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**Regular paper**

**Benchmark studies on Optimal  
Reactive Power Dispatch (ORPD)  
Based Multi-Objective Evolutionary  
Programming (MOEP) using Mutation  
Based on Adaptive Mutation Operator  
(AMO) and Polynomial Mutation  
Operator (PMO)**



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Present days, Power System operates in a stressed condition due to reactive power shortage. Hence, this research involves development of an adaptive mutation algorithm based multi-objective for Optimal Reactive Power Dispatch (ORPD) in a power system in order to minimize the total loss and the improved voltage stability simultaneously. The performance of a Multi-Objective Evolutionary Programming (MOEP) is significantly dependent on the parameter setting of the operator. These parameters tend to change the characteristic of adaptive in different stages of evolutionary process. The intention of this paper is to create adaptive controls for each parameter existing in MOEP where it is able to improve even more the performance of the evolutionary programming. Hence, in this paper, an adaptive mutation operator based multi-objective evolutionary programming is presented. A computer program was written in MATLAB. At the end, the result was compared with the Polynomial Mutation Operator.

**Keywords:** Optimal Reactive Power Dispatch, Multi-Objective Evolutionary Programming, Adaptive Mutation Operator, Polynomial Mutation Operator

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## **1. Introduction**

The ORPD plays an important role in today's power system. Hence, ORPD problem is considered as a very challenging optimization problem. The ORPD problem can be solved using intelligent methods. There are numerous intelligent methods have been developed in recent years. There are Artificial Neural Networks (ANN), Genetic Algorithms (GA), Particle Swarm Optimization (PSO) and Evolutionary Programming (EP). Evolutionary Programming (EP) has been increasingly applied over the past years for the solution of optimization problems with multi-objective. There are a lot of MOEP has been recommended in recent years due to its capability to discover Pareto-optimal solutions in one single simulation run [2].

Multi-objective problems are more complicated to solve compared to the single objective since there is no unique solution. Moreover, there are two objective function implemented in multi-objectives optimization problem where need to optimize simultaneously with a number of equality and inequality constraints. Furthermore, the implementation of Multi Objective able to provide a set of optimal solutions. This is known as Pareto-optimal solutions. The optimal set refers to all possible non-dominated solution while the Pareto Front refers to the corresponding objective function values in the objective space [3]. The typical MOEP utilizes three basic operators. It is included selection,

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crossover and mutation. Evolutionary programming mainly relies mainly on its mutation operator. The mutation operator helps to produce variety in the population. Furthermore, in EP, the step size control is a main significant matter in the design of mutation operator [3] [4].

Furthermore, there are plenty of mutation applies in MOEP. It is included Gaussian mutation, Levy mutation, Cauchy mutation, Polynomial mutation and Adaptive mutation. The AMO depends on its parameter setting which also known as Parameter Control where it allocate initial values of the parameters and then these values adaptively change throughout the implementation of Evolutionary Programming. Besides that, Polynomial mutation depends on Parameter setting called Parameter Tuning. The Parameter Tuning means to set the suitable parameters before the run of algorithms and the parameters remain constant during the execution of algorithms [4] [5]. So, the performance of MOEP is significantly dependent on the parameters setting of these operators. The most desired control of such parameters presents the characteristic of adaptive.

Hence this paper presents, a new approach for MOEP using AMO applied to ORPD problem. Lastly, the result is compared to two types of mutation namely AMO and PMO. The author analyses the result of the MOEP-AMO and MOEP-PMO with three ZDT tests, that is ZDT1, ZDT2 AND ZDT3.

## 2. Notation

The notation used throughout the paper is stated below.

*Indexes:*

<i>MOEP</i>	Multi-Objective Evolutionary Programming
<i>AMO</i>	Adaptive Mutation Operator
<i>PMO</i>	Polynomial Mutation Operator
<i>EP</i>	Evolutionary Programming

*Constants:*

$M$	objective functions
$I_j$	solution index of the $j^{th}$ in the sorted list
$f^{max}$	maximum population
$f^{min}$	minimum population
$C_k$	child
$p_k^u$	parent with upper bound on the parent component
$p_k^l$	parent with lower bound on the parent component
$\delta_k$	small variations
$j$	current generation

## 3. Problem Formulation

### 3.1. Minimization of SVSI

The objective function which also incorporated in the MOEP namely the Static Voltage Stability Index (SVSI) [1] which can estimate the stability margin of the system. The range of SVSI should be in between 0 (no load) and 1 (voltage collapse). The decrement in the

value of *SVSI* indicates that the improvement of voltage stability in the system. Hence, the mathematical formulae of *SVSI* can be written as

$$SVSI_{ji} = \frac{2\sqrt{(X_{ji}^2 + R_{ji}^2)(P_{ji}^2 + Q_{ji}^2)}}{\left| |V_i|^2 - 2X_{ji}Q_{ji} - 2R_{ji}P_{ji} \right|} \quad (1)$$

### 3.2. Minimization of Transmission Loss

Another objective function considered in the proposed method is minimizing the transmission power losses in the transmission network, while satisfying a set of physical and operation, subjected to a set of equality and inequality constraints in the power system. The mathematical equation of transmission loss can be written as,

$$\begin{aligned} \min f_p &= \sum_{k \in N_E} P_{k_{Loss}}(V, \theta) \\ &= \sum_{\substack{k \in N_E \\ k=(i,j)}} g_k (V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij}) \end{aligned} \quad (2)$$

### 3.3. Non-domination sorting

The initialized population is sorted according to the level of non-domination. Each solution should be compared with every other solution in the population to find if it is dominated. Each solution assigned a fitness or rank equal to its non-domination level (1 is the best level, 2 is the next best level and so on). Furthermore, the first set/ rank belongs to the most excellent non-dominated set in the population [5].

### 3.4. Crowding Distance

Once the non-domination sort is completed, the crowding distance is assigned. Crowding Distance also known as a fitness value of an individual [6]. The purpose of crowding distance is to provide the diversity in the population. Then, the individual solutions are sorted in descending order based on the magnitude of the crowding distance values calculated as follows:

$$d_{I_j^m} = d_{I_j^m} + \frac{f_m^{(I_{j+1}^m)} - f_m^{(I_{j+1}^m)}}{f_m^{\max} - f_m^{\min}} \quad (3)$$

### 3.5. Flow chart of MOEP

This segment presents necessary information concerning the development of Evolutionary Programming (EP). The Figure 1 shows a flow chart of ORPD based MOEP.

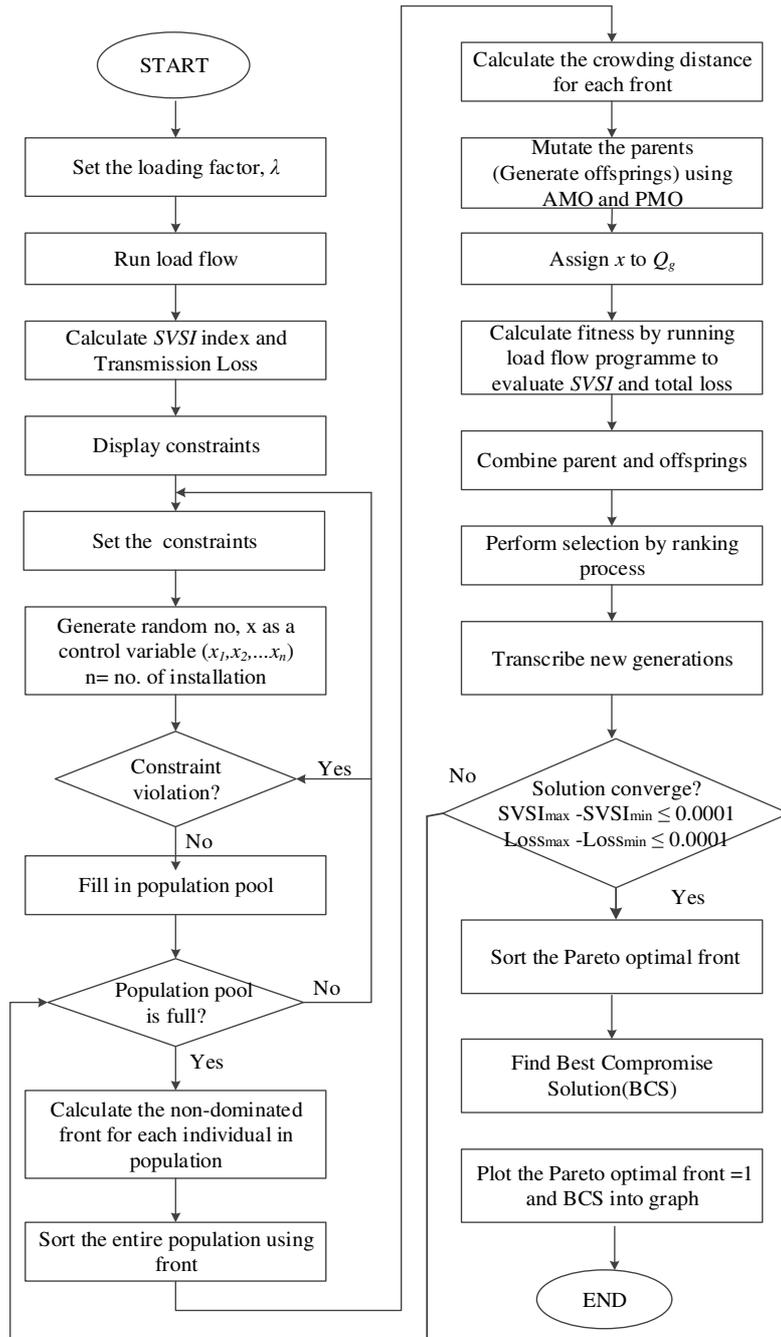


Figure 1: Flow Chart of MOEP

### 3.6. Description of ZDT Test Problem

There are three Zitzler-Deb-Thiele (ZDT1, ZDT2 and ZDT3) test problems which are chosen for evaluating the performance EP on AMO and also PMO. The description of all test functions is shown in Table 1.

Table 1: Summary of Benchmark test problems

Test	n	Limit	Objective Function	Pareto Solutions	Types
ZDT1	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x) [1 - (f_1(x) / g(x))^{1/2}]$ $g(x) = 1 + 9 (\sum_{i=2}^n x_i / (n-1))$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, \dots, n$	Convex
ZDT2	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x) [1 - (f_1(x) / g(x))^{1/2}]^2$ $g(x) = 1 + 9 (\sum_{i=2}^n x_i / (n-1))$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, \dots, n$	Non-convex
ZDT3	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x) [1 - (x_1 / g(x))^{1/2}] - ((x_1 / g(x)) \sin(10x_1 g(x)^{1/2}))$ $g(x) = 1 + 9 (x_i / (n-1))$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, \dots, n$	Discontinuous

### 3.7. Adaptive Mutation Operator (AMO)

The AMO is types of one of the mutation used to solve multi-objective optimization problem based Evolutionary Algorithm which is proposed by Deb and Goyal in 2001. Hence, this section briefly explains about method proposed by Deb and Goyal for improving even more the performance of the Evolutionary programming. The mutation operator changed its current value of a continuous variable to a neighboring value using Polynomial Probability Distribution and this is a basic procedure of any genetic operator. However, there are two empirical facts are observed in order to change the variance of the probability distribution in adaptive ways.

First, the individual from initial solutions are discrete in the search space and crowding distance from the Pareto optimal front was obtained. Moreover, the difference between the maximum crowding distance value (not infinite) and the minimum crowding distance value is lifted. This is because to apply a strong mutation in order to ensure a quick convergence to the optimal Pareto front and a fast achievement of discrete value. Furthermore, at the end of the evolutionary process the solution should be nearer to the Optimal Pareto front and this would be second empirical facts to change the mutation in adaptive ways. Moreover, the difference between the maximum (not infinite) and the minimum crowding distance value is reduced. In this way soft mutation was applied to ensure solutions from destroying which is generated previously and for trying to approximate them to the Pareto optimal front [6].

Hence, the main idea of the adaptive control is to use information about the difference between the maximum (not infinite) and the minimum crowding distance value and the information from the current stage of the evolutionary process. The mathematical formulae of different in crowding distance,  $\Delta$  are shown as :

$$\Delta = \max (\text{Crowding Distance}) - \min (\text{Crowding Distance}) \tag{4}$$

The next operation would be use information from the current evolutionary process. The mathematical equation can be written as :

$$Sigm(j) = 1 / (1 + e^{(-0.07j)}) \quad (5)$$

The main idea of this function is to apply a strong mutation in the early stage of the evolutionary programming and slowly reduced its value during the process. The constant value -0.07 is used because the value of  $e^{-0.07t}$  will be approximately 0 when  $t$  approaches infinity. At point, where  $t$  is greater than 100 the function  $Sigm(j)$  will stop influencing the mutation operator because its value reaches 1. Furthermore, the new value of the parameter  $n$  has to be inversely proportional to  $\Delta$  and be directly proportional to the  $sigm(t)$ . This is because soft mutation needed higher value of  $t$  for  $n$  to reduce the variance of the probability distribution. At the end of the genetic operator, the last step taken by the controller is to update  $n$ , before applying a mutation in the current generation, as follows:

$$n = sigm(j) / \Delta \quad (6)$$

### 3.8. Polynomial Mutation Operator (PMO)

In 1996 Deb and Goyal proposed a variation mechanism called PMO [5]. The mathematical equation (5) of PMO shown as :

$$C_k = p_k + (p_k^u - p_k^l) \delta_k \quad (7)$$

The small variations,  $\delta_k$  is obtained from the following mathematical equation as shown below :

$$\begin{aligned} \delta_k &= (2r_k)^{1/(\eta_m + 1)} - 1 && \text{if } r_k < 0.5 \\ \delta_k &= 1 - [2(1 - r_k)]^{1/(\eta_m + 1)} && \text{if } r_k > 0.5 \end{aligned} \quad (8)$$

### 3.9. Tournament Selection

The offspring produces from the mutation process are combined with the clone parent to undergo a selection process in order to identify the candidates have the chance to be transcribed in the following generation. The best individual from the offspring population will be selected according to a selection scheme in order to form the parent population for the following generation. The offsprings are chosen using tournament scheme. Moreover, the populations of individuals with improved fitness function are sorted in increasing order. The first half of the population would be retained as the new individuals or parent for the next generation and the others will be removed from the pool. The progression continues until a convergence is reached. The convergence criterion is duly specified by the difference between the maximum and minimum objective function (fitness) to be less than 0.0001. The mathematical equation of tournament selection given as shown below.

$$fitness_{max} - fitness_{min} \leq 0.0001 \quad (9)$$

#### 4. Result and Discussion

The performances of MOEP were analysed with three ZDT test function namely ZDT1, ZDT2 and ZDT3. The algorithm developed using MATLAB. The following parameters are used in the EP and the result of EP shown in Table 2 and Table 3 respectively.

Population Size	=	100
Generation	=	1000
Mutation Probability	=	5

Table 2 and Table 3 show the fitness value of the PMO and AMO respectively, for 100 generations with population size 100. From the table, the analysis shown that ZDT1 test function of AMO outperformed well than ZDT1 test function PMO. The convergence metric of ZDT1 for AMO is 0.0050 which is less than PMO that is 0.0144. Besides that the crowding distance value of AMO which is 0.0084 is lower compared to the PMO that is 0.0251 for ZDT1. However, for the ZDT2 test function the convergence metric and crowding distance of PMO lower than AMO, where 0.0007 and 0.0053 for PMO while 0.0267 and 0.0091 for AMO respectively. The convergence metric of ZDT3 for PMO test function worse compared with ZDT3 test function for AMO and also with other two test function.

Figure 2, Figure 4 and Figure 6 show non-dominated solutions obtained by EP using PMO on three ZDT test problem. The presented results are the outcomes of a single run of an EP. Moreover, Figure 3, Figure 5 and Figure 7 shows the Pareto front for EP using AMO. Based on the analysis, the Figure 2 and Figure 3 illustrates a convex Pareto front characteristic while Figure 4 and Figure 5 shows non-convex Pareto front feature. Besides that, Figure 6 and Figure 7 demonstrate the characteristic of discontinuous Pareto front.

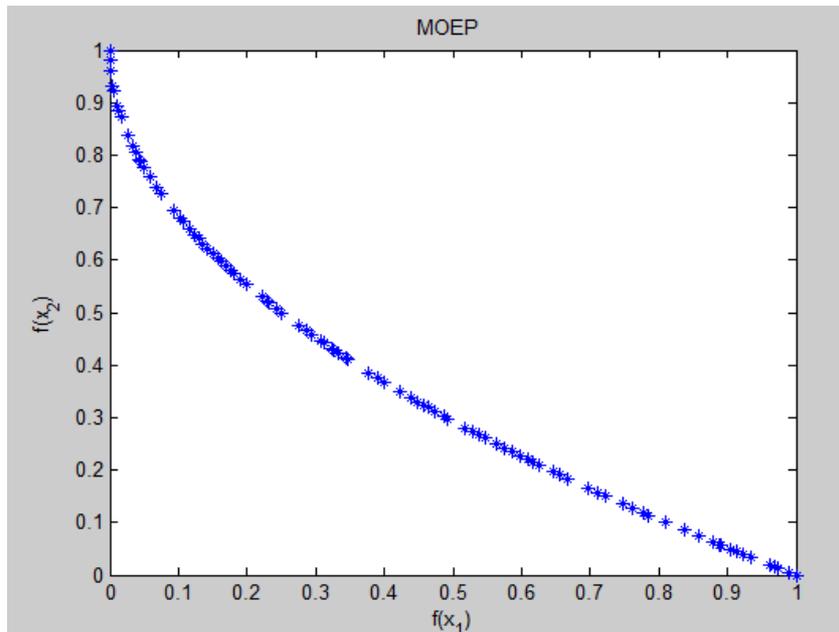


Figure 2 : ZDT1 based PMO

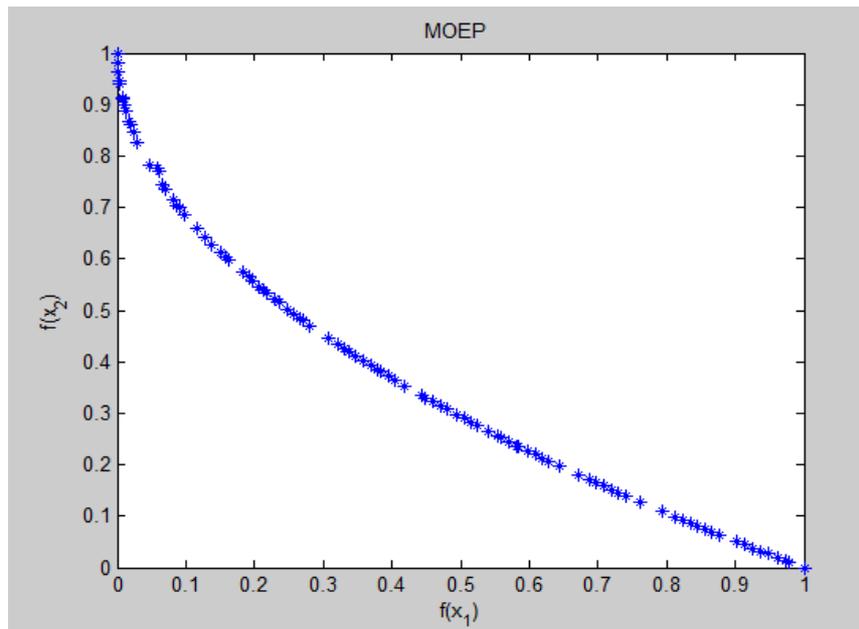


Figure 3 : ZDT1 based AMO

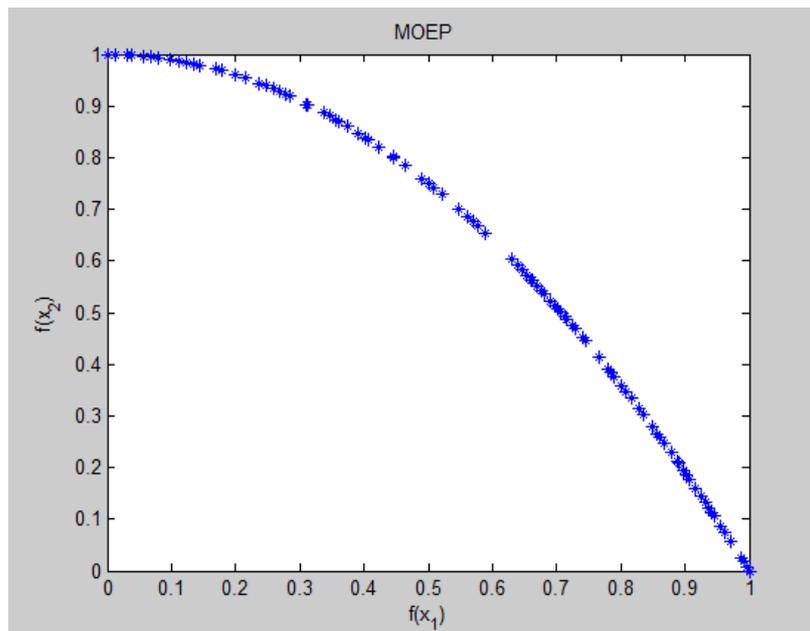


Figure 4 : ZDT2 based PMO

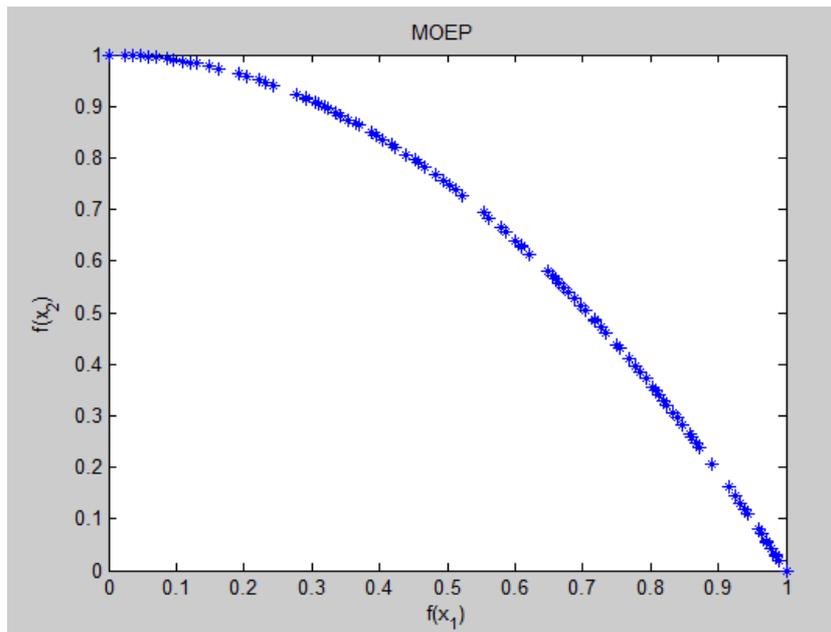


Figure 5 : ZDT2 based AMO

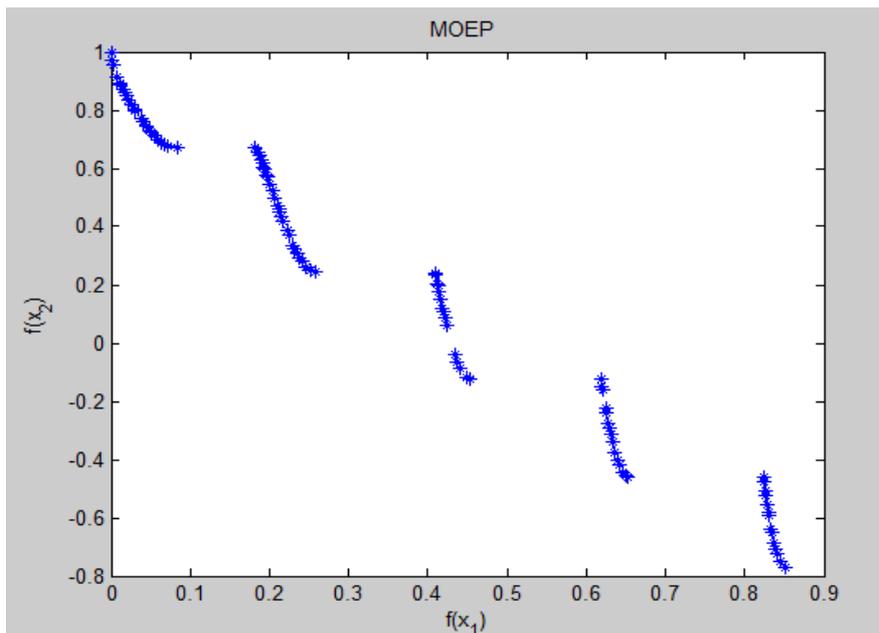


Figure 6 : ZDT3 based PMO

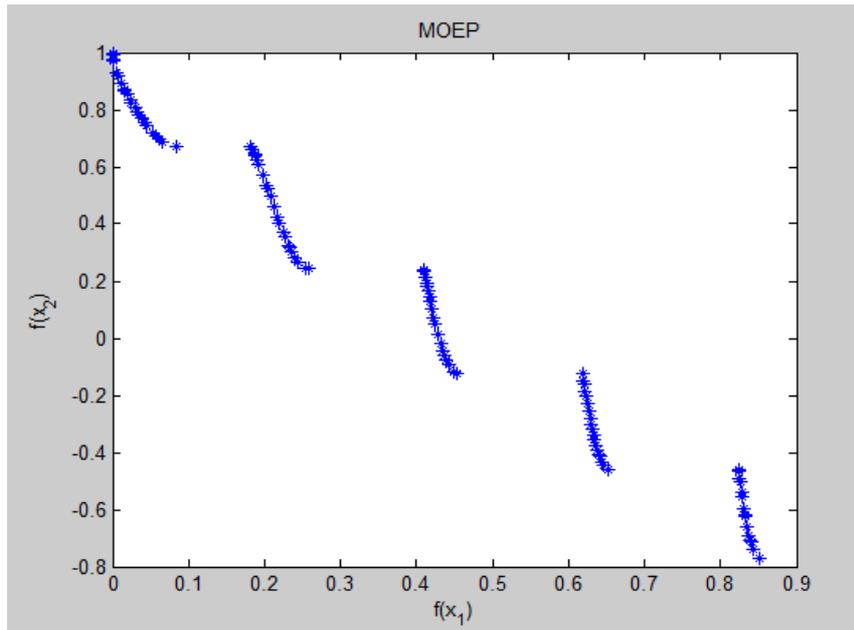


Figure 7 : ZDT3 based AMO

#### 4.1. Comparative Studies

The comparative studies made between two types of mutation operator, namely AMO and PMO. The results are shown in Table 4. The table shows the number of non-dominated solution presented at the Pareto front using AMO as well as PMO based EP. The results are verified with three different test functions. It is based on test function ZDT1, ZDT2 and ZDT3.

The AMO outperformed PMO for every ZDT test function. Based on analysis, it shows that only 85 non dominated solutions present along the Pareto front using PMO for ZDT1 test while for AMO 93 non-dominated solutions present along the Pareto front. However, ZDT3 test function using AMO able to present 100 non-dominated solutions compared with PMO where 90 non-dominated solution only. Hence, as highlighted in the table it is observed that EP is using AMO better than EP using PMO since MOEP based AMO able to present more non-dominated solutions along the Pareto front in one single run compared to EP based PMO.

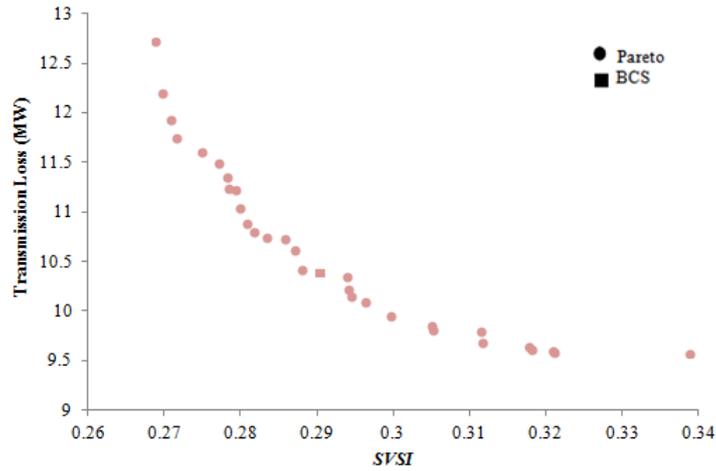


Figure 8: Pareto front for SVSI and Transmission loss obtained using MOEP-AMO for ORPD

Table 4: Non-dominated solutions present along the Pareto Front

Types of Mutation	Non-Dominated Solution No		
	ZDT1	ZDT2	ZDT3
PMO	85	89	90
AMO	93	95	100

#### 4.2. Comparative Studies, Applied for ORPD

The developed algorithm of MOEP using AMO and PMO has been tested with bus 30 reactively loaded to 25 MVar in the IEEE 30-bus RTS in order to verify the Benchmarks Studies. Both *SVSI* and transmission loss were optimized simultaneously. Table 5 tabulates the result of MOEP using AMO and PMO. In this table, the results are verified from three aspects in terms of *SVSI* value, transmission loss and amount of non-dominated solutions. As highlighted in the table, it is observed that MOEP-AMO is better than MOEP-PMO since MOEP-AMO managed to improve the *SVSI* value and decreased the transmission loss value as compared to MOEP-PMO in the system.

Figure 8 shows the Pareto front for *SVSI* and transmission losses obtained using MOEP-

Table 5: Results of MOEP for AMO and PMO when bus 30 reactively loaded with 25 MVar

Variable	<i>SVSI</i>	Transmission Loss (MW)	Non Dominated Solution No.
MOEP-PMO	1.2235	8.6623	57
MOEP-AMO	0.2934	5.0085	90

AMO for ORPD when bus 30 relatively loaded. It is observed that the *SVSI* and transmission loss values, decreased with respect to loading factor after the implementation of MOEP-AMO in the system.

## 5. Conclusion

Multi-Objective Evolutionary Programming (MOEP) is a new implementation dealing with three ZDT test function. The EP-AMO uses ZDT function enables fast convergence to the Pareto front and the number of non-dominated solution present along the Pareto front more compared to MOEP-PMO. Furthermore, in order to verify the result of Benchmark studies, ORPD scheme has been applied to test the Power System. From the result, it can be concluded that as compare with both mutations, it was found out that MOEP-AMO is outperformed MOEP-PMO in most cases.

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