Model based Predictive Control (MPC) is an efficient control technique for the control of power converters. It has several advantages for this kind of controls, such as robustness to parameters variations and fast dynamic responses. The MPC considers the model of the converter-based system to forecast possible future behavior of the controlled variable. It allows selecting the optimal voltage vector to be applied so that a predefined cost function is minimized. This paper focuses on an MPC algorithm for three-phase grid connected converter. Several simulation results (under Matlab-Simulink software environment tool) and experimental results (under developed prototyping platform) are presented in order to show the effectiveness of the considered control strategy.

Keywords: Wind power generation, Three-phase grid connected converter, Model Predictive Control.

1. Introduction

During last years, Distributed Generations (DG) of electrical power integrating renewable energy sources are increasingly used because of their environmental and economic interest compared to traditional generators. Recently, distributed power generation based on wind energy has become one of the world’s fastest growing energy sectors, helping to satisfy the global energy demands [1]-[2]-[3]. In grid connected wind energy systems, the three-phase grid connected converter play a key role to connect the wind energy generator to the utility grid. To achieve high control performances, several issues have been studied in the literature, including converter topologies, modulation strategies and control strategies [4]-[5]. For this reason, the development of high performance control strategies for three-phase grid connected converter becomes of prime importance, since it allows to meet standards and grid interconnection requirements [6]-[7]. The control scheme of a grid-connected converter is usually divided into three parts:

- Grid synchronization part: This part is generally used to compute the grid voltage position $\theta_{dq}$. It is based on the use of Phase Locked Loop (PLL) [8] and is particularly aimed to synchronize the control with regard to the grid voltage vector.
- dc-link voltage control part: This part is generally aimed to control the dc-link voltage $V_{dc}$ in the dc-side of the grid connected converter. In most cases, the control of the dc-link voltage is based on a simple PI controller.
- Current control part: This part is generally aimed to control the grid currents $i_{gabc}$ in the grid side of the grid connected converter.
The current control part is the most important one to meet new grid codes specified by national standards for the connection of power converters to the grid. Moreover, it allows controlling the power flow through the converter, namely the active power $P$, the reactive power $Q$ and in some cases the distorted power $D$. So, robust and efficient current control algorithms should be used in order to ensure required performances. To this purpose, several control strategies based on different current control algorithms have been studied and developed [9]-[10]. Among them, we can quote the Direct Power Control (DPC) [11] and the DPC-SVM [12]. For the DPC algorithm, the current control part is based on nonlinear hysteresis controllers associated to a switching table. For the DPC-SVM algorithm, the current control part is based on linear PI controllers associated to a Space Vector Modulation (SVM) process. Nowadays, a growing interest is also paid for Model Predictive Control (MPC) algorithms due to their robustness and performances [13]-[14]-[15]-[16]. During last decade, their implementation on digital solutions was limited by computation delays that affect the predictive control performances [17]. The recent revolution in digital electronic technologies allowed performing real time computation of predictive control algorithms despite their complexity. Compared to the DPC algorithm, MPC algorithms provide lower current THD with the same dynamic performances. Besides, they are characterized by a mean variable switching frequency lower than the case of the DPC algorithm. Compared to the DPC-SVM algorithms, MPC algorithms provide higher current THD with faster dynamic performances [18]. In addition, they are less sensitive to parameters variations.

This work is aimed to present an MPC algorithm used for the control of three-phase grid connected converter. The considered MPC algorithm is based on prediction and optimization tasks. The prediction task is performed based on the mathematical model of a three-phase grid connected converter, while the optimization one is performed based on a predefined cost function. During each sampling period, the optimal converter voltage vector is computed as follows. Firstly, the grid current trajectories are computed based on the mathematical model of the three-phase grid connected converter. These trajectories are computed for all switching signal combinations that lead to seven different converter voltage vectors. The optimization task is then achieved by selecting the switching signals combination that minimizes a cost function. This last is defined so that minimal error between the grid current reference vector and the predicted one is obtained [19]-[20]-[21]-[22]-[23].

This paper is organized as follows. In section II, the model of three-phase grid connected converter is firstly presented. Then, the MPC algorithm is detailed and discussed according to the obtained simulation results. Finally, section III presents numerous experimental results in order to show the effectiveness of the proposed MPC algorithm.

2. Three-phase grid connected converter model

The power circuit of a three-phase grid connected converter is presented in Fig.1. As depicted in this figure, an $L$ filter is used to interface between the grid and the converter, where $L_g$ denotes the inductor value and $R_g$ its equivalent serial resistance.
Fig. 1. (a) Power circuit of the three-phase grid connected converter through $L$ filter (b) Single phase equivalent circuit

$V_{g_i,(i=a,b,c)}$ (respectively $i_{g_i,(i=a,b,c)}$) refer to the grid voltage vector components (respectively the grid current vector components) in the stationary reference frame, while $V_{dc}$ refer to the dc-link voltage. The mathematical model of the three-phase grid connected converter in the $dq$ synchronous reference frame is characterized by the following equations.

$$\frac{di_{gd}}{dt} = \frac{1}{L_g} \left( V_{gd} - R_g i_{gd} + \omega_s L_g i_{gq} - V_{convd} \right) \quad (1)$$

$$\frac{di_{gq}}{dt} = \frac{1}{L_g} \left( V_{gq} - R_g i_{gq} - \omega_s L_g i_{gd} - V_{conqv} \right) \quad (2)$$

$$P = V_{gd} i_{gd} + V_{gq} i_{gq} \quad (3)$$

$$Q = V_{gq} i_{gd} - V_{gd} i_{gq} \quad (4)$$

Where $di_{gd}/dt$ and $di_{gq}/dt$ are the instantaneous grid current time derivatives. $V_{gd}$ and $V_{gq}$ (respectively $i_{gd}$ and $i_{gq}$) are the $d$ and $q$ components of the grid voltage vector (respectively the grid current vector) and $\omega_s$ is the angular frequency of the grid voltage. $V_{convd}$ and $V_{conqv}$ are the $d$ and $q$ components of the converter output voltage vector. $L_g$ and $R_g$ are respectively the resistor and inductor of the used $L$ filters. Taking into account that the $d$ axis is linked to the grid voltage vector ($V_{gq}=0$), equations (3) and (4) can be simplified as follows

$$P = V_{gd} i_{gd} \quad (5)$$

$$Q = -V_{gd} i_{gq} \quad (6)$$

3. Model based predictive control

The proposed MPC algorithm is based on the discrete system model, which is used to predict system response during the next sampling period [24]. The main objective of the MPC algorithm is to control the active and reactive power through the control of the $d$ and $q$ grid current components.
The MPC-based control for three-phase grid connected converter is presented on Fig.2. As shown in this figure, a cascade control structure is used. In one hand, a PI-based external control loop controls the dc-link voltage $V_{dc}$. On the other hand, an MPC-based internal control loop controls the grid currents in the $dq$ synchronous reference frame, where the $d$ axis is linked to the grid voltage vector. According to equations (1) and (2), and using the forward Euler discretization method, the digital prediction equations are deduced as in (7) and (8).

$$i_{gd}[k+1] = a_0 \left(V_{gd}[k] - V_{convd}[k]\right) + a_1 i_{gd}[k] + a_2 i_{gq}[k]$$  \hspace{1cm} (7)

$$i_{gq}[k+1] = a_0 \left(V_{gq}[k] - V_{convq}[k]\right) + a_1 i_{gq}[k] - a_2 i_{gd}[k]$$  \hspace{1cm} (8)

Where $a_0 = T/L_g$, $a_1 = (1 - R_g T/L_g)$, and $a_2 = \omega_g T_s$; $T_s$ is the sampling period. $i_{gd}[k+1]$ and $i_{gq}[k+1]$ (respectively $i_{gd}[k]$ and $i_{gq}[k]$) are the predicted $d$ and $q$ grid current components at the $(k+1)^{th}$ sampling period (respectively measured $d$ and $q$ grid current components during the $k^{th}$ sampling period).

During each sampling period, the evolution of the $d$ and $q$ grid current components depends on the applied converter voltage components $V_{convd}[k]$ and $V_{convq}[k]$ at the $k^{th}$ sampling period. These voltage components used for the current prediction are expressed in the $dq$ synchronous reference frame and can be determined through the application of a rotation operation (with an angle equal to $\theta_{dq}$) to the $\alpha\beta$ components of the converter voltage vectors. The $\theta_{dq}$ position is the grid voltage vector position and is computed through a PLL.

It should be noted that the voltage vectors $V_{convd}^j[k]$ and $V_{convq}^j[k]$ depend also on the dc-link voltage $V_{dc}^*$ level as shown in equations (9) and (10). For simulation reasons, the dc-link voltage is assumed equal to its reference $V_{dc}^*$.

$$\begin{bmatrix} V_{convd}^j[k] \\ V_{convq}^j[k] \end{bmatrix} = \begin{bmatrix} \cos(\theta_{dq}[k]) & \sin(\theta_{dq}[k]) \\ -\sin(\theta_{dq}[k]) & \cos(\theta_{dq}[k]) \end{bmatrix} \begin{bmatrix} V_{convd}[k] \\ V_{convq}[k] \end{bmatrix}$$  \hspace{1cm} (9)
\[
\begin{bmatrix}
V_{\text{conv}}^j[k] \\
V_{\text{comp}}^j[k]
\end{bmatrix} = \frac{2}{3} V_{dc}^j \begin{bmatrix}
1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix}
\begin{bmatrix}
S_a[k] \\
S_b[k] \\
S_c[k]
\end{bmatrix}
\]

(10)

Where \( V_{\text{conv}}^j[k] \) and \( V_{\text{comp}}^j[k] \) are the output converter voltage vectors in the stationary coordinates \( \alpha\beta \), and \( S_a[k] \), \( S_b[k] \) and \( S_c[k] \) are the switching signals for each phase of the converter.

Hence, to perform the prediction process, seven cases must be taken into account since the number of switching states combinations is equal to eight with two combinations that lead to null converter voltage vector as presented in equations (11) and (12), where \( (i_{gd}^j[k+1])(j=0..6) \) and \( (i_{gq}^j[k+1])(j=0..6) \) are the predicted \( d \) and \( q \) grid current components at the \( (k+1)^{th} \) sampling period when the \( (V_{\text{conv}}^j[k])(j=0..6) \) and \( (V_{\text{comp}}^j[k])(j=0..6) \) voltage vector components are applied during the \( k^{th} \) sampling period.

\[
(i_{gd}^j[k+1] = a_0 [V_{gd}^j[k] - V_{\text{conv}}^j[k]] + a_1 i_{gd}^j[k] + a_2 i_{gq}^j[k]) \quad (j=0..6)
\]

(11)

\[
(i_{gq}^j[k+1] = a_0 [V_{gq}^j[k] - V_{\text{conv}}^j[k]] + a_1 i_{gd}^j[k] - a_2 i_{gq}^j[k]) \quad (j=0..6)
\]

(12)

**Table 1:** Converter switching states and corresponding output voltage vectors \( V_{\text{conv}}^j[k][j=0..6] \)

<table>
<thead>
<tr>
<th>( S_a )</th>
<th>( S_b )</th>
<th>( S_c )</th>
<th>( V_{\text{conv}}^j )</th>
<th>( V_{\text{comp}}^j )</th>
<th>( V_j )</th>
<th>( V_{\text{conv}}^j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( V_{\text{conv}}^0 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2( V_{dc}/3 )</td>
<td>0</td>
<td>( V_1 )</td>
<td>( V_{\text{conv}}^1 )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( V_{dc}/3 )</td>
<td>( V_{dc}/\sqrt{3} )</td>
<td>( V_2 )</td>
<td>( V_{\text{conv}}^2 )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-( V_{dc}/3 )</td>
<td>( V_{dc}/\sqrt{3} )</td>
<td>( V_3 )</td>
<td>( V_{\text{conv}}^3 )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-( V_{dc}/3 )</td>
<td>-( V_{dc}/\sqrt{3} )</td>
<td>( V_5 )</td>
<td>( V_{\text{conv}}^5 )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( V_{dc}/3 )</td>
<td>-( V_{dc}/\sqrt{3} )</td>
<td>( V_6 )</td>
<td>( V_{\text{conv}}^6 )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( V_7 )</td>
<td>( V_{\text{conv}}^7 )</td>
</tr>
</tbody>
</table>

As mentioned previously, the MPC algorithm is aimed to achieve a minimum grid current error in the next sampling period. To this purpose, it is possible to firstly predict the \( dq \) grid current error vector components \( (\Delta i_{gd}^j[k+1])(j=0..6) \) and \( (\Delta i_{gq}^j[k+1])(j=0..6) \). These errors are defined as the difference between the reference grid current vector at the \( k^{th} \) sampling period and the predicted one at the next sampling period \((k+1)^{th}\) when the converter voltage vector \( (V_{\text{conv}}^j[k])(j=0..6) \) is applied. So, the grid current error vector components \( (\Delta i_{gd}^j[k+1])(j=0..6) \) and \( (\Delta i_{gq}^j[k+1])(j=0..6) \) are expressed as in (13) and (14).

\[
(\Delta i_{gd}^j[k+1] = i_{gd}^j[k] - i_{gd}^j[k+1]) \quad (j=0..6)
\]

(13)

\[
(\Delta i_{gq}^j[k+1] = i_{gq}^j[k] - i_{gq}^j[k+1]) \quad (j=0..6)
\]

(14)

Then, a cost function \( g \) is applied to the obtained grid current error components. This cost function is defined in equation (15).
The core of the predictive algorithm is the cost function minimization [13]-[14]. The optimal switching signals $S^{\text{opt}}_{(a,b,c)}$ that lead to the minimal cost function value $\text{Min}(g^0_{j=0..6})$ are selected and applied to the converter. Note that for the case of null voltage vector, the selected $S^{\text{opt}}_{(a,b,c)}$ switching signals depend on the applied switching signals during the previous sampling period so that the switching frequency is reduced.

A set of simulation studies developed under Matlab-Simulink environment tool were performed in order to show the effectiveness of the proposed MPC-based control for three-phase grid connected converter. It should be noted that during simulation procedure, the following operation conditions were used:

- The dc-link capacitor was initially charged to 500V.
- The dc-link voltage reference $V^*_{dc}$ was set to 600V.
- The $q$ axis grid current reference $i^*_{gq}$ was set equal to zero for unit power factor operation.

Also, it should be noted that simulation and experimental results, are presented only for the case where the grid connected converter works as three phases PWM boost rectifier. The achieved control is of course also available for the case when the converter works as a three-phase inverter.

Fig.3. Response to a step reference of 600V applied to the reference dc-link voltage and load step change at 0.8s (a) dc-link voltage response (b) Grid current response

Fig.3.a presents the response of the dc-link voltage to a step reference $V^*_{dc}$ equal to 600V. The start-up of the MPC control was done at no load. Then, at 0.8s, a step load change was applied by connecting a resistive load to the DC side. The obtained simulation results show that during steady state operation, the error of the dc-link voltage $V_{dc}$ becomes equal to zero. A small backward overshoot appears when the load is connected. Then, the PI
controller compensates the disturbance due to load connection. Fig.3.b shows the grid current $i_{ga}$ response. It can be noted that $i_{ga}$ increases when the load is connected and has a sinusoidal waveform. Fig.4 shows the waveform of the grid current $i_{ga}$ with regard to the grid voltage $V_{ga}$ during steady state operation. As expected, a unit power factor operation was achieved with a sinusoidal grid current absorption. The obtained THD for the grid current $i_{ga}$ is equal to 7.8%.

![Fig.4. Grid voltage $V_{ga}$ and current $i_{ga}$ waveforms during steady state operation](image)

Based on the simulation results of the MPC algorithm and the results presented in [18], we can deduce a comparison with other standard control algorithms as detailed in Table 2.

<table>
<thead>
<tr>
<th>Criteria/ Control</th>
<th>DPC</th>
<th>DPC-SVM</th>
<th>MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching frequency</td>
<td>Variable</td>
<td>Constant</td>
<td>Variable</td>
</tr>
<tr>
<td>Current THD</td>
<td>10.8%</td>
<td>3.46%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Robustness</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

These results demonstrate that the considered MPC algorithm achieves lower current THD than the DPC algorithm and faster dynamic performances than the DPC-SVM algorithm. In addition, it is less sensitive to parameters uncertainties.

4. Experimental set-up

In order to illustrate the performances of the MPC algorithm, experiments were carried out on a prototyping platform based on STM32F4 microcontroller that controls a three-phase PWM boost rectifier. This prototyping platform is presented in Fig.5 and includes two main parts:

The first one is a power part, which is composed of:
- The grid source
- A 20 kVA three-phase converter.
- An autotransformer is used to vary the voltage peak magnitude of the grid voltages in the grid side converter.
- Three $L$ filters (50mH/20A) placed between the autotransformer and the converter.
- A dc-link capacitor (1100µF/800V).
- A variable resistive load connected to the dc-link capacitor.

The second one is a control part, which is composed of:
- Measurement board that provides current and voltage measurements.
- The STM32F4 microcontroller where the MPC-based algorithm was implemented. This microcontroller is based on Cortex-M4-ARM processor, which is associated to a Floating Point Unit (FPU) and works with system clock frequency equal to 168 MHz.
- An Interface board to adapt the voltage level between STM32F4 and the converter drivers.
- Host PC used to configure STM32F4 microcontroller.

During experimentation, the following operation conditions were taken considered:
- The dc-link capacitor was initially charged by acting on the autotransformer to 150V.
- The dc-link voltage reference $V_{dc}^*$ was set to 200V.

Fig. 6. presents the timing diagram of the implemented control. This diagram presents the computation cycles of the different modules during each sampling period, where $t_{PI}$ is the computation time of the PI controller and $T_{MPC}$ is the computation time of the predictive controller. By adding the A/D conversion time, the whole execution time $T_{ex}$ of the control architecture is equal to 114 $\mu$s. Taking into account the execution time $T_{ex}$ of the MPC-based algorithm, the used sampling period $T_s$ was set equal to 125 $\mu$s. It can be noted that the switching states $S_{(a,b,c)opt}$ are refreshed every sampling period. However, the application of the switching states is not instantaneously, but after an execution time $T_{ex}$. 
Fig. 6. Timing diagram of the MPC-based algorithm

<table>
<thead>
<tr>
<th>Module</th>
<th>Computation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A/D conversion</td>
<td>( t_{\text{A/D}} = 14.2 \mu s )</td>
</tr>
<tr>
<td>PI</td>
<td>( t_{\text{PI}} = 7.8 \mu s )</td>
</tr>
<tr>
<td>MPC</td>
<td>( t_{\text{MPC}} = 92 \mu s )</td>
</tr>
<tr>
<td>( T_{\text{ex}} )</td>
<td>( = t_{\text{A/D}} + t_{\text{PI}} + t_{\text{MPC}} = 114 \mu s )</td>
</tr>
</tbody>
</table>

The obtained experimental results for the MPC algorithm are presented on Fig. 7, Fig. 8 and Fig. 9. The experimental tests were achieved in three steps. In the first step, the switching signals of the three-phase converter were tied low and the load is disconnected. In this case, the converter works as a simple three-phase diode bridge rectifier. Then, in the second step, the switching states computed by the MPC-based algorithm were applied to the three-phase converter. Fig. 7 shows the response of the dc-link voltage \( V_{\text{dc}} \) after switching signals application. It can be noted that \( V_{\text{dc}} \) voltage reaches its reference with zero error during steady state operation.
Finally in the third step, a load is connected to the dc-link. Fig.8 shows the dc-link voltage $V_{dc}$ and the grid current $i_{ga}$ waveforms.

As depicted in this figure, the dc-link voltage $V_{dc}$ decreases slightly due to a sudden connection of the load, then the PI-based controller compensates the load disturbance and the $V_{dc}$ voltage remains equal to its reference $V_{dc}^*$. It can be noted also that the grid current increases with sinusoidal waveform after the load connection.

Fig.9 shows the waveform of the grid voltage $V_{ga}$ with regard to the grid current $i_{ga}$ and grid voltage position $\theta_{dq}$ during steady state operation. It can be noted that the three phase PWM boost rectifier operates at unity power factor with sinusoidal current absorption. The obtained THD value is equal to 12.10%.
5. Conclusion

A Model based Predictive Control (MPC) for three-phase grid-connected converter in wind power systems has been presented. The control was divided into two control loops: the external one control the dc-link voltage $V_{dc}$ and is based on a PI controller while the second one focuses on the control of the grid current and is based on the MPC algorithm. In each sampling period, the proposed strategy calculates the converter switching time, which minimizes the cost function defined as a sum of the absolute values of the current errors. The used control strategy is validated through simulation and experimental tests. The obtained results gave proof of the effectiveness and the performances of the implemented MPC based algorithm especially low harmonic distorted grid current waveforms.

APPENDIX

Table3: Parameters of Three-phases grid connected converter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>1100uF/800V</td>
<td>dc-link capacitor</td>
</tr>
<tr>
<td>$L_{g}$</td>
<td>50mH/20A</td>
<td>Filter inductance</td>
</tr>
<tr>
<td>$R_{g}$</td>
<td>3 Ω</td>
<td>Filter resistance</td>
</tr>
<tr>
<td>$R_{load}$</td>
<td>250 Ω</td>
<td>Load resistance</td>
</tr>
<tr>
<td>$T_s$</td>
<td>125μs</td>
<td>Sampling period</td>
</tr>
<tr>
<td>$T_{ex}$</td>
<td>114 μs</td>
<td>Execution time</td>
</tr>
</tbody>
</table>

Acknowledgment

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The notation used throughout the paper is stated below.

Nomenclature

$L$ Inductance  
$R$ Resistance  
$V_g$ Grid voltage
References


