3 - \sin\theta w_1\tilde{P}_3\tilde{P}_4 + \ddot{\tilde{P}}_2 - \frac{d^3y_r}{d\sigma^3}$$

The representation in (14) is analogous to the Local generalized controllable canonical (LGCC) form [15], in the sense that it differs from the basic LGCC form since it is also affected by uncertainties. With reference to system (14), the following assumption (which is an alternative form of Assumption 1) is introduced.

Assumption 2: To simplify the control design, consider that

$$\begin{aligned} D_{u3}(\zeta,\tilde{w}_1,\tilde{p},\sigma) + D_{u2}(\zeta,\tilde{w}_1,\tilde{p},\sigma) + D_{u1}(\zeta,\tilde{w}_1,\tilde{p},\sigma) \cong \Delta\Psi_1(\zeta,\tilde{w}_1,\tilde{p},\sigma) \\ D_{u6}(\zeta,\tilde{w}_1,\tilde{p},\sigma) + D_{u5}(\zeta,\tilde{w}_1,\tilde{p},\sigma) + D_{u4}(\zeta,\tilde{w}_1,\tilde{p},\sigma) \cong \Delta\Psi_2(\zeta,\tilde{w}_1,\tilde{p},\sigma) \end{aligned} \quad (16)$$

Furthermore, we assume that

$$\forall(\zeta,\sigma)\in IR^6\times IR^+,\ \exists\,\alpha_i,\tau_i>0 \ and\ \ i=1,2,\ \ \left|\gamma_i(\zeta)D_m(\zeta,\tilde{w}_1,\tilde{P},\sigma)\right|<\alpha_i, \left|\Delta\Psi_i(\zeta,\tilde{w}_1,\tilde{p},\sigma)\right|<\tau_i.$$

The nominal system corresponding to system (14) is given by

$$\begin{aligned} \dot{\zeta}_1 &= \zeta_2 \\ \dot{\zeta}_2 &= \zeta_3 \\ \dot{\zeta}_3 &= \phi_1(\zeta,\tilde{w}_1) + \gamma_1(\zeta)W \\ \dot{\zeta}_4 &= \zeta_5 \\ \dot{\zeta}_5 &= \zeta_6 \\ \dot{\zeta}_6 &= \phi_2(\zeta,\tilde{w}_1) + \gamma_2(\zeta)W \end{aligned} \quad (17)$$

For the nominal system, the dynamic feedback that linearize the system (17) is as follows

$$W = \frac{1}{\gamma(\zeta)}\left[v - \phi(\zeta,\tilde{w}_1)\right] = \begin{cases} \dfrac{d^2w_1}{d\sigma^2} = v_1\cos\theta + v_2\sin\theta + w_1^3\left(\dfrac{\tan\varphi}{L}\right)^2 \\ w_2 = \left(\dfrac{L\tan\varphi}{w_1}\right)^2\left(v_2\cos\theta - v_1\sin\theta - 3w_1\dfrac{dw_1}{d\sigma}\dfrac{\tan\varphi}{L}\right) \end{cases} \quad (18)$$

With $v(\zeta,t)\in IR^2$ is the new control.

Now the original control problem (Problem 1) is transformed from a system (4) under assumption 1 of the system (14) under assumption 2. The new problem (Problem 2) is that of steering the state vector ζ of the system (14) to the small neighborhood of zero asymptotically in spite of matched and unmatched perturbation; that is, a robust state regulation problem is now considered. Clearly, the solution to Problem 2 implies the solution to Problem 1.

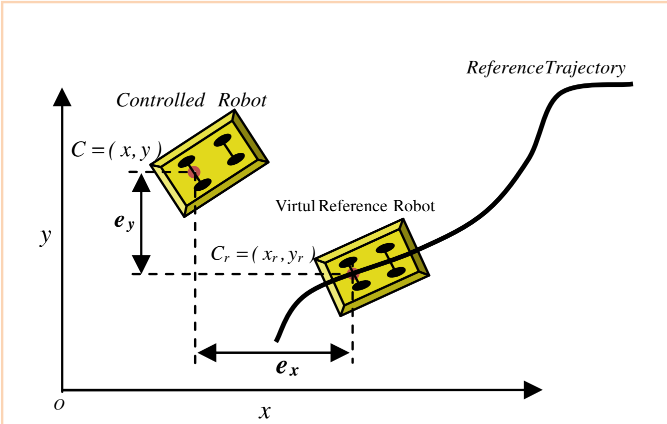

Fig. 3. Tracking the reference trajectory of a car-like mobile robot.

3. Control design

We will construct a robust output feedback controller, which makes the system (14) asymptotically stable. More precisely, for a given known stabilizing control for the nominal system (17); we want to redesign another robust stabilizing feedback control of the perturbed system (14). We can realize that we want to robustify an existing feedback control of the nominal system.

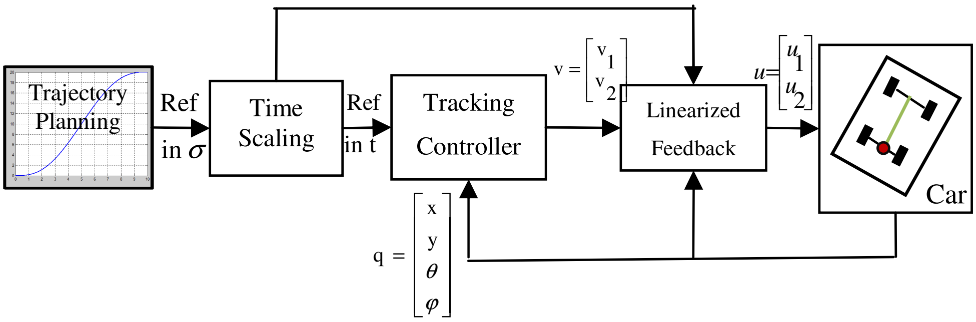

Fig. 4. Closed-loop control diagram.

In analogy with Khan et al.[15], the proposed control law of dynamic nature can be expressed as

$$v(\zeta,t) = v_c(\zeta,t) + v_{disc}(\zeta,t) \tag{19}$$

The first part $v_c \in IR^2$ is continuous control that stabilizes the nominal system (17) at the equilibrium point and designed at the sliding motion. The second part $v_{disc} \in IR^2$ is discontinuous in nature and considered in this paper as an integral SMC. Its role is to reject uncertainties from the initial condition. In the next subsections, the design of $v_c(\zeta,t)$ and $v_{disc}(\zeta,t)$ will be discussed.

3.1. Design of $v_c(\zeta,t)$

Applying the feedback linearization to the nominal system (17) yield the following multivariable Brunovsky form

$$\dot{\zeta}=\begin{bmatrix}0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}\zeta_1\\\zeta_2\\\zeta_3\\\zeta_4\\\zeta_5\\\zeta_6\end{bmatrix}+\begin{bmatrix}0&0\\0&0\\1&0\\0&0\\0&0\\0&1\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix} \tag{20}$$

To stabilize (20) we design linear feedback via pole placement; that is,

$$v_c(\zeta,t)=-K\zeta=\begin{cases}v_{1c}=-k_1\,\zeta_3-k_2\,\zeta_2-k_3\,\zeta_1\\v_{2c}=-k4\,\zeta_6-k_5\,\zeta_5-k_6\,\zeta_4\end{cases} \tag{21}$$

Where $K=\begin{bmatrix}k_1&k_2&k_3&0&0&0\\0&0&0&k_4&k_5&k_6\end{bmatrix}^T$ is the control gain, which can then be chosen such that all the roots of the characteristic equations of the closed-loop $p^3+k_{1,4}\,p^2+k_{2,5}\,p+k_{3,6}=0$ lie in the left half-plane of the complex plane to ensure exponential stability. This controller makes the origin of the system (20) globally asymptotically stable [13], [14].

3.2. Design of $v_{disc}(\zeta,t)$

The enhancement of the robustness of the feedback control $v_c(\zeta,t)$ in (19) is done by adding an integral sliding mode controller to compensate the perturbations while eliminating the reaching phase. Hence, we select the following sliding manifold of integral type[16] [17].

$$s(\zeta,t)=s_0(\zeta,t)+\mathrm{Z}(\zeta,t) \tag{22}$$

where $s_0=[s_1\quad s_2]^T\in IR^2$ is a conventional sliding surface defined by

$$s_0(\zeta,t)=\begin{bmatrix}\zeta_3+\lambda_2\zeta_2+\lambda_1\zeta_1\\\zeta_6+\lambda_2\zeta_5+\lambda_1\zeta_4\end{bmatrix} \tag{23}$$

With λ_1,λ_2 are chosen such that $p^2+\lambda_2\,p+\lambda_1=0$ is a Hurwitz polynomial. $\mathrm{Z}\in IR^2$ is an unknown integral function of the state to be determined such that the reaching phase is eliminated.

For the nominal system (17), the invariance conditions of the integral sliding manifold with continuous control $W_c(\zeta,t)$ are given by

$$\begin{aligned}&s(\zeta,t)=0\quad\forall\mathrm{t}>0\\&\frac{ds(\zeta,t)}{d\sigma}=0\ \ \forall\mathrm{t}>0\Rightarrow\frac{d\mathrm{Z}}{d\sigma}=-\begin{bmatrix}\lambda_1\zeta_2+\lambda_2\zeta_3+\phi_1(\zeta,\tilde{w}_1)+\gamma_1(\zeta)W_c\\\lambda_1\zeta_5+\lambda_2\zeta_5+\phi_2(\zeta,\tilde{w}_1)+\gamma_2(\zeta)W_c\end{bmatrix}\end{aligned} \tag{24}$$

To satisfy this invariance condition from the initial time, we obtain from the above equations, the dynamics of the variable Z

$$\begin{aligned}&\frac{d\mathrm{Z}}{d\sigma}=-\begin{bmatrix}\lambda_1\zeta_2+\lambda_2\zeta_3+\phi_1(\zeta,\tilde{w}_1)+\gamma_1(\zeta)W_c\\\lambda_1\zeta_5+\lambda_2\zeta_5+\phi_2(\zeta,\tilde{w}_1)+\gamma_2(\zeta)W_c\end{bmatrix}\\&\mathrm{Z}(0)=-s_0(\zeta(t_0))\end{aligned} \tag{25}$$

Taking into account the reachability condition defined as follows [6],

$$\frac{ds}{d\sigma} = -M \,.\, sign(s) \tag{26}$$

The discontinuous control is,

$$v_{disc}(\zeta, t) = -M \,.\, sign(s) \tag{27}$$

Where $M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \in IR^{2\times 2}$ is a positive definite diagonal matrix. Thus, the final control law becomes

$$W(\zeta, t) = \frac{1}{\gamma(\zeta)}\left[-K\zeta - M \,.\, sign(s) - \phi(\zeta, \tilde{w}_1)\right] \tag{28}$$

Note that this control law can be implemented by integrating the derivative of the control $W(\zeta, t)$, so that the control input actually applied to the system is continuous. This can be a benefit for the car-like robot, for which a discontinuous control action could be disruptive.

4. Stability analysis

In this section, we analyze the proposed control law applied to the uncertain system (14). To prove that the sliding mode can be enforced infinite time, differentiating (22) along the dynamics of (14) and then substituting (28), one has for one component of the sliding surface

$$\frac{ds_1}{d\sigma} = \lambda_1\zeta_2 + \lambda_2\zeta_3 - K_1\zeta - M_1 sign(s_1) + \gamma_1(\zeta)D_m + \Delta\Psi_1 + \frac{dZ}{d\sigma} \tag{29}$$

Using (25) in (29), it yields

$$\frac{ds_1}{d\sigma} = -M_1 sign(s_1) + \gamma_1(\zeta)D_m + \Delta\Psi \tag{30}$$

Now, by considering as a Lyapunov candidate function $V_1 = \frac{1}{2}s_1^2$, the time derivative of this function becomes

$$\frac{dV_1}{d\sigma} = s_1 \frac{ds_1}{d\sigma} \tag{31}$$

By using (30) in (31), one has

$$\frac{dV_1}{d\sigma} \le -M_1 |s_1| + |\gamma_1(\zeta)D_m| + |\Delta\Psi_1| \tag{32}$$

In view of Assumption 2, the above expression can be written as

$$\begin{aligned} \frac{dV_1}{d\sigma} &\le |s_1|\left[-M_1 + \alpha_1 + \tau_1\right] \\ \frac{dV_1}{d\sigma} &\le -\eta_1 |s_1| < 0 \end{aligned} \tag{33}$$

It is clear the Lyapunov derivative is negative if the η_1 is a strictly positive constant. To this end, the gain M_1 is chosen to enforce the sliding motion, such as

$$M_1 > \alpha_1 + \tau_1 + \eta_1 \tag{34}$$

And the same for the gain M_2

$$M_2 > \alpha_2 + \tau_2 + \eta_2 \tag{35}$$

This indicates that the system will reach the integral sliding manifold infinite time and remain on it. On the other hand, from (24) we can see the zero initial value of $s(\zeta, t)$ at $t = 0$. Therefore the sliding mode exists $\forall t \geq 0$.

5. Simulation results

To assess the effectiveness of the proposed controller, computer simulations using MATLAB/SIMULINK are implemented. The objective of the controller is to steer the car from an initial condition given at $t = t_0 : q_0 = (0m, 0m, 0^\circ, 0^\circ)$ to a final point in time $t = t_f : q_f = (20m, 10m, 0^\circ, 0^\circ)$.

To show the robustness of our controller, the simulation was run with initialization error $(x_0, y_0, \theta_0, \varphi_0) = (2m, 2m, 10°, 0°)$ and with disturbances $P(q,t)$ such as

$$P(q,t) = \begin{bmatrix} 1.5\sin(5t)\sin\theta\cos\theta \\ 1.5\cos(5t)\cos\theta\sin\theta \\ 0.1\sin(20t) \\ 0.01\sin(10t) \end{bmatrix} \tag{36}$$

Leading to $\rho_1 = \rho_2 = 0.8, \rho_3 = 0.2, \rho_4 = 0.02$, $\alpha_1 = 0.7, \alpha_2 = 0.8$, $\tau_1 = 2, \tau_2 = 1.5$ and the nonlinear time-varying controller and the integral sliding mode controller are defined with a constant gain value as follows

Constants	k_1	k_2	k_3	λ_1	λ_2	M_1	M_2
$v_c(q,t)$	26.5	105.5	90	-	-	-	-
$v_{disc}(q,t)$				25	9	50	10

Table 1. The parameter values of the control law used in the simulations.

The following figure shows the trajectory tracked by blue solid line and the reference trajectory is shown in red dotted.

5.1. With $v_c(\zeta,t)$ only

Fig. 5 and 6 show the simulation results with initialization error and disturbances using the nonlinear time-varying controller only, we note that the robot doesn't track the reference trajectory and the position error of the car in Fig. 7 doesn't converge to zero; so this control has poor performance, it is not robust against disturbances. For that, we will add the proposed integral sliding mode control in the next simulation to ensure a high robustness for the trajectory tracking of the car.

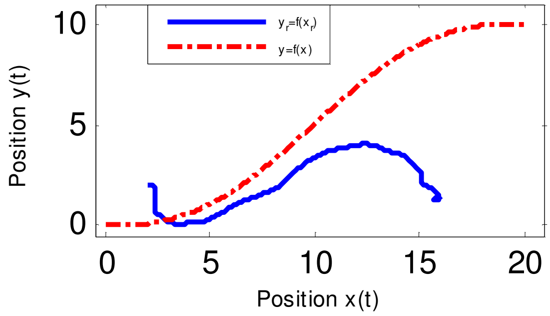

Fig. 5. The path of the car x-y with initialization error and disturbances, using the nonlinear time-varying controller only.

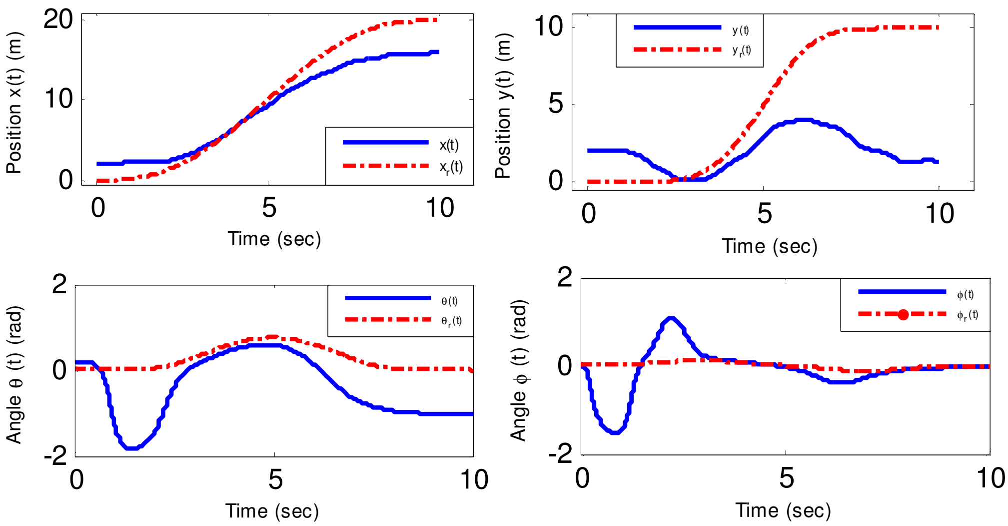

Fig. 6. Tracking the reference trajectories *x(t), y(t), θ(t)* and *φ(t)* respectively, with initialization error and disturbances, using the nonlinear time-varying controller only.

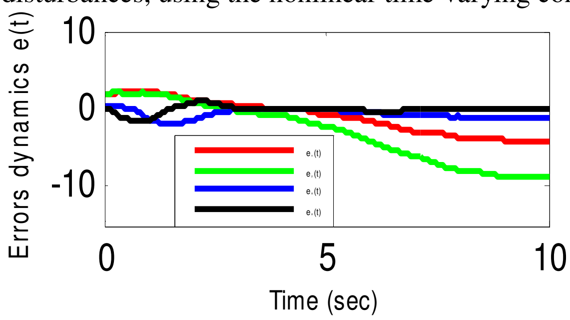

Fig. 7. The posture error of the car with initialization error and disturbances, using the nonlinear time-varying controller only.

5.1. With integral sliding mode control $v(\zeta,t)$

Fig. 8 and 9 show the simulation results of the proposed integral sliding mode controller with initialization error and disturbances, we note that the robot track the reference trajectory infinite time and the position error of the car in Fig. 10 converges quickly to zero; In addition, the reaching phase has been eliminated. The enlarged graphs have been provided in fig. 9 shows that the chattering phenomenon and the disturbances is minimized.

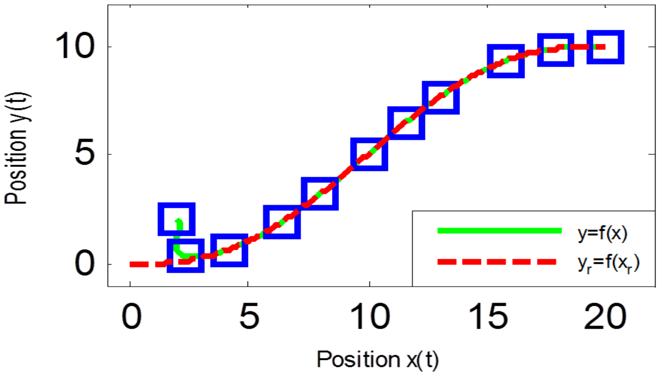

Fig. 8. The path of the car x-y with initialization error and disturbances, using the integral sliding mode controller (ISMC).

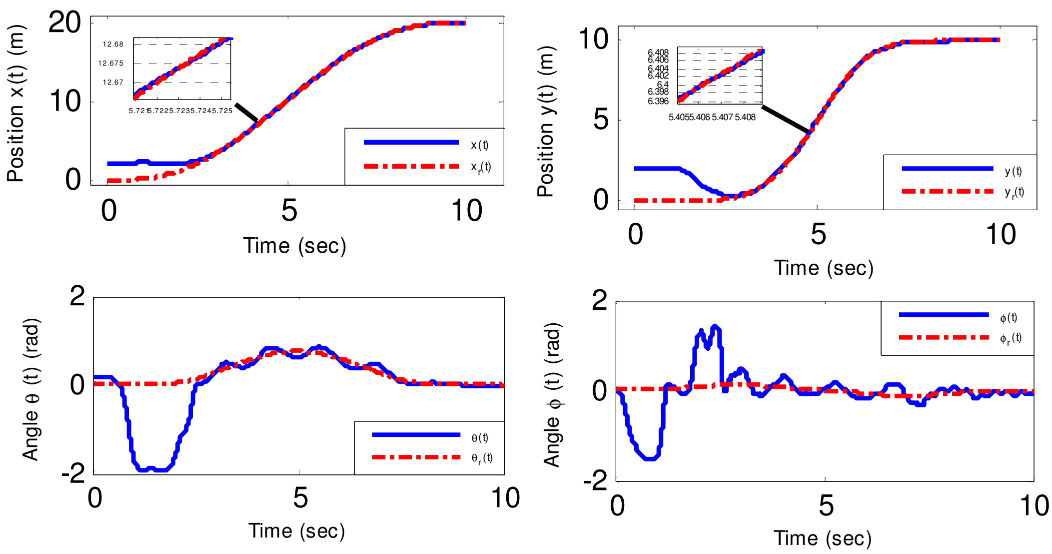

Fig.9. Tracking the reference trajectories *x (t), y (t)*, *θ(t)* and *φ(t)* respectively, with initialization error and disturbances, using ISMC.

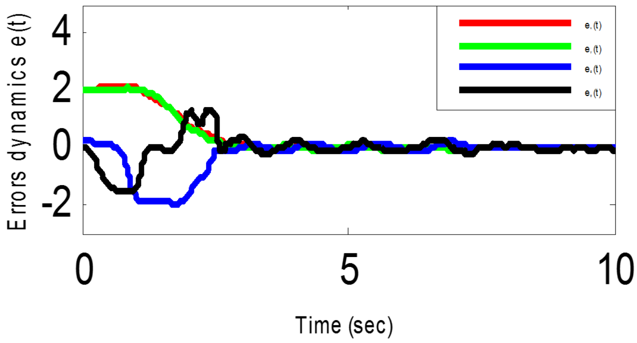

Fig. 10. The posture error of the car with initialization error and disturbances, using an Integral sliding mode controller.

Fig. 11 present the control inputs of the car, we can see that the chattering has been eliminated in the linear velocity due to the second derivative of dynamic linearizing feedback in (18). The time evolution of the components of the sliding manifold ***s*** is steered to zero as shown in Fig. 12, since the sliding mode is enforced from the initial instance.

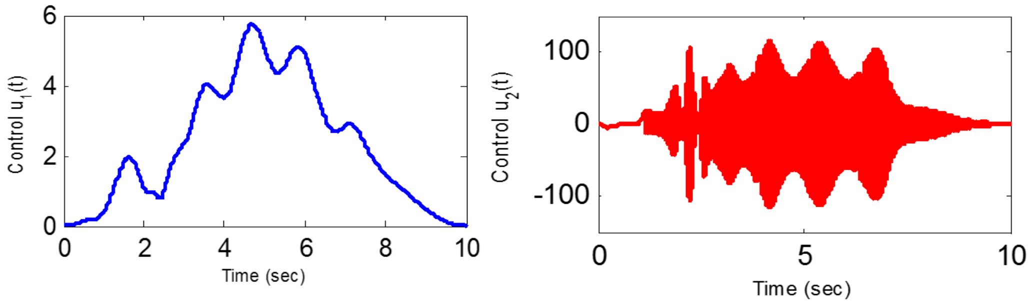

Fig. 11. The control inputs u_1 and u_2 of the car using Integral sliding mode controller in presence of disturbances.

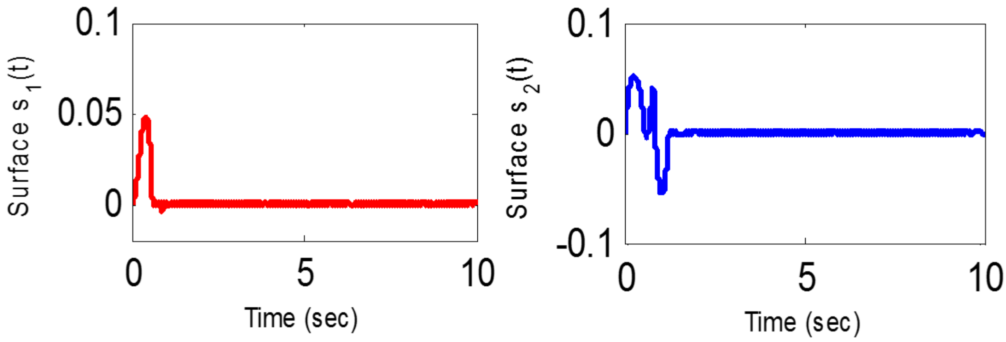

Fig. 12. Time evolution of the components of sliding manifold s_1 and s_2, for ISMC with initialization error and disturbances.

Under uncertainty the tracking error in the nonlinear time-varying controller only is increased and response is not desired at all, but in integral sliding mode controller we get the desired response. From this result, it is clear that the proposed integral sliding mode control has a faster responded, higher robustness, global stability, higher tracking precision.

6. Conclusions

In automatic control, the sliding mode control improves the system performance by allowing the successful completion of a task even in the presence of perturbations.

In this work, we presented the differential flatness-based integrated planning and control for a class of non-holonomic WMR to achieve full state controllability. We first showed that the kinematic model of the car-like robot under consideration is deferentially flat. The planning problem can then be achieved by polynomials of an appropriate order to satisfy the specified terminal conditions in the flat output space. The uncertain system output trajectories are asymptotically regulated to zero inspite, while a sliding mode is enforced infinite time along an integral manifold from the initial condition. The use of the integral sliding manifold allows one to subdivide the control design procedure into two steps. First a linear control component is designed by pole placement and then a discontinuous control component is added so as to cope with the presence of uncertainty. The design procedure is relying on a suitably transformed system which generally appears in a linear system and the control input appears with k time derivatives. As a consequence, the control acting on the original system is obtained as the output of a chain of integrators and is, accordingly, continuous. The performance of closed loop system has been verified using simulation results for the car-like robot. In a future work, we will try to verify the obtained simulation results on the real test bench.

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