This paper presents voltage assessment indices for unbalance radial feeder in smart grid application. These indices are voltage stability margin, voltage deviation index, voltage profile and voltage unbalance factor. Voltage assessments are based on unique two-bus π-network model having time varying composite voltage sensitive load for every 15 minutes. Each node of feeder have three different time varying load categories consumers (industrial, commercial and residential), which is more practical to actual feeder. Characteristics and composition of residential, industrial and commercial consumers are accounted through time varying static composite load models. Composite load modelling is combination of constant impedance, constant current and constant power. The load flow solution is based on two port network modeling, load modeling and forward- backward sweep technique. Developed indices were calculated for modified IEEE 37 node test feeder with consideration of 15 minutes characteristics time interval for load modeling to meet out requirement of smart grid implementation. Simulation results demonstrate the validity and effectiveness of proposed indices for smart grid applications.

Keywords: Stochastic load modeling; distribution system modeling; two-bus network pi model; voltage stability margin; voltage assessment.

1. Introduction

In a deregulated environment, proper voltage assessment is a challenging task for utilities. To evaluate feasibility of smart grid implementation, there is a need to identify voltage security margin, voltage deviation, voltage profile and voltage unbalance. Voltage instability is a major threat for security of power systems and has top priority indice among other voltage assessment indices as poor voltage stability margin leads the blackout of feeder.

The major cause of voltage instability is continuously increased demand, high penetration of renewable energy sources, high powers between several interconnected areas and economic constraints for renovation and augmentation of existing systems. This scenario puts an increasing threat to power system security and reliability. In order to prevent the occurrence of voltage collapse, a proper analysis of voltage stability is essential for successful operation and planning of the power system. Thus, voltage stability is a very crucial and hot point for both utilities and researchers. Several techniques have been proposed in the literature for predicting the voltage instability and collapse of the power system. A fast method to determine the voltage stability limit of power system was proposed by Haque [1]. Moghavvemi, et al. [2] proposed bus/line stability indices for reduced two-bus equivalent network using line receiving end power equation. Voltage stability [3] and security indicator [4] is developed using load flow study under steady state.
operation. Voltage instability has become a challenging problem with unbalanced multiphase distribution networks. Voltage stability of a system depends on load models, network topology and settings of reactive compensation devices. Distribution feeder comprises of loads like industrial, commercial and residential loads, which causes high resistance to reactance ratio. Thus power-flow solution must be able to convergence problems. Backward and/or forward sweeps methods usually have faster convergence and a reduced number of equations, in comparison with conventional methods, [2, 3–5]. Sumit Banerjee, et al. [5] proposed a reactive loading index to identify heavily loaded branch of radial distribution system.

S. Gunalan, et al. [6] has investigated the impact of different load models on voltage stability for the unbalanced radial distribution system. Yanling Wang et al.[7] has determined voltage static stability limit by considering the singularity of the Jacobin matrix for different load category consumers having different load compositions. Generally radial distribution feeders have three categories of consumers: residential, commercial and industrial respectively. Each category consumer has statics ZIP load models. Characteristics and composition of theses static load models are unpredictable. Nagarajuet, et al.[8] presents a line loadability index to assess voltage instability and the line loading margin of radial distribution system (RDS) with different realistic load models. This index enables online monitoring and predicts a proper measure to prevent the system from collapse. Jinquan Zhao, et al. [9] has carried out comprehensive review and evaluation of voltage stability. It is also concluded that for a real large power system on line voltage stability monitoring can are not suitable. Pei-Hwa Huang, Ta-Hsiu Tseng [10] has investigated the effect of load characteristics of system voltage stability analysis with the static ZIP load models. Static ZIP load models have three different characteristics i.e. constant power, constant current and constant impedance. In a practical system, load characteristics and composition of load models varies throughout the day. So, it became inevitable to analysis stochastic behaviors of feeder, which requires a more detailed customer load modeling with a daily time varying load profile at each node. Thus voltage stability assessment is one of the important factors that indicate the how far the present operating condition is from the system voltage collapse.

Researchers have paid very little attention to voltage stability assessment of unbalanced radial feeder with voltage depended and time varying load. The objective of this paper is to develop indices for voltage assessment of unbalance radial feeder with variable load pattern using two port parameters, which takes into account the detailed and extensive modeling necessary for the smart grid environment. The load flow solution is based on backward and forward sweeps method and has reduced number of equations, in comparison with conventional methods, [11, 12–14].Network topology is an important aspect in distribution networks studies. In this paper, node numbering method is utilized as reported in [15]-[16]. Stochastic voltage assessment is carried out on modified IEEE 37 node test feeder. This paper has incorporated following features.

- Composite voltage dependent load modeling with time variable pattern to meet smart meter measurement criterion.
- Stochastic voltage assessment indices are voltage stability margin, voltage deviation index, voltage profile and voltage unbalance factor.
2. Nomenclature

\[
\begin{bmatrix}
A_{eq} & B_{eq} \\
C_{eq} & D_{eq}
\end{bmatrix}
\]

ABCD parameters of the two-bus \( \pi \)-equivalent

\[S_{m}, P_{m}, Q_{m} \]

Apparent, real and reactive load at bus \( m \)

\[Z_{s_{eq}}, Y_{s_{eq}}\]

Apparent, real and reactive power input to bus \( m \)

\[WR_{abc}^{k}(h), WC_{abc}^{k}(h), WI_{abc}^{k}(h)\]

Equivalent series impedance, shunt admittance of two-bus \( \pi \)-equivalent network

\[WR_{abc}^{k}(h), WC_{abc}^{k}(h), WI_{abc}^{k}(h)\]

Relevant factors for active industrial, residential, and commercial load models at bus \( k \) during \( h \)

\[P_{abc}^{k}(h), Q_{abc}^{k}(h)\]

Relevant factors for reactive industrial, residential, and commercial load models at bus \( k \) during \( h \)

\[P_{abc}^{k}(h), Q_{abc}^{k}(h), P_{abc}(h), Q_{abc}(h)\]

Active and reactive power of node \( k \) at nominal voltage

\[Q_{abc}^{k}(h), Q_{abc}(h)\]

Active power for industrial, residential, and commercial load connected at bus \( k \) during \( h \)

\[Q_{abc}^{k}(h), Q_{abc}(h)\]

Reactive power for industrial, residential, and commercial load connected at bus \( k \) during \( h \)

\[C_{abc}^{k}(h), C_{abc}(h)\]

ZIP active load composition at bus \( k \) during \( h \)

\[D_{abc}^{k}(h), D_{abc}(h)\]

ZIP reactive load composition at bus \( k \) during \( h \)

\[V_{abc}^{k}(h), V_{abc}^{k}(h)\]

Nominal and three phase voltage at \( k \) node

\[[V_{abc}], [I_{abc}]\]

Three phase voltage and current at bus \( m \)

\[PC_{abc}^{t}, QC_{abc}^{t}\]

Transformer core losses

\[h, n_{t}\]

15 minutes characteristics time interval, Turn ratio of transformer

\[V_{(+abc)}^{h}(k), V_{(-abc)}^{h}(k), V_{abc}^{h}(k)\]

Positive, negative and zero sequence voltage at bus \( k \) during \( h \)

\[N, NL\]

Total node, total line Segment

\[T_{abc}^{dl}\]

Voltage unbalance factor

\[V_{abc}^{h}(k)\]

Phase wise voltage at bus \( k \) during \( h \)
3. Mathematical Formulations for Voltage Assessment

3.1. Voltage Stability Margin Indice

Voltage stability is defined as the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and/or after being subjected to a disturbance [1] and [2]. For smooth and reliable operation of feeder, it is always like to know the location of bus having voltage collapse. In this paper a developed voltage stability margin is used to detect the voltage instability in real-time scenarios of unbalance radial feeder by considering different composition and characteristics of static load at a regular 15 minute time. In establishing the analytical basis for the distribution line loadability, let us consider an equivalent two $\pi$ bus model for all series feeder components (Line, Transformer, and Voltage regulator) modeled as two-port network elements as shown below in figure.

![Two $\pi$ bus model](image)

Therefore, power balance equation for the two-bus equivalent network can be written as

$$
S_m = P_m + jQ_m = V_m I_{se}^* = S_{L_m} + S_{sh} + S_n
$$

(1)

Where

$$
S_{sc} = \left(V_m - V_n\right) I_{sc}^* , S_{sh} = V_m I_{shm}^* + V_n I_{shn}^*
$$

$$
I_{se}^* = \left(\frac{S_m - S_{L_m}}{V_m}\right) \cdot S_{sh} \left(\frac{V_m^*}{|V_m|^2 + |V_n|^2}\right) = IB_m + S_{sh} \left(\frac{V_m^*}{|V_m|^2 + |V_n|^2}\right)
$$

Parameters of equivalent two-bus power network can easily be determined as follows:

$$
Z_{seq} = \left(\frac{V_m - V_n}{I_{se}}\right), Y_{seq} = \frac{I_{shm}}{V_m} = \frac{I_{shn}}{V_n}
$$

The ABCD matrix of two-bus $\pi$-equivalent system is used relate voltages and currents at sending node in terms of voltages and currents at receiving node.

$$
\begin{bmatrix}
V_m \\
I_m
\end{bmatrix}
= 
\begin{bmatrix}
A_{eq} & B_{eq} \\
C_{eq} & D_{eq}
\end{bmatrix}
\begin{bmatrix}
V_n \\
I_n
\end{bmatrix}
$$

(2)
The ABCD parameters of the two-bus $\pi$-equivalent system are given by

\[
A_{eq} = D_{eq} = 1 + Y_{sh_{eq}} Z_{se_{eq}}, \quad B_{eq} = Z_{se_{eq}}, \quad C_{eq} = Y_{sh_{eq}} \left(2 + Y_{sh_{eq}} Z_{se_{eq}} \right)
\]

\[
A_{eq} = |A_{eq}| \angle \alpha, \quad B_{eq} = |B_{eq}| \angle \beta
\]

The complex power $S_n$ delivered to the receiving end is

\[
S_n = P_n + jQ_n = V_n r^* = V_n \left(\frac{V_m - A_{eq} V_n}{B_{eq}}\right)^*
\]

\[
S_n = |V_n| \delta_n \left(\frac{V_m}{B_{eq}} \delta_m - A_{eq} |V_n| \beta \right) = |V_n| \delta_n \left(\frac{V_m}{B_{eq}} \beta - A_{eq} |V_n| \alpha - \delta_n\right)
\]

\[
P_n = \frac{|V_n||V_m|}{B_{eq}} \cos(\delta_n + \beta - \delta_m) - \frac{A_{eq}|V_n|^2}{B_{eq}} \cos(\beta - \alpha)
\]

\[
Q_n = \frac{|V_n||V_m|}{B_{eq}} \sin(\delta_n + \beta - \delta_m) - \frac{A_{eq}|V_n|^2}{B_{eq}} \sin(\beta - \alpha)
\]

\[
\frac{\partial P_n}{\partial \delta_n} = \frac{|V_n||V_m|}{B_{eq}} \sin(\delta_n + \beta - \delta_m)
\]

\[
\frac{\partial P_n}{\partial V_n} = \frac{|V_m|}{B_{eq}} \cos(\delta_n + \beta - \delta_m) - 2\frac{|V_n||A_{eq}|}{B_{eq}} \cos(\beta - \alpha)
\]

\[
\frac{\partial Q_n}{\partial \delta_n} = \frac{|V_n||V_m|}{B_{eq}} \cos(\delta_n + \beta - \delta_m)
\]

\[
\frac{\partial Q_n}{\partial V_n} = \frac{|V_m|}{B_{eq}} \sin(\delta_n + \beta - \delta_m) - 2\frac{|V_n||A_{eq}|}{B_{eq}} \sin(\beta - \alpha)
\]

The Jacobian matrix $[22]$ of above power flow equation is given by

\[
\begin{bmatrix}
\frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_n} \\
\frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial V_n}
\end{bmatrix} = \begin{bmatrix}
-\frac{|V_n||V_m|^2}{B_{eq}^2} + 2\frac{|A_{eq}|}{B_{eq}} \frac{|V_n|^2}{B_{eq}} \cos(\delta_n - \alpha - \delta_n) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_n} \\
\frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial V_n}
\end{bmatrix} = \begin{bmatrix}
-\frac{|V_n||V_m|^2 + 2\frac{|A_{eq}|}{B_{eq}} |V_n|^2 \cos(\delta_n - \alpha - \delta_n)}
\end{bmatrix}
\]

The determinant of the Jacobian matrix $\Delta [J]$ is defined as voltage stability margin. The value of Jacobian matrix $\Delta [J]$ for each line can be computed by load flow calculations. The line with the minimal $\Delta [J]$ is the weakest line, and its receiving bus is the weakest bus. At critical point of voltage stability, the determinant of Jacobian matrix $\Delta [J]=0$. Two port parameter employed enables the fast assessment of voltage stability and so useful for the practical on-line monitoring of power systems. The complex power at load bus has the
different load relative compositions and critical characteristics that vary throughout the 24 hours.

3.2. Voltage Unbalance Indice

Unbalanced three-phase loads or no uniformly spread single-phase loads, time-varying operations will lead to voltage variation and unbalance at three-phase equipment terminals. The effect of voltage unbalance is quite severe and voltage unbalance indice [23] expressed as below.

\[
T_{do_{abc}}^h = \sqrt{\frac{1}{N} \sum_{k=1}^{N} \left( \frac{V_{oabc}^h(k)}{h} \right)^2 }, \quad T_{d2_{abc}}^h = \sqrt{\frac{1}{N} \sum_{k=1}^{N} \left( \frac{V_{-abc}^h(k)}{h} \right)^2 }
\]

\[
T_{d2_{abc}}^h = T_{do_{abc}}^h + T_{d4_{abc}}^h
\]

3.3. Voltage Deviation Indice

Voltage deviation indice [24] is defined for the purpose of observing the effectiveness of the voltage correction in the system. Voltage deviation is the difference between the nominal voltage and the actual voltage. Minimizing the voltage deviation at every node of the system can make the voltage profile of the network better. A voltage deviation indice (VDI) for each 15 minute interval is defined as normalized maximum voltage difference at node k.

\[
VDI_{abc}^h = \max \left[ \left( 1 - \frac{V_{abc}^h(k)}{VR} \right)^N \right]_{k=1}
\]

4. Modeling and Computation

4.1. Load Modeling

This paper has considered that each load point has a mix of time varying load for industrial, residential and commercial consumers in a random proportion. In this paper IEEE 37 node test feeder assumed to be supplying power to a delta connected load at each node having a mix of industrial, residential, and commercial type consumers.

\[
P_{abc}^h = WIP_{abc}^h \cdot P_{0_{abc}}^h \cdot Q_{I_{abc}}^h = WIQ_{abc}^h \cdot Q_{0_{abc}}^h
\]

\[
Q_{abc}^h = WCP_{abc}^h \cdot P_{0_{abc}}^h \cdot QC_{abc}^h = WCQ_{abc}^h \cdot Q_{0_{abc}}^h
\]

\[
PR_{abc}^h = WRF_{abc}^h \cdot P_{0_{abc}}^h \cdot QR_{abc}^h = WRO_{abc}^h \cdot Q_{0_{abc}}^h
\]

The spot load at each node k shared among industrial, residential and commercial consumers and participation of each category load is characterized by relevant factors. The specific value of aforesaid relevant factor is generated by normalization of normally distributed pseudorandom numbers at each 15 minutes characteristics time interval h such that following condition must be satisfied for all load buses.
Critical load characteristics for each type of consumers may express by means of the sum of constant impedance (Z), constant current (I) and constant power (P) load models. In this paper voltage dependency of active and reactive power consumption are modelled by three components: constant impedance (Z), a constant current (I) and a constant power (P) injections. The participation of each category load is characterized following equations.

\[ WRP_{abc}^k (h) + WCP_{abc}^k (h) + WIP_{abc}^k (h) = 1 \]  \hspace{1cm} (17)

\[ WRO_{abc}^k (h) + WCQ_{abc}^k (h) + WIQ_{abc}^k (h) = 1 \]  \hspace{1cm} (18)

ZIP load composition [26] for each load category consumers have consumer constant Impedance [Z], constant current [I], and constant power [P] components [19]. The calculation of load current is carried out as mentioned in W. H. Kersting [20]. ZIP active and reactive load compositions for each node \( k \) at each 15 minutes characteristics time interval \( h \) are characterized by following relevant factors.

\[ p_{abc}^k (h) = C_{1abc}^k (h) + C_{2abc}^k (h) \frac{V_{abc}^k (h) \cdot V_{abc}^k (h)}{V_{abc}^k (h)} + C_{3abc}^k (h) \frac{V_{abc}^k (h)}{V_{abc}^k (h)} \]  \hspace{1cm} (19)

\[ q_{abc}^k (h) = D_{1abc}^k (h) + D_{2abc}^k (h) \frac{V_{abc}^k (h) \cdot V_{abc}^k (h)}{V_{abc}^k (h)} + D_{3abc}^k (h) \frac{V_{abc}^k (h)}{V_{abc}^k (h)} \]  \hspace{1cm} (20)

ZIP load composition [26] for each load category consumers have consumer constant Impedance [Z], constant current [I], and constant power [P] components [19]. The calculation of load current is carried out as mentioned in W. H. Kersting [20]. ZIP active and reactive load compositions for each node \( k \) at each 15 minutes characteristics time interval \( h \) are characterized by following relevant factors.

\[ C_{1abc}^k (h) = 0.8*p_{abc}^k (h) + 0.6*p_{abc}^k (h) + 0.8*p_{abc}^k (h) \]  \hspace{1cm} (21)

\[ C_{2abc}^k (h) = 0.2*p_{abc}^k (h) + 0.4*p_{abc}^k (h) + 0.19*p_{abc}^k (h) \]  \hspace{1cm} (22)

\[ C_{3abc}^k (h) = 0*p_{abc}^k (h) + 0.19*p_{abc}^k (h) + 0.01*p_{abc}^k (h) \]  \hspace{1cm} (23)

\[ D_{1abc}^k (h) = 0.8*q_{abc}^k (h) + 0.6*q_{abc}^k (h) + 0.8*q_{abc}^k (h) \]  \hspace{1cm} (24)

\[ D_{2abc}^k (h) = 0.2*q_{abc}^k (h) + 0.4*q_{abc}^k (h) + 0.19*q_{abc}^k (h) \]  \hspace{1cm} (25)

\[ D_{3abc}^k (h) = 0*q_{abc}^k (h) + 0.19*q_{abc}^k (h) + 0.01*q_{abc}^k (h) \]  \hspace{1cm} (26)

Modeling load patterns of residential, commercial and industrial customer are carried out by using daily load profiles reported in [17]. To incorporate the complexity of load, these load profiles are fitted in MATLAB 7th order polynomial equation as shown on Fig. 2.
4.2. Load Flow Algorithm

The forward/backward sweep (FBS) methods[26] are used to compute load flow. Error and convergence criterion for source node are below.

\[
E_{abc} = \left| \begin{bmatrix} V_{abc} \end{bmatrix}_{\text{specified}} - \begin{bmatrix} V_{abc} \end{bmatrix}_{\text{calculated}} \right| 
\]

\[
\left| E_{abc} \right|_{\text{itr}+1} - \left| E_{abc} \right|_{\text{itr}} \leq \varepsilon
\]

If above convergence criterion is not met than change in apparent power at last nodes of main feeder, laterals and sub laterals are calculated using below equations.

\[
\begin{bmatrix}
\partial S_a(k) \\
\partial S_b(k) \\
\partial S_c(k)_{\text{itr}}
\end{bmatrix} = 
\begin{bmatrix}
V_a(k)I_a(k) - S_a(k) \\
V_b(k)I_b(k) - S_b(k) \\
V_c(k)I_c(k) - S_c(k)_{\text{itr}} 
\end{bmatrix}
\]

Update each last node k voltage using equation (29) for the next iteration

\[
\begin{bmatrix} V_{abc} \end{bmatrix}_{\text{itr}+1} = \begin{bmatrix} V_{abc} \end{bmatrix}_{\text{itr}} + \begin{bmatrix} \partial S_{abc} \end{bmatrix}_{\text{itr}} \ast \left( \frac{V_{\text{BASE}}}{P_{\text{BASE}}} \right) \ast \begin{bmatrix} V_{abc} \end{bmatrix}_{\text{itr}} - V_{\text{nominal}}_{\text{itr}}
\]

5. Simulation Results and Analysis

Modified IEEE 37 node test feeder [25] is used to study the impact of load characteristics and composition on voltage stability margin, voltage deviation, voltage profile and voltage unbalance factor. The proposed method is programmed in MATLAB on a PC Pentium IV, 2.8-GHz computer with 4 GB of RAM. For load flow base voltage and base MVA are chosen as 4.8 KV and 2.5 MVA respectively. Original test network is modified to introduce twenty-four hourly random load scenarios at each load node with ZIP characteristics and removal of voltage regulator.

![Fig. 3 Voltage stability margin of phase A](image-url)
Voltage stability margin for phase A, phase B and phase C are shown in figure 3, figure 4 and figure 5 respectively. It is observed from above figures that voltage stability margin for phase C is higher than phase A and phase B. Voltage stability margin for Phase A of branch 3 has negative value for whole day and considered as most critical branch from voltage stability point. The receiving point of this branch may consider as point of coupling for distributed generation placement. The branch can also use as tie switch for reconfiguration of feeder. Voltage stability margin for Phase B of branch 12 is always minimum during whole day and varies from 0.9358 to 0.9015 at 5 and 56 characteristics time interval, respectively. During whole day, minimum voltage stability margin for phase C occurs for branch 35 and 36. During whole day, branch 35 and 36 have minimum voltage stability margin for 1050 and 390 mintues respectively. The least value of voltage stability margin found 0.8902 for branch 35 at 48 characteristics time interval.
Voltage deviation profile of all buses for 24-hour are shown in figure 6 to figure 8. Phase A of bus 34 has always voltage deviation maximum during whole day as shown figure 7. Phase B of bus 33 have maximum value at 56 characteristics time interval as shown in figure 8. It observed that voltage deviation for phase A exceeds from its maximum value during 35 to 73 characteristics time interval. Figure 8 also reveal that voltage deviation for phase C occurred maximum for node 31 and 34 during whole day. Maximum voltage deviation for node 31 and 34 was found 0.7599 and 0.7445 respectively. These nodes are receiving node of branch 35 and 36. It is also observed that period for which maximum voltage deviation and voltage stability margin for phase C are same. The maximum value of voltage deviation found 0.7599 for node 31 at 48 characteristics time interval.
Phase voltage profile of all buses for 24-hour are shown in figure 9 to figure 11. It was found the instant at which voltage deviation attained maximum value, on same instant voltage profile have minimum value. Minimum phase voltage for phase A, phase B and phase C are on node 34, 33 and 31 respectively. These node can used for shunt compensation.

A general trend of deviation of voltage unbalance from their nominal values was observed along the feeders, especially with feeders supplying greater distances, loads, PV generation, and their unequal connection between phases. As per IEEE Standards on Power Quality the allowable limit for voltage unbalance is limited to 2% in low voltage and in medium voltage networks. We have also found voltage unbalance factor varies between 0.5536 and 0.8609.

6. Conclusion

To ensure stable, secure and consistent supply of voltage, proper voltage assessment for each feeder component is required. This paper can successfully address the problem of voltage assessment in term of voltage stability margin, voltage deviation, voltage profile and voltage unbalance for the unbalance radial distribution feeder having time varying realistic load. Practical application of the developed indices was tested on modified IEEE 37 test feeder and simulation results reveal the effectiveness and feasibility of the presented
indices for smart grid application. In load modeling, characteristics and composition of residential, industrial and commercial consumers are accounted through voltage dependent load for each fifteen minute characteristics time interval, which plays an important role in the planning and operation of distribution networks. Low computational time, storage of all data in vector form and connectivity matrixes has great potential to be used in on-line operation.

References