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Robust Stability Analysis of PMSM with Parametric Uncertainty using Kharitonov Theorem

The permanent magnet synchronous motors (PMSM) are used as servo motor for precise motion control and are used as generator to generate electrical energy driven by wind energy. There is large variation in inertia due to varying load and parametric uncertainty in PMSM. The design objective of this paper is to analytically determine the relative robust stability of PMSM with parametric uncertainty using Kharitonov theorem and Routh stability criterion. The conventional integral controller (IC) and two robust internal model controllers (IMCs) are used for relative robust stability analysis of speed control of PMSM. The frequency domain performance specifications like gain margin (GM) and phase margin (PM) are taken for relative robust stability analysis, and the effect of controllers on time domain performance specifications such as settling time (ST), rise time (RT) and overshoot (OS) is also studied.

Key Words- Gain and Phase Margin, Kharitonov Theorem, IMC, PMSM, Robust Stability, Routh Stability Criterion

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Nomenclature

Symbol	Description (SI unit)	Symbol	Description (SI unit)
B_t, B_l, B_1	Viscous friction coefficient	T_l,	load torque (in Newton meters)
(Nm/rad/sec)		T_1,T_2,T_i,	time constant (sec)
i_d, i_q	d, q axis stator currents (in	u_d, u_q	d, q axis voltages (in volts)
amperes)		V_{dc}	dc voltage input to the inverter
J	moment of inertia, kg -m^2	(in volts)	
k_{in}, k_i	Inverter gain	V_{cm}	Maximum Control voltage
L_d, L_q	d, q axis inductances (in henrys)		
p	derivative operator	λ_{af}	mutual flux due to magnet (in
P	number of poles	Weber's)	
R	stator resistance (in ohms)	ω_r	rotor speed (in rad./ sec)
T_e	electric torque (in Newton	θ_r	rotor angular position (in degree)
meters)		*	Superscript indicating reference
		value	

1. Introduction

Permanent magnet synchronous motor (PMSM) gives high dynamic performance and widely used in industrial applications such as machine tools, rolling mills, paper mills, sugar mills and electric winding machines [1] - [4] and now days widely used in electric vehicles [5]. It has wide acceptance in motion control applications due to its high performance, compact structure, high air-gap flux density, high power density, high torque to inertia ratio, and high efficiency. Permanent magnet synchronous generators (PMSG) with prime mover as hydro turbine and another PMSG with a variable speed wind turbine are used for generation of electrical energy from renewable energy resources such as wind energy [6]. The control techniques such as model predictive control [1], [4], sliding mode, adaptive control [7], fuzzy logic control (FLC) neural network (NN) based control observer

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based control [3], and IMC [8] have been applied for the desired speed and position control of PMSM.

In PMSM there are various nonlinearities such as hysteresis, backlash, saturation etc. and uncertainties that are structured and unstructured, moreover parameters are changed such as the inertia of electric winding machine may increase to more than several times of the original inertia [8], under loaded or running condition; therefore, it is necessary to analyze the robust stability of PMSM. The parametric uncertainty may also occur in PMSM or any other system; while modeling, changes in operating conditions and lack of precise knowledge of the actual system parameters etc. [9]. Hence robustness determination of PMSM or other system is the major concern of control engineers and researchers for wide range of applications or for better performance. Robust stability analysis may also be used to determine the maximum allowable range of load inertia and other uncertain PMSM parameters.

In real physical systems the parameters of the system are not known exactly and cannot be fixed or may vary with time. Due to parameter variations the characteristic equation is subjected to minimum and maximum levels. Hence, in such cases, it is difficult to perform classical stability tests for many possibilities of parameter combinations in the characteristic equation. In 1978 the Russian mathematician Kharitonov came to the rescue, he published a paper in a mathematical journal without knowing the problem of robust control and stability [10] that was later applied for the same. As this theorem is widely used in various control systems like mechanical systems [11], electrical system such as PWM push-pull DC-DC converter [12], [13], hybrid electric vehicle [20] and may also be used in PMSM drive.

Generally Routh stability criterion is used for determination of absolute stability of system. In this paper relative robust stability of the PMSM analyzed through analytical analysis of gain margin (GM) and phase margin (PM) that makes the process simpler. The aspect of this paper is not only to analyze the relative robust stability of PMSM with parametric uncertainty but also synthesize the controller which improves the relative robust stability margins. The conventional integral controller (IC) and robust IMC in conjunction with the worst value of GM and PM of Kharitonov polynomials are used for the analysis of relative robust stability of uncertain or perturb PMSM. Internal model control (IMC) method was introduced by Garcia and Morari [14], it has good abilities of tracking disturbance rejection, and robustness [8]. It also provides an effective framework for the analysis of control system performance especially for the stability and robustness issue hence it is applied for speed control of PMSM in this paper [8], [14]-[17].

The effect on relative robust stability, using speed controllers such as IC and two types of IMC for PMSM is also calculated by analytical method in terms of GM and PM. The worst values of GM and PM are actual robust stability margin of the system [18]. The results obtained from proposed analytical techniques are also verified through MATLAB. Analytical analysis is important because in graphical technique, the response is stable up to some period of time but in due course of time, sometimes the system becomes unstable i.e. system doesn't asymptotically stable. The effect of controllers on time domain performance parameters such as settling time (ST), rise time (RT) and overshoot (OS) using graphical technique is also studied.

2. Modeling of PMSM

This section describes the mathematical modeling, general structure and simplified block diagram of PMSM for speed control and stability analysis. The general structure of the PMSM drive system is given in fig. 1. This consists of a PMSM, current control voltage-source inverter (CCVSI) and speed controller [3]. In this paper IC and IMC are used as speed controllers taken one at a time. For the purpose of control design ω_r as output variable and i_q^* is chosen as control input in this paper. The PMSM system can be written in the following explicit form [1], [2], [4], and the description of symbols are given in nomenclature.

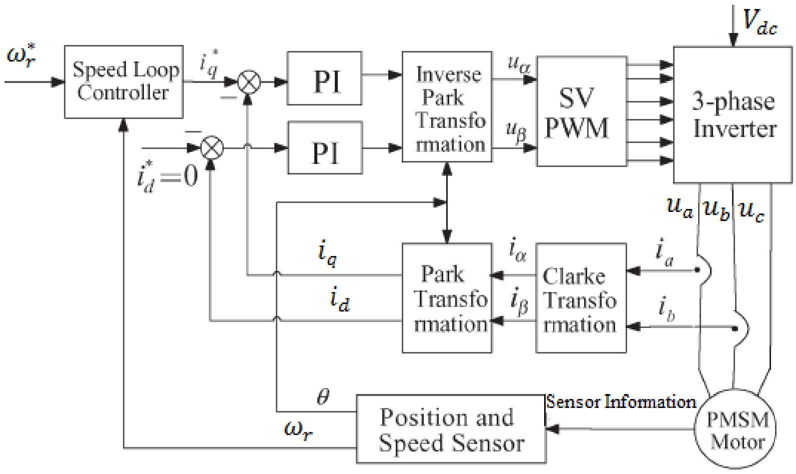

Fig.1. General Structure of the PMSM control system

$$u_d = Ri_d + p\lambda_d - \omega_r\lambda_q \quad (1)$$
$$u_q = Ri_q + p\lambda_q + \omega_r\lambda_d \quad (2)$$

where $\lambda_q = L_q i_q$ and $\lambda_d = L_d i_d + \lambda_{af}$

$$p\omega_r = (T_e - B_1\omega_r - T_l)/J \text{ and } p\theta_r = \omega_r \quad (3)$$

The electric torque $T_e = \frac{3}{2} \times \frac{P}{2}\left[\lambda_{af} i_q + (L_d - L_q) i_d i_q\right]$ (4)

For constant flux operation $i_d = 0$, hence modified equations are

$$u_q = (R + L_q p) i_q + \omega_r \lambda_{af} \quad (5)$$

and the electromechanical equation is

$$\frac{P}{2}(T_e - T_l) = Jp\omega_r + B_1\omega_r \quad (6)$$

Where electric torque

$$T_e = \frac{3}{2} * \frac{P}{2} * \lambda_{af} i_q \quad (7)$$

and if the load is assumed to be frictional, than

$$T_l = B_l \omega_r \quad (8)$$
$$(Jp + B_t)\omega_r = \left\{\frac{3}{2}\left(\frac{P}{2}\right)^2 \lambda_{af}\right\} i_q = k_t * i_q \quad (9)$$

where $B_t = \frac{P}{2} B_l + B_1$, and Motor torque constant $k_t = \frac{3}{2}\left(\frac{P}{2}\right)^2 \lambda_{af}$

if $\frac{1}{B_t} = k_m, \frac{J}{B_t} = T_m$ then $G_m(s) = \frac{k_m}{1 + sT_m}$ (10)

The approximate current loop transfer function is [2]: $G_i(s) = \frac{k_i}{1 + sT_i}$ (11)

where $k_i = \frac{T_m k_{in}}{T_2 k_b}$, $k_{in} = 0.65 \times \frac{V_{dc}}{V_{cm}}$, $k_b = k_t k_m \lambda_{af}$.

Fig. 2 is drawn using (1) to (11) that is simplified block diagram of speed control of PMSM. The numerical nominal values with range of uncertainty in PMSM parameters for robust stability analysis are given in table 1.

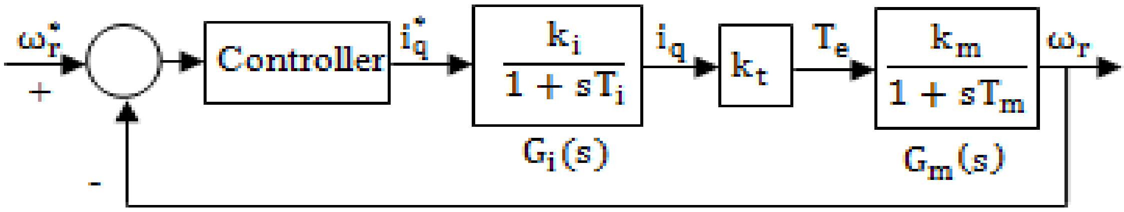

Fig.2 Simplified speed control loop of PMSM

Table 1: Values of PMSM parameter with uncertainty

Symbol	Values±Uncertainty (SI unit)	% Variation
R	$1.5 \pm 0.3\ \Omega$	20%
$L_d = L_q$	0.0056 ± 0.00112 H	20%
J	0.006 ± 0.001 kg-m^2	16.67%
B_t	0.01 ± 0.002N.m/rad/sec	20%
P	6	-
$\boldsymbol{\lambda}_{af}$	0.15 ± 0.05 wb-turn	33.3%
T_1	0.00058 ± 0.00008 sec	13.8%
T_2	0.3 ± 0.1 sec	33.3%
V_{dc}	285 ± 35 V	12.3%
V_{cm}	10 ± 5 V	50%

The simplified transfer function *G(s)* of PMSM is written as:

$$G(s) = \frac{\omega_r(s)}{i_q^*} = \frac{\frac{k_i k_m k_t}{T_i T_m}}{s^2 + \frac{(T_i + T_m)}{T_i T_m} s + \frac{1}{T_i T_m}} \tag{12}$$

Fig.3 shows the variation in singular values of nominal values of parameters as shown in blue and range of uncertain PMSM parameters as shown in green color as given in table 1, with low to high frequency. From fig. 3, uncertain PMSM system has 20% deviation in singular values from its nominal values.

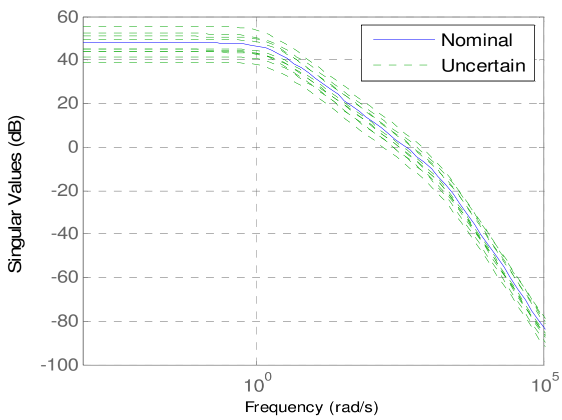

Fig.3 Singular values of nominal and uncertain PMSM system

After putting the nominal values from table 1 in (12) the transfer function of PMSM is as

$$G(s) = \frac{7.098 \times 10^5}{s^2 + 1726\, s + 2874} \tag{13}$$

The nominal values of PMSM parameters are used to design the controllers and range of uncertainty limits as given in table 1 are used for robust stability analysis.

3. Kharitonov's Theorem

The polynomials

$P(s) = \sum_{i=0}^{n} a_i\, s^{n-i} = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n$, (14)

where $\alpha_i \leq a_i \leq \beta_i$, $0 \leq i \leq n$.

According to Kharitonov's theorem, the interval polynomial P(s) is stable if and only if the following four polynomials are stable.

$p_1(s) = \alpha_0 s^n + \alpha_1 s^{n-1} + \beta_2 s^{n-2} + \beta_3 s^{n-3} + \alpha_4 s^{n-4} \ldots$

$p_2(s) = \alpha_0 s^n + \beta_1 s^{n-1} + \beta_2 s^{n-2} + \alpha_3 s^{n-3} + \alpha_4 s^{n-4} \ldots$

$p_3(s) = \beta_0 s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \beta_3 s^{n-3} + \beta_4 s^{n-4} \ldots$

$p_4(s) = \beta_0 s^n + \beta_1 s^{n-1} + \alpha_2 s^{n-2} + \alpha_3 s^{n-3} + \beta_4 s^{n-4} \ldots$ (15)

Corollary of Kharitonov's theorem [18]: For low order uncertain systems, the robust stability can be checked by testing only one Kharitonov polynomial out of four in (15) for $n = 3$, and two Kharitonov polynomials for $n = 4$. This is also explained below: For $n = 3$; check only $P_3(s)$

$p_3(s) = \beta_0 s^3 + \alpha_1 s^2 + \alpha_2 s + \beta_3$ (16)

For $n = 4$; check $P_3(s)$ and $P_4(s)$

$p_3(s) = \beta_0 s^4 + \alpha_1 s^3 + \alpha_2 s^2 + \beta_3 s + \beta_4$ & $p_4(s) = \beta_0 s^4 + \beta_1 s^3 + \alpha_2 s^2 + \alpha_3 s + \beta_4$ (17)

Condition for stability in frequency domain (See appendix A): For uncertain systems, the robust stability can be checked by using ω_{cg} and ω_{cp}. For a stable system ω_{cp} must be greater than ω_{cg}.

4. Robust Gain Margin and Phase Margin

The closed loop characteristic equation (CLCE) with controller *C(s)* and unity feedback *H(s)*=1 is written as

$$1 + G(s)C(s) = 0 \quad (18)$$

In order to determine the gain margin, a virtual gain k_v is introduced in series with *G(s)*.

4.1. Calculation of GM and PM for *n=3*

The CLCE in the form of polynomial can be written as;

$$P(s) = a_0 s^3 + a_1 s^2 + a_2 s + a_3 k_v \quad (19)$$

The GM of system is calculated simply from the s^1 row of the Routh array, the ω_{cp} is determined using auxiliary equation which is derived from the s^2 row. Using (19) and (15) all four Kharitonov polynomial are formulate as below;

$$\begin{aligned} p_1(s) &= \alpha_0 s^3 + \alpha_1 s^2 + \beta_2 s + \beta_3 k_{v1} \\ p_2(s) &= \alpha_0 s^3 + \beta_1 s^2 + \beta_2 s + \alpha_3 k_{v2} \\ p_3(s) &= \beta_0 s^3 + \alpha_1 s^2 + \alpha_2 s + \beta_3 k_{v3} \\ p_4(s) &= \beta_0 s^3 + \beta_1 s^2 + \alpha_2 s + \alpha_3 k_{v4} \end{aligned} \quad (20)$$

Design a Routh table for $p_1(s)$ of (20) and taking higher order coefficient=1 i.e. $\alpha_0 = 1$

s^3	1	β_2
s^2	α_1	$\beta_3 k_{v1}$
s^1	$(\alpha_1\beta_2 - \beta_3 k_{v1})/\ \alpha_1$	0
s^0	$\beta_3 k_{v1}$	0

From s^1 row

$$(\alpha_1\beta_2 - \beta_3 k_{v1})/\alpha_1 = 0 \tag{21}$$

$$k_{v1} = \frac{\alpha_1\beta_2}{\beta_3} = \frac{\text{coefficient of } s^2 \times \text{coefficient of } s^1}{\text{coefficient of } s^0} \tag{22}$$

The corresponding gain margin (GM_1) in dB=$20\ log\ k_{v1}$. Similarly other three remaining polynomials give

$$k_{v2} = \frac{\beta_1\beta_2}{\alpha_3},\ k_{v3} = \frac{\alpha_1\alpha_2}{\beta_3} \text{ and } k_{v4} = \frac{\beta_1\alpha_2}{\alpha_3} \tag{23}$$

The auxiliary equation, from s^2 row: $\alpha_1 s^2 + \beta_3 k_{v1} = 0$ (24)

In order to determine ω_{cp}, put $s = j\omega$ and $k_{v1} = \frac{\alpha_1\beta_2}{\beta_3}$ in (24)

get: $\omega = \omega_{cp1} = \sqrt{\beta_2} = \sqrt{\text{coefficient of } s^1}$ (25)

Similarly three remaining polynomial gives:

$$\omega_{cp2} = \sqrt{\beta_2},\ \omega_{cp3} = \sqrt{\alpha_2}, \omega_{cp4} = \sqrt{\alpha_2} \tag{26}$$

The ω_{cg} is calculated using empirical formula [18], [19]: $\omega_{cg} = \omega_{cp}\left(\frac{1}{k_v}\right)^{0.5}$ (27)

Sometimes it is observed that the empirical formula (27) gives large deviation in PM calculation using ω_{cg} for GM $> 50dB$. Hence the need was felt to develop the same for better results and (28) is proposed to minimize the deviation further. The new proposed empirical formula is (28), gives better result as compare to (27)

$$\omega_{cg} = \begin{Bmatrix} \omega_{cp}\left(\frac{1}{k_v}\right)^{0.5} \text{ if GM} < 50dB \\ \omega_{cp}\left(\frac{1}{2k_v}\right)^{0.5} \text{ if GM} > 50dB \end{Bmatrix} \tag{28}$$

A comparative analysis is performed to validate the outcome. For $\omega_{cg} = \omega_{cp}\left(\frac{1}{k_v}\right)^{0.5}$ the ω_{cg} of all four Kharitonov polynomials are as fallows;

$$\omega_{cg1} = \sqrt{\frac{\beta_3}{\alpha_1}}, , \omega_{cg2} = \sqrt{\frac{\alpha_3}{\beta_1}}, \omega_{cg3} = \sqrt{\frac{\beta_3}{\alpha_1}}, \omega_{cg4} = \sqrt{\frac{\alpha_3}{\beta_1}} \tag{29}$$

The PMs of all four polynomials are given as fallows;

$$PM_1 = 180^0 - 90^0 - tan^{-1}\left(\frac{\sqrt{\alpha_1^3.\beta_3}}{\alpha_1\beta_2 - \beta_3}\right), PM_2 = 180^0 - 90^0 - tan^{-1}\left(\frac{\sqrt{\beta_1^3.\alpha_3}}{\beta_1\beta_2 - \alpha_3}\right),$$

$$PM_3 = 180^0 - 90^0 - tan^{-1}\left(\frac{\sqrt{\alpha_1^3.\beta_3}}{\alpha_1\alpha_2 - \beta_3}\right) \text{ and } PM_4 = 180^0 - 90^0 - tan^{-1}\left(\frac{\sqrt{\beta_1^3.\alpha_3}}{\beta_1\alpha_2 - \alpha_3}\right) \tag{30}$$

Now the proposed empirical formula $\omega_{cg} = \omega_{cp}\left(\frac{1}{2k_v}\right)^{0.5}$ is considered and ω_{cg} for first Kharitonov polynomial is $\omega_{cg1} = \sqrt{\frac{\beta_3}{2\alpha_1}}$

and corresponding PM is: $PM_1 = 180^0 - 90^0 - tan^{-1}\left(\frac{\sqrt{2\alpha_1^3.\beta_3}}{2\alpha_1\beta_2 - \beta_3}\right)$ (31)

Similarly the PM of all remaining second, third and fourth polynomials are determined.

4.2. Calculation of GM and PM for *n* = 4

Considering the Kharatinov polynomial

$$\begin{aligned}
p_1(s) &= \alpha_0 s^4 + \alpha_1 s^3 + \beta_2 s^2 + \beta_3 s + \alpha_4 k_{v1} \\
p_2(s) &= \alpha_0 s^4 + \beta_1 s^3 + \beta_2 s^2 + \alpha_3 s + \alpha_4 k_{v2} \\
p_3(s) &= \beta_0 s^4 + \alpha_1 s^3 + \alpha_2 s^2 + \beta_3 s + \beta_4 k_{v3} \\
p_4(s) &= \beta_0 s^4 + \beta_1 s^3 + \alpha_2 s^2 + \alpha_3 s + \beta_4 k_{v4}
\end{aligned} \tag{32}$$

Designing the Routhtable for $p_1(s)$ of (32) and taking higher order coefficient=1 i.e.$\alpha_0 = 1$

$$\begin{array}{llll}
s^4 & 1 & \beta_2 & \alpha_4 k_{v1} \\
s^3 & \alpha_1 & \beta_3 & 0 \\
s^2 & (\alpha_1\beta_2 - \beta_3)/\,\alpha_1 & \alpha_4 k_{v1} & 0 \\
s^1 & \dfrac{[(\alpha_1\beta_2 - \beta_3)/\,\alpha_1]\,\beta_3 - \alpha_1\alpha_4 k_{v1}}{(\alpha_1\beta_2 - \beta_3)/\,\alpha_1} & 0 & 0 \\
s^0 & \alpha_4 k_{v1} & 0 & 0
\end{array}$$

From s^1 row of above Routh array: $\frac{[(\alpha_1\beta_2-\beta_3)/\,\alpha_1]\,\beta_3-\alpha_1\alpha_4 k_{v1}}{(\alpha_1\beta_2-\beta_3)/\,\alpha_1} = 0$ (33)

After simplification of (33), we get: $k_{v1} = \frac{(\alpha_1\beta_2-\beta_3)\,\beta_3}{\alpha_1^2\alpha_4}$ (34)

Similarly other three remaining polynomial gives:

$k_{v2} = \frac{(\beta_1\beta_2-\alpha_3)\,\alpha_3}{\beta_1^2\alpha_4}$,$k_{v3} = \frac{(\alpha_1\alpha_2-\beta_3)\,\beta_3}{\alpha_1^2\beta_4}$ and $k_{v4} = \frac{(\beta_1\alpha_2-\alpha_3)\,\alpha_3}{\beta_1^2\beta_4}$ (35)

The auxiliary equation, from s^2 row of Routh array: $\frac{\alpha_1\beta_2-\beta_3}{\alpha_1} s^2 + \alpha_4 k_{v1} = 0$ (36)

Put $s = j\omega$ in (36), we get ω_{cp} as: $\omega = \omega_{cp1} = \sqrt{\frac{\beta_3}{\alpha_1}} = \sqrt{\frac{\text{coefficient of } s^1}{\text{coefficient of } s^3}}$ (37)

Similarly rest of the other three polynomials give:

$\omega_{cp2} = \sqrt{\frac{\alpha_3}{\beta_1}}$, $\omega_{cp3} = \sqrt{\frac{\beta_3}{\alpha_1}}$ and $\omega_{cp4} = \sqrt{\frac{\alpha_3}{\beta_1}}$ (38)

For the calculation of ω_{cg} empirical formula (27) is used, which gives

$\omega_{cg1} = \sqrt{\frac{\alpha_1\alpha_4}{(\alpha_1\beta_2-\beta_3)}}$,$\omega_{cg2} = \sqrt{\frac{\beta_1\alpha_4}{(\beta_1\beta_2-\alpha_3)}}$,

$\omega_{cg3} = \sqrt{\frac{\alpha_1\beta_4}{(\alpha_1\alpha_2-\beta_3)}}$, and $\omega_{cg4} = \sqrt{\frac{\beta_1\beta_4}{(\beta_1\alpha_2-\alpha_3)}}$ (39)

The PMs are given as;

$$PM_1 = 180^0 - 90^0 - tan^{-1}\left(\frac{\beta_2 - \frac{\alpha_1\alpha_4}{(\alpha_1\beta_2-\beta_3)}}{\beta_3 * \sqrt{\frac{(\alpha_1\beta_2-\beta_3)}{\alpha_1\alpha_4}} - \alpha_1 * \sqrt{\frac{\alpha_1\alpha_4}{(\alpha_1\beta_2-\beta_3)}}}\right) \tag{40}$$

Since (40) is complex, for simplicity it may expressed in term of ω_{cg1} and polynomial coefficients that is given as

$$PM_1 = 180^0 - 90^0 - tan^{-1}\left(\frac{\beta_2 - \omega_{cg1}^2}{\beta_3 * \frac{1}{\omega_{cg1}} - \alpha_1 * \omega_{cg1}}\right) \tag{41}$$

$$PM_2 = 180^0 - 90^0 - tan^{-1}\left(\frac{\beta_2 - \omega_{cg2}^2}{\alpha_3 * \frac{1}{\omega_{cg2}} - \beta_1 * \omega_{cg2}}\right) \tag{42}$$

Similarly PM_3 and PM_4 in terms of polynomial coefficients and ω_{cg} may be calculated. Considering proposed empirical formula (28), for the calculation of ω_{cg}:

$\omega_{cg1} = \sqrt{\frac{\alpha_1\alpha_4}{2(\alpha_1\beta_2-\beta_3)}}$, $\omega_{cg2} = \sqrt{\frac{\beta_1\alpha_4}{2(\beta_1\beta_2-\alpha_3)}}$,

$$\omega_{cg3} = \sqrt{\frac{\alpha_1\beta_4}{2(\alpha_1\alpha_2-\beta_3)}} \text{ and } \omega_{cg4} = \sqrt{\frac{\beta_1\beta_4}{2(\beta_1\alpha_2-\alpha_3)}} \quad (43)$$

The PM may be calculated as, $PM_1 =$

$$180^0 - 90^0 - tan^{-1}\left(\frac{\beta_2 - \frac{\alpha_1\alpha_4}{2(\alpha_1\beta_2-\beta_3)}}{\beta_3 * \sqrt{\frac{2(\alpha_1\beta_2-\beta_3)}{\alpha_1\alpha_4}} - \alpha_1 * \sqrt{\frac{\alpha_1\alpha_4}{2(\alpha_1\beta_2-\beta_3)}}}\right) \quad (44)$$

PM may be expressed in the terms of ω_{cg} and polynomial coefficients which is given by (45).

$$PM_1 = 180^0 - 90^0 - tan^{-1}\left(\frac{\beta_2 - \omega_{cg1}^2}{\beta_3 * \frac{1}{\omega_{cg1}} - \alpha_1 * \omega_{cg1}}\right) \quad (45)$$

Similarly PM_2, PM_3 and PM_4 in terms of polynomial coefficients and ω_{cg} are calculated.

5. Design and Implementation of Controllers

In this section IC and two IMCs for speed control of PMSM are designed and implemented, these controllers are then applied to CLCE (18) for robust stability analysis.

5.1 Integral controller

The transfer function of Integral controller is defined as: $C(s) = \frac{K_I}{s}$ (46)

The CLCE of PMSM with integral controller (IC) is

$$s^3 + \frac{(T_i+T_m)}{T_iT_m}s^2 + \frac{1}{T_iT_m}s + \frac{k_ik_mk_t}{T_iT_m}.K_I = 0 \quad (47)$$

The selection procedure of IC gain K_I is described in appendix B. For the formulation of all four Kharitonov polynomials, the lower and upper level values α_1 and β_1 are determined from (15), table 1 and (47) as below:

$\alpha_0 = \beta_0 = 1$ and $K_I = 0.01$

$\alpha_1 = min_q\left\{\frac{(T_i+T_m)}{T_iT_m}\right\} = 2002$, $\beta_1 = max_q\left\{\frac{(T_i+T_m)}{T_iT_m}\right\} = 1517$, $\alpha_2 = min_q\left\{\frac{1}{T_iT_m}\right\} = 3200$,

$\beta_2 = max_q\left\{\frac{1}{T_iT_m}\right\} = 2597$

$$\alpha_3 = min_q\left\{\frac{k_ik_mk_t}{T_iT_m} * K_I\right\} = 35100 \text{ and } \beta_3 = max_q\left\{\frac{k_ik_mk_t}{T_iT_m} * K_I\right\} = 2462 \quad (48)$$

In (48) the vector q represents the vector of the uncertain PMSM parameters. As seen the minimum values are greater than the maximum value that is the value $\alpha_1 > \beta_1$, therefore, interchanging the limits in accordance with Kharitonov theorem, and representing (47) as a polynomial in various lower and upper limits values:

$$P(s) = s^3 + [1517, 2002\,]s^2 + [2597, 3200\,]s + [2462, 35100] \quad (49)$$

Formulate all four Kharatinov polynomial using (49)

$$\begin{aligned} p_1(s) &= s^3 + 1517s^2 + 3200s + 35100 \\ p_2(s) &= s^3 + 2002\,s^2 + 3200\,s + 2462 \\ p_3(s) &= s^3 + 1517s^2 + 2597s + 35100 \\ p_4(s) &= s^3 + 2002\,s^2 + 2597\,s + 2462 \end{aligned} \quad (50)$$

After applying Routh criterion on (16) that is also corollary of Kharitonov theorem, the robust stability condition is obtained as: $\alpha_1 \times \alpha_2 > \beta_3 \times \beta_0$ (51)

where $\beta_0 = 1$. The corresponding values from (50) are put in (51) to satisfy the condition of robust stability: $1517 \times 2597 > 3510$. Hence for PMSM with IC speed controller the robust stability is proved for parametric uncertainty. The GMs and PMs are calculated using (22) to (31) and (50). The GMs of all four polynomials are 42.8dB, 68.3dB, 41dB, 66.5dB and PMs are 23.5, 63.86, 19.43 and 58.84. The worst value of GM is minimum of {42.8, 68.3, 41, 66.5} that is 41dB and the worst value of PM is minimum of {23.5, 63.86, 19.43, 58.84} that is 19.43 degree. The values from analytical method and bode plots are given in table 2. The corresponding values for bode plots are taken from MATLAB simulation results as in fig. 11.

TABLE 2: GM and PM values with IC

Polynomials	Analytical method				Bode Plots			
	GM (dB)	ω_{cp} (rad./s)	ω_{cg} (rad./s)	PM (deg.)	GM (dB)	ω_{cp} (rad./s)	ω_{cg} (rad./s)	PM (deg.)
$P_1(s)$	42.8	56.57	4.81	23.5	42.8	56.6	4.59	24.6
$P_2(s)$	68.3	56.57	0.784	63.86	68.3	56.6	0.704	66.2
$P_3(s)$	41	50.96	4.81	19.43	41	51	4.66	20
$P_4(s)$	66.5	50.96	0.784	58.84	66.5	51	0.806	58.2

It is seen from table 2 that the analytical technique for calculation of GM gives similar values within negligible error. But for PM the values are different in both the cases.PM for GM > 50dB is calculated with reduced error, considering p_2 of (50) for the verification of proposed empirical formula given in (28), as this has largest GM as shown in table 2. If old empirical formula is used as in (27), then p_2 of (50) gives the value of PM as 55.24 degree, which has 16.56% error as obtained from bode plots in table 2. Using proposed empirical formula as in (28) for GM>50dB, p_2 of (50) gives PM as 63.86 degree that is shown in table 2;error is reduced to 3.53% from 16.56% as obtained from bode plots in PM by using proposed empirical formula as given in (28) for those which have GM >50dB.

5.2 Internal Model Control

IMC has various properties or features such as noise and disturbance rejection capability, perfect traking of reference values and insensitive to parameter variations i.e. it makesrobust system [15-17]. The focus in this paper is on property of robustness. Another important merit is that there is only a single parameter μ in controller that is required to be tuned as given in (54) as compared to PI/PID controllers where two/three parameters are required to be tuned. Using this advantage of IMC, in this paper two IMCs, one for $\mu = 0.1$ and the other for $\mu = 10$ are designed to see the effectiveness of μ on PMSM speed control performance. IMC may have various configrations, one of them is presented in fig. 4 [16]. It consists of a controller *C(s),* system *G(s)* and system model *$G_0(s)$.* The selection procedure of suitable values of μ for PMSM is discussed in appendix-C.

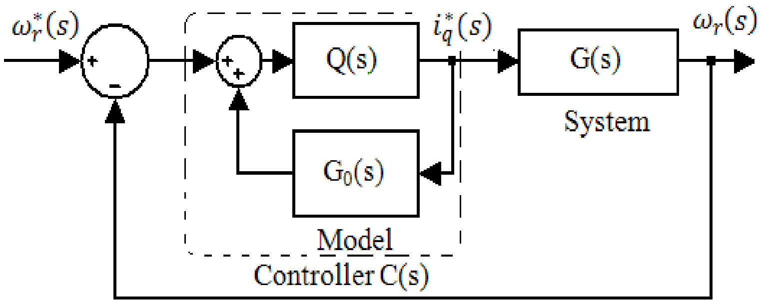

Fig.4 Block diagram of IMC

From fig. 4

$$\omega_r(s) = \frac{G(s)C(s)\omega_r^*(s)}{1+C(s)G(s)} \tag{52}$$

$$C(s) = \frac{Q(s)}{1-G_0(s)*Q(s)} \tag{53}$$

The condition for perfect reference point tracking and effect of parameter variations is obtained, if $G_0(s) = G(s)$ and $Q(s) = G(s)^{-1}$. If $G(s)$ is strictly proper, then its inverse model $G(s)^{-1}$ becomes improper i.e. $Q(s)$ [17]. Hence in order to make a proper transfer function one low pass filter is incorporated as a part of $Q(s)$. Hence $Q(s)$ is defined as

$$Q(s) = \frac{1}{(\mu s+1)^n} G(s)^{-1} \tag{54}$$

Here n is suitably taken to make $Q(s)$ proper. The low pass filter parameter μ represents trade off between traking performance and robustness of system. Put $n=2$, $\mu = 0.1$ in (54), IMC-1 is in the form of transfer function is as:

$$C(s) = \frac{1.4*10^{-4}s^2+0.243*s+0.4}{s^2+20s} \tag{55}$$

The CLCE of PMSM with IMC-1 is

$$s^4 + \left(\frac{T_i+T_m}{T_iT_m} + 20\right)s^3 + \left(20 * \frac{T_i+T_m}{T_iT_m} + \frac{1}{T_iT_m} + 1.4 * 10^{-4} \times \frac{k_ik_mk_t}{T_iT_m}\right)s^2 + \left(20 * \frac{1}{T_iT_m} + 0.243 * \frac{k_ik_mk_t}{T_iT_m}\right)s + 0.4 * \frac{k_ik_mk_t}{T_iT_m} = 0 \tag{56}$$

For the formulation of all four Kharitonov polynomials, the lower and upper level values α_1 and β_1 etc. are determined from (15), table 1 and (56) as below:

$$\alpha_0 = \beta_0 = 1, \alpha_1 = min_q\left\{\frac{T_i+T_m}{T_iT_m} + 20\right\} = 2022, \beta_1 = max_q\left\{\frac{T_i+T_m}{T_iT_m} + 20\right\} = 1537$$

$$\alpha_2 = \min_q\left\{20 * \frac{T_i+T_m}{T_iT_m} + \frac{1}{T_iT_m} + 1.4 * 10^{-4} \times \frac{k_ik_mk_t}{T_iT_m}\right\} = 43731.4$$

$$\beta_2 = \max_q\left\{20 * \frac{T_i+T_m}{T_iT_m} + \frac{1}{T_iT_m} + 1.4 * 10^{-4} \times \frac{k_ik_mk_t}{T_iT_m}\right\} = 32971.468$$

$$\alpha_3 = \min_q\left\{20 * \frac{1}{T_iT_m} + 0.243 * \frac{k_ik_mk_t}{T_iT_m}\right\} = 91693,$$

$$\beta_3 = \max_q\left\{20 * \frac{1}{T_iT_m} + 0.243 * \frac{k_ik_mk_t}{T_iT_m}\right\} = 111766.6$$

$$\alpha_4 = min_q\left\{0.4 * \frac{k_ik_mk_t}{T_iT_m}\right\} = 1404000 \text{ and } \beta_4 = max_q\left\{0.4 * \frac{k_ik_mk_t}{T_iT_m}\right\} = 98460 \tag{57}$$

In (57) the vector q represents the vector of the uncertain PMSM parameters. As seen the minimum values are greater than the maximum values that is the value $\alpha_1 > \beta_1$, therefore, interchanging the limits in accordance with Kharitonov theorem, and (56) is now representing as polynomial in various lower and upper limits values.

$P(s) = s^4 + [1537, 2022]s^3 + [32971.5, 43731.4]s^2 + [\,111766.6, 916930]s + [98460, 1404000]$ (58)

All four Kharitonov polynomial are formulated using (58) as follows:

$p_1(s) = s^4 + 1537s^3 + 43731.4s^2 + 916930s + 98460$
$p_2(s) = s^4 + 2022s^3 + 43731.4s^2 + 111766.6s + 98460$
$p_3(s) = s^4 + 1537s^3 + 32971.5s^2 + 916930s + 1404000$
$p_4(s) = s^4 + 2022s^3 + 32971.5s^2 + 111766.6s + 1404000$ (59)

According to Corollary as given in (17), only two polynomials are required to be checked for stability of (59). After applying Routh stability criterion on p_3 and p_4 of (17), it gives robust stability condition as under

$\alpha_1\alpha_2\beta_3 > \alpha_1{}^2\beta_4 + \beta_0\beta_3{}^2$ and $\beta_1\alpha_2\alpha_3 > \beta_1{}^2\beta_4 + \beta_0\alpha_3{}^2$ (60)

where $\beta_0 = 1$

The corresponding values from (59) are put in (60) and that completely satisfies the condition of robust stability. Hence the robust stability of PMSM with IMC-1 is proved. For relative robust stability it is necessary to determine the GM and PM that are calculated using (34) to (45) and (59). The GMs for all four polynomials of (59) are 48.3 dB, 27.8 dB, 22.8 dB, 2.25 dB and PMs are 85.87, 58.52, 75.7, 6.77. The worst value of GM is minimum of {48.3, 27.8, 22.8, 2.25} that is 2.25dB, and the worst value of PM is minimum of {85.87, 58.52, 75.7, 6.77} that is 6.77 deg. The values obtained using analytical method and bode plots are given in table 3. The corresponding values are obtained from bode plots of fig. 12, that is simulated result using MATLAB.

TABLE 3: GM and PM values with IMC-1

Polynomials	Analytical Method				Bode Plots			
	GM (dB)	ω_{cp} (rad./s)	ω_{cg} (rad./s)	PM (deg)	GM (dB	ω_{cp} (rad./s)	ω_{cg} (rad./s)	PM (deg)
$P_1(s)$	48.3	24.42	1.51	85.87	48.3	24.4	0.11	89.7
$P_2(s)$	27.8	7.43	1.5	58.52	27.8	7.43	0.846	71.5
$P_3(s)$	22.8	24.42	6.58	75.7	22.8	24.4	1.53	86.8
$P_4(s)$	2.25	7.43	6.53	6.77	2.25	7.43	6.51	6.97

It is seen from table 3 that the analytical technique for calculation of GM gives the values within negligible error, while the proposed technique for calculation of PM gives the values within 18.2% error. The error is due to empirical formula of ω_{cg} and the values for $GMs < 50dB$, hence (27) only may be applied.

Now putting *n=2* and $\mu = 10$ in (54), IMC-2 controller in form of transfer function is as

$$C(s) = \frac{1.4*10^{-8}s^2+2.43*10^{-5}s+4*10^{-5}}{s^2+0.2s} \quad (61)$$

The CLCE of PMSM with IMC-2 is

$$s^4 + \left(\frac{T_i+T_m}{T_iT_m} + 0.2\right)s^3 + \left(0.2 * \frac{T_i+T_m}{T_iT_m} + \frac{1}{T_iT_m} + 1.4 * 10^{-8} \times \frac{k_ik_mk_t}{T_iT_m}\right)s^2 + \left(0.2 * \frac{1}{T_iT_m} + 2.43 * 10^{-5} * \frac{k_ik_mk_t}{T_iT_m}\right)s + 4 * 10^{-5} * \frac{k_ik_mk_t}{T_iT_m} = 0 \quad (62)$$

For the formulation of all four Kharitonov polynomials, the lower and upper level valuesα_1 and$\beta_1$etc. are determined from (15), table 1 and (62) as below

$\alpha_0 = \beta_0 = 1, \alpha_1 = min_q \left\{\frac{T_i+T_m}{T_iT_m} + 0.2\right\} = 2002.2, \beta_1 = max_q \left\{\frac{T_i+T_m}{T_iT_m} + 0.2\right\} = 1517.2$

$\alpha_2 = \min_q \left\{0.2 * \frac{T_i+T_m}{T_iT_m} + \frac{1}{T_iT_m} + 1.4 * 10^{-8} \times \frac{k_ik_mk_t}{T_iT_m}\right\} = 3600.5$

$\beta_2 = \max_q \left\{0.2 * \frac{T_i+T_m}{T_iT_m} + \frac{1}{T_iT_m} + 1.4 * 10^{-8} \times \frac{k_ik_mk_t}{T_iT_m}\right\} = 2900.4$

$\alpha_3 = min_q \left\{0.2 * \frac{1}{T_iT_m} + 2.43 * 10^{-5} * \frac{k_ik_mk_t}{T_iT_m}\right\} = 725.3, \quad \beta_3 = max_q \left\{0.2 * \frac{1}{T_iT_m} + 2.43 * 10^{-5} * \frac{k_ik_mk_t}{T_iT_m}\right\} = 525.38$

$$\alpha_4 = min_q \left\{4 * 10^{-5} * \frac{k_ik_mk_t}{T_iT_m}\right\} = 140.4 \text{ and } \beta_4 = max_q \left\{4 * 10^{-5} * \frac{k_ik_mk_t}{T_iT_m}\right\} = 9.85 \quad (63)$$

In (63) the vector q represents the vector of the uncertain PMSM parameters. As seen the minimum values are greater than the maximum values that is the value $\alpha_1 > \beta_1$, therefore, interchanging the limits in accordance with Kharitonov theorem, and (62) is now representing as polynomial in various lower and upper limits values.

$$P(s) = s^4 + [1517, 2002]s^3 + [2900.4, 3600.5]s^2 + [525.4, 725.3]s + [9.85, 140.4] \quad (64)$$

The all four Kharatinov polynomials are as under using (64)

$$\begin{aligned} p_1(s) &= s^4 + 1517s^3 + 3600.5s^2 + 725.3s + 9.85 \\ p_2(s) &= s^4 + 2002s^3 + 3600.5s^2 + 525.4s + 9.85 \\ p_3(s) &= s^4 + 1517s^3 + 2900.4s^2 + 725.3s + 140.4 \\ p_4(s) &= s^4 + 2002s^3 + 2900.4s^2 + 525.4s + 140.4 \end{aligned} \quad (65)$$

According to Corollary as given in (17), only two polynomials are required to check for stability of (65) after applying Routh stability criterion on corresponding polynomials, the condition is satisfied. Hence speed control performance of PMSM with IMC-2 is robustly stable. For relative robust stability it is necessary to determine the GM and PM which is calculated using (34) to (45) and (65). The GMs for all four polynomial in dB are 44.8, 39.6, 19.89 and 14.7.The PMs in degree are 75.36, 70.1, 45.6, and 33.87.

The worst value of GM is minimum of {44.8, 39.6, 19.89, 14.7} that is 14.7dB and the worst value of PM is minimum of {75.36, 70.1, 45.6, 33.87} that is 33.87. The values obtained using analytical method and bode plots are given in table 4. The corresponding values are obtained from bode plots of fig. 13, that is simulated result of bode plots using MATLAB.

TABLE 4: GM and PM values with IMC-2

Polynomials	Analytical method				Bode Plots			
	GM (dB)	ω_{cp} (rad./s)	ω_{cg} (rad./s)	PM (deg.)	GM (dB)	ω_{cp} (rad./s)	ω_{cg} (rad./s)	PM (deg.)
$P_1(s)$	44.8	0.69	0.0523	75.36	44.8	0.69	0.014	86.1
$P_2(s)$	39.6	0.512	0.0523	70.1	39.6	0.512	0.0186	82.7
$P_3(s)$	19.9	0.69	0.22	45.6	19.9	0.69	0.17	54.6
$P_4(s)$	14.7	0.512	0.22	33.87	14.7	0.512	0.195	38.5

It is seen from table 4 that the analytical techniques for calculation of GM gives values within negligible error, while the analytical technique for calculation of PM gives values within 15.2% error.

6. Results and Discussion

The results obtained from analytical technique are verified by the results obtained from MATLAB simulation. This section also presents the response for speed control of PMSM under uncertainty of parameters. Fig.5, fig.6 and fig.7 show the step response for speed control of PMSM under nominal and uncertain or perturb with IC, IMC-1 and IMC-2 respectively. The figures 5 to 10 are plotted for 20 random uncertain samples between lower, nominal and upper limit values of parameters of PMSM as given in table 1.

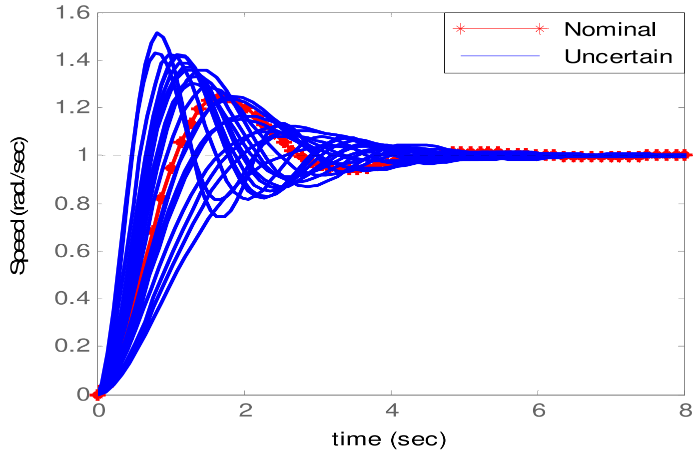

Fig.5 Step response of nominal and perturbed PMSM with IC

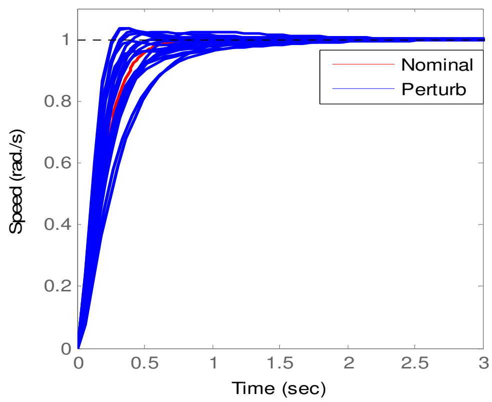

Fig. 6 Step response of nominal and perturbed PMSM with IMC-1

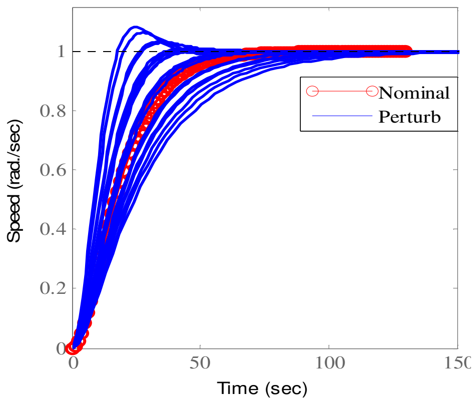

Fig.7 Step response of nominal and perturbed PMSM with IMC-2

The performance index of nominal values of parameters and with uncertain/perturbed parameters of PMSM in time domain is shown in table 5. It shows the percentage overshoot, settling time and rise time for the same controllers.

TABLE 5: Performance index of nominal and perturbed PMSM in time domain

Controller		Overshoot (%)	Settling time (s)	Rise time (s)
IC	Nominal	24.2	4.14	0.735
	Perturbed	3.58-54.2	3.2-5.45	0.303-1.77
IMC-1	Nominal	≅ 0	0.597	0.338
	Perturbed	0.164-3.73	0.307-1.39	0.18-0.595
IMC-2	Nominal	≅0	59.4	34.1
	Perturbed	0.01-8.16	32.3-105	11.9-58.1

From fig.5 to 7 and table 5, it is evident that IMC-1 gives neglagible overshoot, least settling time and rise timewith respect to IC and IMC-2. IMC-2 gives less OS and large RT and ST as compared to IC. Hence IMC-1gives better time domain performance specification as compared to IC and IMC-2.

Fig.8 to 10 show the bode plot of uncertain PMSM with IC, IMC-1 and IMC-2 respectively. The performance index of perturbed PMSM in frequency domain is given in table 6 which is completed using fig.8 to 10.

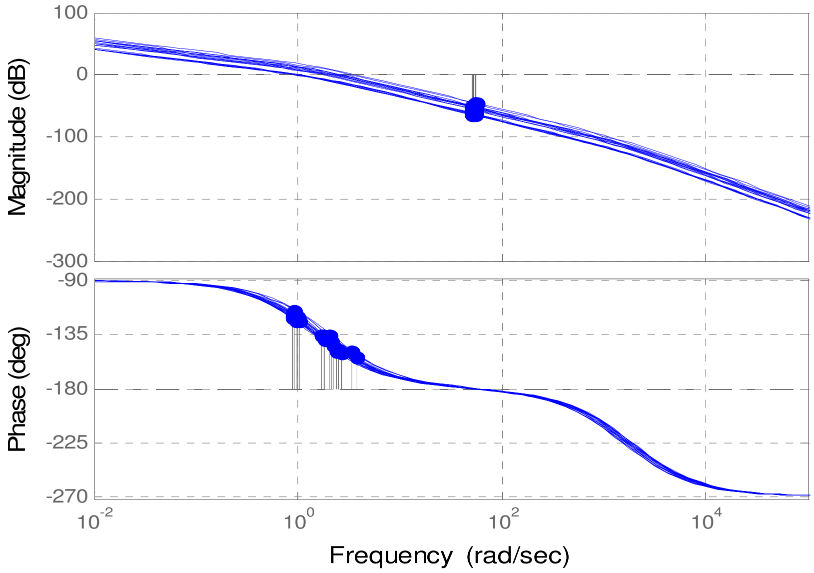

Fig.8 Bode plot of perturbed PMSM with IC

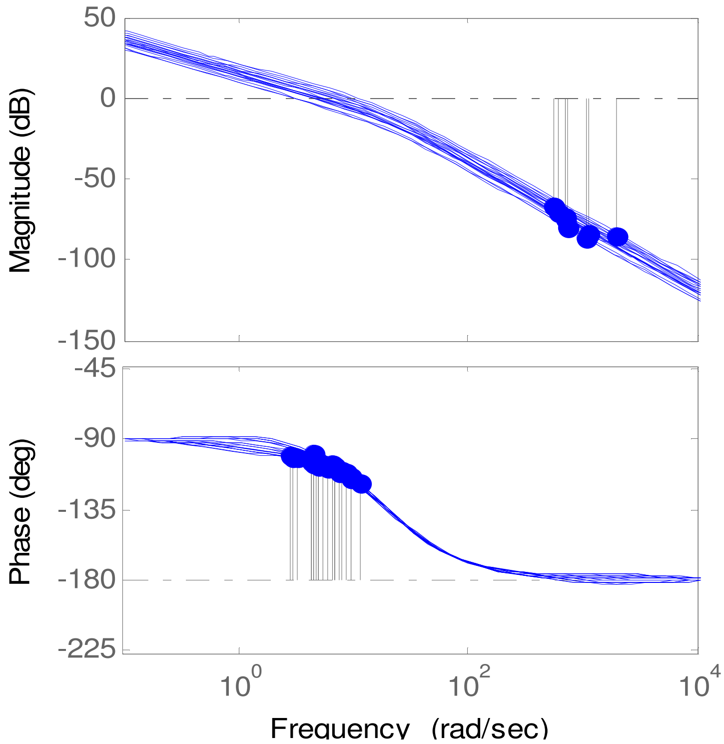

Fig.9 Bode plot of perturbed PMSM with IMC-1

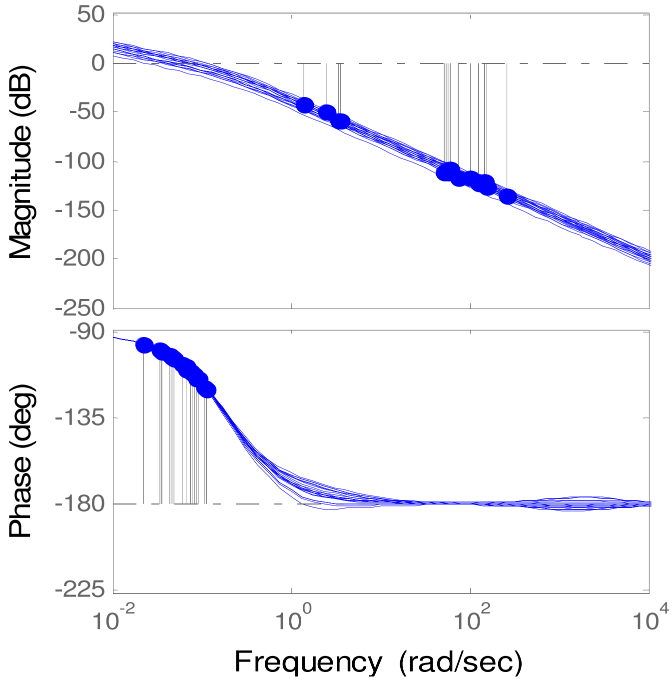

Fig.10 Bode plot of perturbed PMSM with IMC-2

TABLE 6: Performance index of perturbed PMSM in frequency domain

Controller	Range of GM (dB)	Range of PM(deg.)
IC	49.4-63.9	27.5-63.4
IMC-1	66.9-86.5	61.4-80.2
IMC-2	41.9-137	60-83.4

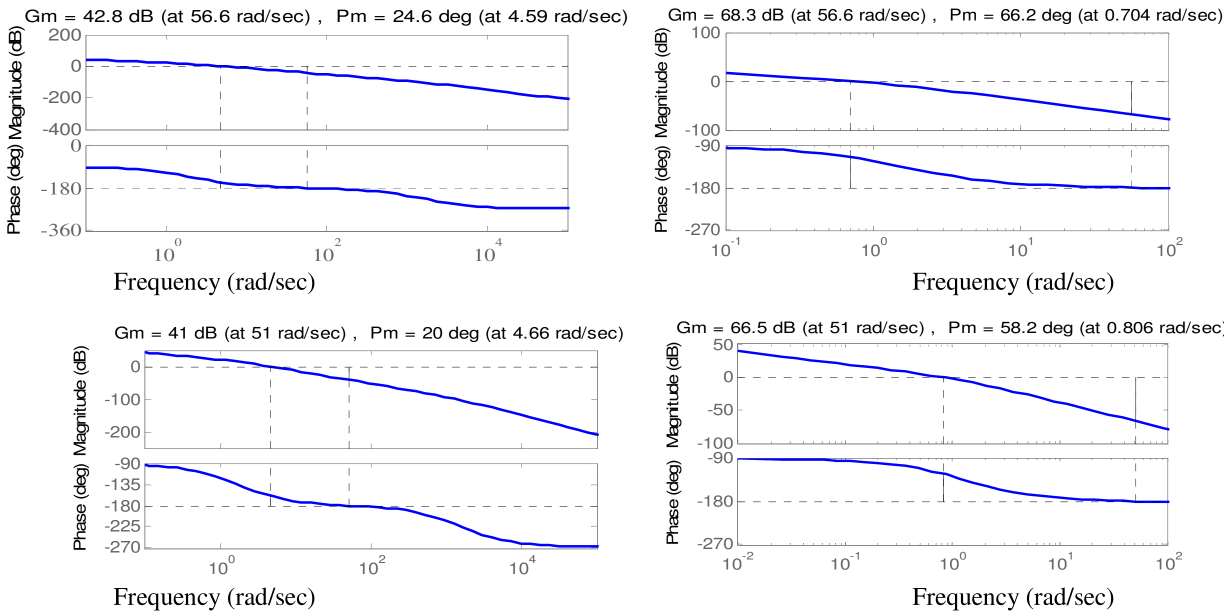

Fig.11 Bode plot of all four Kharitonov polynomials with IC

From table 6, it is seen that IMC-1 gives least GM and PM bandsas compared to IC and IMC-2 andwith least GM and PM bandsthe performance remains unaffected under parameter variations i.e. robustness of the PMSM towords the parameter variation is achieved with IMC-1. Fig.11 to fig.13 show the bode plots of all four Kharitonov polynomial of PMSM with IC, IMC-1 and IMC-2 respectively. The objective of these plots is to determine the GM and PM to compare the values using analytical techniques that is given in table 2 to 4.

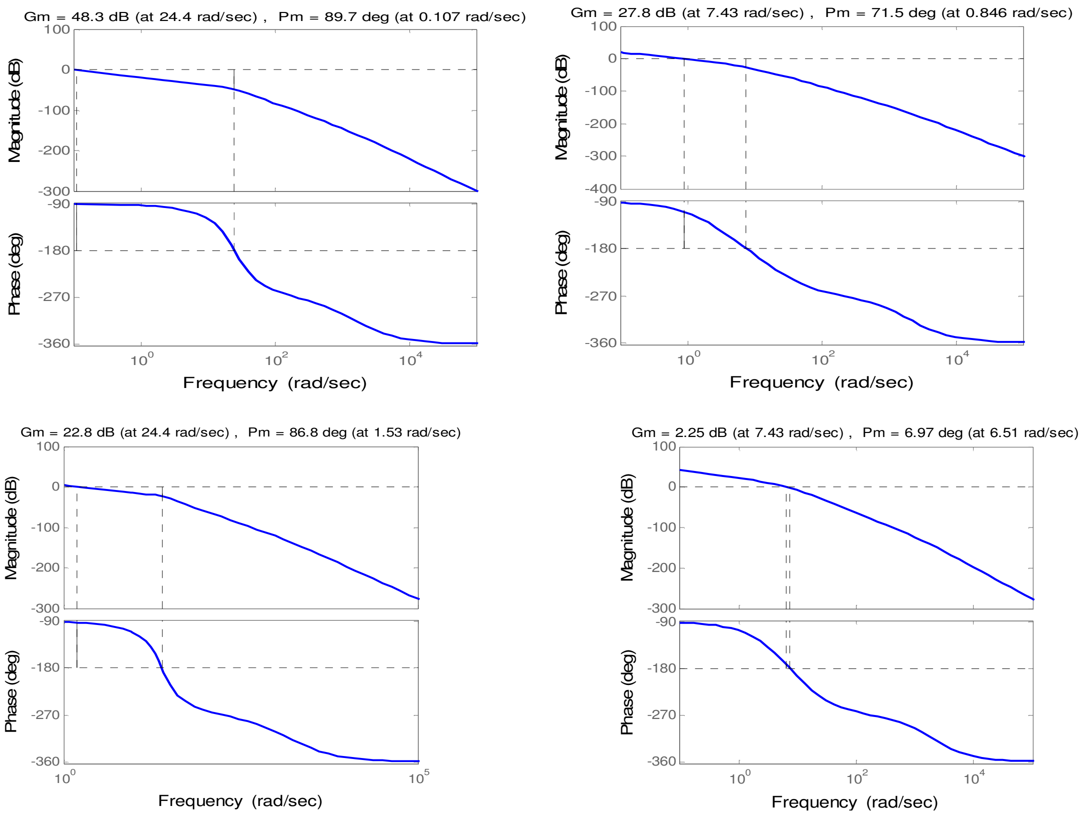

Fig.12 Bode plot of all four Kharitonov polynomials with IMC-1

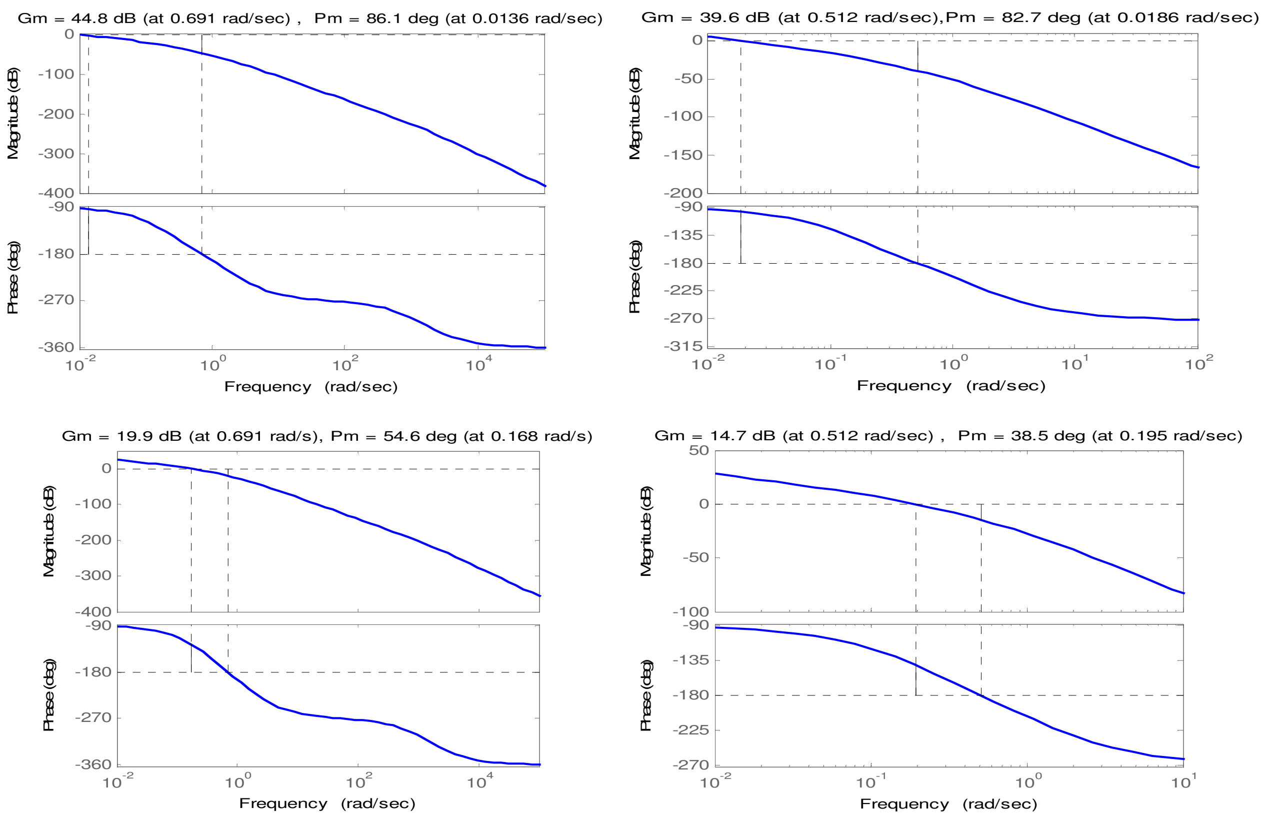

Fig.13 Bode plot of all four Kharitonov polynomials with IMC-2

Fig.14 shows the step response in of all four kharitonov polynomial with IC and the performanc index is given in table 7.

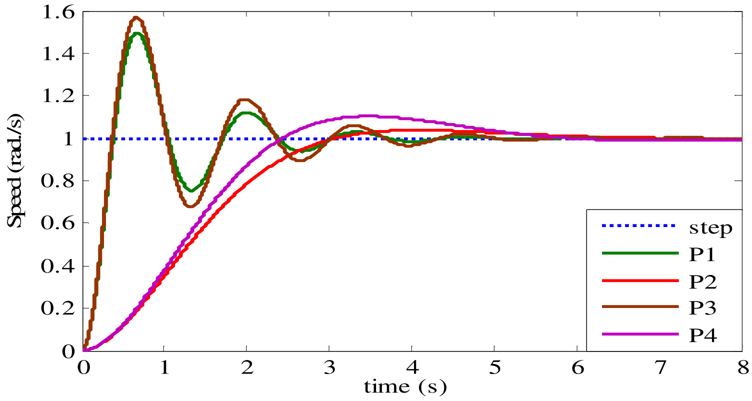

Fig.14 step response of all four kharitonov polynomial with IC

TABLE 7: Performance index of all four kharitonov polynomial with IC in time domain and frequency domain

Polynomial	Time domain			Frequency domain	
	OS (%)	ST (s)	RT (s)	GM(dB)	PM(deg.)
$P_1(s)$	49.6	2.92	0.38	42.8	23.5
$P_2(s)$	3.8	4.8	3.09	68.3	63.86
$P_3(s)$	56.9	4.1	0.37	41	19.43
$P_4(s)$	11	5.12	2.44	66.5	58.84

In table 7 the GM and PM are taken from table 2.It is seen that if the GM and PM are improved, the system becomes sluggish and high RT and ST is obtained, and if the system has fast response i.e. less RT and ST then OS increases and GM and PM decrease. Hence it is required to have proper balance in time domain and frequency domain performance specifications and better performance may be obtained with suitable tuning of the gain of IC and proper values of μ in IMCs to get better control over wide range of speed in PMSM. With domain, frequency and time the analysis obtained gives wider aspects to improve the performance and may be done for uncertain parameter variations in all control system problems. The worst values of GM with IC, IMC-1 and IMC-2 are 41, 2.25 and 14.7dB respectively. IC gives relatively high stability margin in terms of GM hence IC gives higher robust stability. The worst values of PM with IC, IMC-1 and IMC-2 are 19.43, 6.77 and 33.87degree respectively and IMC-2 gives relatively high stability in terms of PM.

7. Conclusion

The objective of this paper is to determine the relative robust stability of uncertain/perturbed PMSM with IC and two IMC speed controllers using Kharitonov theorem and with Routh stability criterion. The analytical technique used for evaluating GM and PM is simple, less time consuming and computationally efficient and in GM evaluation it gives negligible error and gives some error in PM evaluation of 3.53% for $n = 3$ for IC and for $n = 4$, error of 18.2% for IMC. This large error is not affecting the relative robust stability analysis, if done for two or more systems in order to find the most robust system among them. This technique is used for relative robust stability analysis for uncertain PMSM and may also be applied for other similar types of systems mainly for low order up to $n \leq 4$ more effectively. According to proper balance in time and frequency domain performance specification it is concluded that IMC with μ= 0.1 gives better time domain performance and IMC with μ=10 gives better frequency domain performance specification for speed control of PMSM. Hence, Kharitonov theorem along with Routh stability criterion for determination of relative robust stability may also be effectively applied to control wide range of control applications such as converters, HEV, aircraft and electromechanical systems, process control like temperature, flow, level, pressure etc. and other systems, where parametric uncertainties have significant role.

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APPENDIX-A

Verification of Condition for stability in frequency domain

For third order polynomial, n=3;

Equation (16) gives $\omega_{cg} = \sqrt{\frac{\beta_3}{\alpha_1}}$ and $\omega_{cp} = \sqrt{\alpha_2}$, according to condition for stability in frequency domains $\omega_{cp} > \omega_{cg} \Rightarrow \sqrt{\alpha_2} > \sqrt{\frac{\beta_3}{\alpha_1}} \Rightarrow \alpha_1.\alpha_2 > \beta_3$ (66)

Condition (66) is similar to (51), hence condition for stability in frequency domain is verified for $n = 3$.

For fourth order polynomial n=4;

Polynomial p_3 of (17) gives $\omega_{cg} = \sqrt{\frac{\alpha_1\beta_4}{(\alpha_1\alpha_2-\beta_3)}}$ and $\omega_{cp} = \sqrt{\frac{\beta_3}{\alpha_1}}$, According to condition for stability in frequency domain:

$$\omega_{cp} > \omega_{cg} \Rightarrow \sqrt{\frac{\beta_3}{\alpha_1}} > \sqrt{\frac{\alpha_1\beta_4}{(\alpha_1\alpha_2-\beta_3)}} \Rightarrow \alpha_1\alpha_2\beta_3 > {\alpha_1}^2\beta_4 + \beta_0{\beta_3}^2 \tag{67}$$

Similarly p_4 of (17) gives robust stability condition as follows: $\beta_1\alpha_2\alpha_3 > {\beta_1}^2\beta_4 + \beta_0{\alpha_3}^2$ (68)

Conditions (67) and (68) are similar to two conditions given in (60), hence condition for stability in frequency domain is verified for n=4.

APPENDIX-B

This appendix represents the analytical method for the design of integral controller. From (22), the GM (in dB) in terms of coefficient of s is described as:

$$\text{GM in dB} = 20\log\left(\frac{\text{coefficient of } s^2 \times \text{coefficient of } s^1}{\text{coefficient of } s^0}\right) \tag{69}$$

From (13), (18) and (46), the CLCE with IC can be written as:

$$1 + \left(\frac{7.098\times10^5}{s^2+1726\,s+2874}\right)\left(\frac{K_I}{s}\right) = 0 \tag{70}$$

where K_I is integral controller parameter. Formulating polynomial using (70) as:

$$p(s) = s^3 + 1726s^2 + 2874s + 7.098 \times 10^5 * K_I \tag{71}$$

To design a conventional integral controller as robust controller a high GM = 55dB is considered in this paper.

From (69) and (71): $20\log\frac{1726*2874}{7.098\times10^5 K_I} = 55 \Rightarrow K_I = 0.012$ (72)

Equations (72) gives $K_I = 0.012$, for easy calculation and implementation, considering $K_I = 0.01$ in this paper.

APPENDIX-C

This section presents the analytical method for the selection of single adjustable parameter μ of IMC.

Considering second order system of (73): $G(s) = \frac{k}{s^2+d_1s+d_0}$ (73)

and from (53), $C(s)$ is written as: $\mathrm{C(s)} = \frac{\mathrm{Q(s)}}{1-\mathrm{G_0(s)}*\mathrm{Q(s)}}$ (74)

where $Q(s) = \frac{1}{(\mu s+1)^n}G(s)^{-1}$ and $G_0(s) = G(s)$. For n =2, $\mathrm{C(s)} = \frac{s^2+d_1s+d_0}{k(\mu^2s^2+2\mu s)}$ (75)

The closed loop characteristic equation is $1 + G(s)C(s) = 0$, (76) is formed after replacing *G(s)* and *C(s)* from (73) and (75) respectively: $s^2 + \frac{2}{\mu}s + \frac{1}{\mu^2} = 0$ (76)

Comparing (76) with standard second order characteristic equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$. It gives $\xi = 1$ and $\omega_n = \frac{1}{\mu}$. A performance index in table 8 is used for the selection of suitable value of μ.

Table 8: Performance index of IMC for different value of μ

μ	ξ	ω_n	DP = $-\xi\omega_n \pm j\ \omega_n\sqrt{(1-\xi^2)}$	ST =$4/\xi\omega_n$	OS
0.1	1	10	-10	0.4	0
1	1	1	-1	4	0
10	1	0.1	-0.1	40	0

The selection of desired value of μ requires a proper balance of bandwidth, settling time, sensitivity and control effort etc. Hence according to proper balance of performance parameter as derived in table 8, $\mu = 1$ is the best out of the three given values from 0.1, 1 and 10, but for relative robust stability analysis two values of $\mu = 0.1$ and 10 are taken into consideration.