This paper presents a novel approach for optimal estimation of the non-saturated parameters of synchronous machines from the digitized samples of the sudden short-circuit armature current. The proposed technique uses the least error squares (LES) parameter estimation algorithm to identify these parameters. The proposed technique splits the armature current into three components, namely; the steady state armature current, transient current, and the subtransient current. The parameters of each component namely; \( X_d, X'_d, X''_d, T_d, T'_d \) and \( T''_d \) are determined. After identifying the parameters, the algorithm uses them to determine the digitized samples of the d.c. armature transient current, simply by subtracting the armature current from the armature current using these parameters. These samples are then used to identify the armature time constant \( T_a \) as well as the subtransient reactance \( X'_q \). The proposed algorithm is tested using simulated data. Effects of the critical parameters on the algorithm are also studied in this paper.

Keywords: Synchronous machines, parameter identification, short-circuit current..

Article history: Received 16 January 2014, Received in revised form 26 September 2014, Accepted 15 May 2015

1. Introduction

Estimation of the parameters of synchronous machines from short-circuit is the conventional technique for finding only the time constants \( T_d', T_d'', \ldots \), [1,2], which were often determined graphically. Many attempts have been made to modify this technique. Reference 1. Proposes an approach based on the back-solution programs to calculate the parameters of the synchronous machine from sudden short-circuit current. No approximations, such as neglecting the coupling between transient and subtransient period and neglecting the armature resistance, are necessary. Reference 2. Proposes a nonlinear procedure for estimating the operational impedance of a suitable order from a sudden-short circuit data. It assumes that the machine is operating in a quasi linear mode.

A common method of determining parameters of direct and quadrature axis models of synchronous machines is by so-called standstill frequency response test (SSFR) [3-7]. This method requires a sinusoidal source of power at sufficient levels and current over a range of frequencies varying typically from 0.001 Hz to 200 Hz [4]. Parameters obtained from this small-signal test must be modified to account for iron saturation. Furthermore, obtaining the machine parameters from SSFR tests requires some type of curve-fitting procedure [5]. Reference[ 3] proposes a d. c. flux decay test for extracting the synchronous machine parameters, with the machine stationary, direct current is passed through two phases and the decay of the current is observed following short-circuiting of the supply. Reference [4] applies a time-domain parametric identification procedure to determine the machines parameters. This method does not account for saturation. \( T_o \) avoid having derivatives of the measured data, a low-pass filter was introduced.
Reference [14] presents a systematic approach for identification of a three-phase salient-pole synchronous machine rated at 5kVA from standstill time-domain data. The machine time constant models and the equivalent circuit models are identified and their parameters are estimated. The initialization of the estimated parameters is achieved by the Laplace transformation of the recorded standstill time response data and the derivation of the well-known operational inductances. The estimation is performed using the Maximum Likelihood algorithm. Based on the best estimated equivalent circuit models, simulation studies using the measured on-line dynamic responses are performed to validate the identified machine models.

In Reference [15] the measurement results of a series of standstill frequency response tests, performed at different magnetization levels, are discussed. For each data set an individual model is estimated, which allows to see the variation of the different parameters as function of the saturation. Further, an estimator is presented which uses the different data sets to estimate one global model, including the field to armature turns ratio. Finally the expected error level of a more traditional saturated synchronous machine model is studied.

Reference [16] addresses equivalent circuit and magnetic saturation issues associated with synchronous machine modeling. In the proposed synchronous machine model, the rotor equivalent circuits are replaced by arbitrary linear networks. This allows for elimination of the equivalent circuit parameter identification procedure since the measured frequency response may be directly embedded into the model. Magnetic saturation is also represented in both the $\alpha$- and $\beta$-axis. The model is computationally efficient and suitable for dynamic time-domain power system studies.

Reference [17] consists of two parts. The first part suggests a special method of the machine modeling. This model depends on electrical equivalent circuit of the synchronous machine taking into account the existence of dampers and using only reactances and time constants as parameters. The Park’s framework is used. In the second part of this paper, a statistical technique for determining synchronous machine parameters is proposed. This method is based on only electrical quantities (currents and voltages). The method is very efficient and the obtained results suit strongly the expected results. The method allows not only to determine synchronous machine parameters but also to validate the model built on Matlab/SimulinkTM.

Reference [18] proposes using a novel line-to-line voltage perturbation as a technique for online measurement of synchronous machine parameters. The perturbation is created by a chopper circuit connected between two phases of the machine. Using this method, it is possible to obtain the full set of four complex small-signal impedances of the synchronous machine $d-q$ model over a wide frequency range. Typically, two chopper switching frequencies are needed to obtain one data point. However, it is shown herein that, due to the symmetry of the machine equations, only one chopper switching frequency is needed to obtain the information. A 3.7-kW machine system is simulated, and then constructed for validation of the impedance measurement technique. A genetic algorithm is then used to obtain IEEE standard model parameters from the $d-q$ impedances. The resulting parameters are shown to be similar to those obtained by a series of tests involving synchronous reactance measurements and a standstill frequency response.

Reference [19] uses Kalman filter to estimate synchronous machine parameters from noisy measurements of the short circuit current waveform. Fuzzy rule-based logic is used to tune-up measurement noise levels by adjusting their covariance matrix using short circuit measurements. The simulation results show the convergence of the estimated parameters using Kalman filter iterations.

Reference [20], the nonlinear model $H\infty$ identification of a synchronous generator is used. It has been modified to cover the nonlinearities of the system, such as saturation effects in synchronous generators. It has been tested first on a simulated seventh order nonlinear
model of the synchronous machine and then by experiments on a physical machine. Simulation and experimental results show that the proposed method can be used successfully for the identification of the parameters of a synchronous machine model. Reference [21] presents a technique using the decoupled property of the so-called hybrid model from a previous work for them along with the application of the solution of the linear control systems theory to derive time-variant analytical waveforms of the phase voltages and the field current following a generator tripping (load rejection) and an open stator field short-circuit tests in terms of the generator parameters. Time-variant outputs are organized into d- and q-axis estimators for the so-called cross-identification experience. The proposed technique is being successfully applied for the parameter identification of a 4-pole 1.5-kVA, 208-V, and 60-Hz saturated laboratory synchronous generator.

This paper presents a new application of the least error squares parameter estimation algorithm for determining the non-saturated parameters of synchronous machines. The proposed algorithm uses digitized samples of the sudden armature current, for a three phase sudden-short circuit applied to the terminals of the machine running at rated speed on no load. Effects of critical parameters on the algorithm are also studied.

2. Modelling of the Sudden-Short Circuit Current [7,9]

IEEE standard No. 115 [1983] [10] describes in detail the experimental procedure for short-circuit testing. The test consists of a three-phase sudden short circuit of the terminal of a synchronous machine rotating at nominal speed under no load conditions. The open circuit stator voltage can be chosen anywhere within the range authorized by the nominal machine specifications. To obtain the nonsaturated parameters, which leads inevitably to a linear model. It is recommended to chose a value somewhere between 0.1 and 0.4 times the nominal stator voltage. According to this standard, the armature phase current of the synchronous machine can be written as:

\[
i_a(t) = \sqrt{2}E_o \left\{ \frac{1}{X_d} - \left( \frac{1}{X'_d} - \frac{1}{X_d} \right) e^{\frac{t}{T_a}} + \left( \frac{1}{X''_d} - \frac{1}{X_d} \right) e^{\frac{-t}{T_a}} \right\} \cos(\omega_o t + \lambda) \]

\[-\frac{E_o}{\sqrt{2}} \left( \frac{1}{X_d} - \frac{1}{X_q} \right) e^{\frac{-t}{T_a'}} \cos \lambda - \frac{E_o}{\sqrt{2}} \left( \frac{1}{X''_d} - \frac{1}{X_d} \right) e^{\frac{-t}{T_a''}} \cos(2\omega_o t + \lambda) \]

(1)

where

- \( i_a(t) \) is the sudden short-circuit armature current of phase A,
- \( E_o \) is the no load induced armature voltage
- \( X_d \) is the direct-axis synchronous reactance,
- \( X'_d \) is the direct-axis transient reactance,
- \( X''_d \) is the direct-axis subtransient reactance,
- \( X_q \) is the quadrature-axis subtransient reactance,
- \( T'_d \) is the direct-axis subtransient short-circuit time constant,
- \( T_a \) is the armature time constant and finally,
\( \lambda \) is the angle between the axis of phase A and the direct-axis at the instant of short-circuit. In other words \( \lambda \) defines the point in the a.c. cycle at which the short-circuit occurs.

The following relations are also true for the synchronous machines

\[
T_{do}^a = \frac{X_{d}^a}{X_{d}^o} T_{d}^a \\
T_{do}^o = \frac{X_{d}^o}{X_{d}^o} T_{d}^o
\]

where \( T_{do}^a \) is the direct-axis transient open-circuit time constant and \( T_{do}^o \) is the direct-axis subtransient open-circuit time constant.

Equation (1) is a nonlinear equation in the saturated machine parameters, which can be written as

\[
i_a(X,t) = f(X,t)
\]

(4)

where \( X \) is an 8x1 parameter vector to be estimated and is given by

\[
X = \text{col}\left( X_d, X_q, X_{d}^a, X_{q}^a, T_d, T_{d}^a, T_{d}^o, \lambda \right)
\]

Equation (4) is a highly nonlinear equation in the parameter vector to be estimated. In the next section the problem is reformulated to overcome this nonlinearity.

### 3. Problem Formulation

Assume \( m \) samples of the short-circuit armature current are available for the short circited period, this period must be long enough to obtain the three periods of the short-circuit current, namely the subtransient, transient and steady periods.

#### 3.1 The steady-state current

At steady-state, the armature short-circuit current can be written as:

\[
i_{as} (t) = \sqrt{2} \frac{E_o}{X_d} \cos(\omega_o t + \lambda)
\]

(6)

In (6) there is neither transient nor subtransient or d.c. current. Equation (6) can be written as

\[
i_{as} (t) = \frac{\cos \lambda}{X_d} \left( \sqrt{2} E_o \cos \omega_o t \right) - \frac{\sin \lambda}{X_d} \left( \sqrt{2} E_o \sin \omega_o t \right)
\]

(7)

Define the parameters \( X_1 \) and \( X_2 \) as:

\[
X_1 = \frac{\cos \lambda}{X_d}
\]

(8)

\[
X_2 = \frac{\sin \lambda}{X_d}
\]

(9)
and the time-dependent functions $h_1(t)$ and $h_2(t)$ as:

$$h_1(t) = \sqrt{2}E_o \cos(\omega_0 t)$$  \hspace{1cm} (10)

$$h_2(t) = -\sqrt{2}E_o \sin(\omega_0 t)$$  \hspace{1cm} (11)

Then, equation (7) can be rewritten as:

$$i_{ass}(t) = X_1 h_1(t) + X_2 h_2(t)$$  \hspace{1cm} (12)

If the armature current, in the steady state period, is sampled at a preselected rate, say $\Delta T$, then $m$ samples would be obtained at $t_1, t_1 + \Delta T, \ldots, t_1 + (m-1)\Delta T$. Then (12) become

$$\begin{bmatrix}
  i_{ass}(t_1) \\
  i_{ass}(t_2) \\
  \vdots \\
  i_{ass}(t_m)
\end{bmatrix} =
\begin{bmatrix}
  h_1(t_1) & h_2(t_1) \\
  h_1(t_2) & h_2(t_2) \\
  \vdots & \vdots \\
  h_1(t_m) & h_2(t_m)
\end{bmatrix}
\begin{bmatrix}
  X_1 \\
  X_2
\end{bmatrix}$$  \hspace{1cm} (13)

In vector form, equation (13) becomes

$$L_{ass} = H X + \xi$$  \hspace{1cm} (14)

where $L_{ass}$ is $m \times 1$ samples vector of the steady state current, $H$ is $m \times 2$ matrix of measurement, $X$ is $2 \times 1$ parameters vector to be estimated, and $\xi$ is $m \times 1$ errors vector to be minimized. If $m > 2$, equation (14) becomes overdetermined set of equations. The solution to (14) based on least error squares is:

$$X^* = \left[H^T H\right]^{-1} H^T L_{ass}(t)$$  \hspace{1cm} (15)

Having identified the parameters vector $X^*$, then $X_d$ and $\lambda$ can be computed as:

$$X_d = \left[X_1^2 + X_2^2\right]^{1/2}$$  \hspace{1cm} (16)

and

$$\lambda = \tan^{-1}\left(\frac{X_2}{X_1}\right)$$  \hspace{1cm} (17)

Now, the parameters $X_d$ and $\lambda$ are identified using the steady state short-circuit armature current.
3.2 Transient period

The armature short-circuit current in the transient period can be written as:

\[ i_{an} (t) = \sqrt{2} E_o \left[ \frac{1}{X_d'} - \frac{1}{X_d} \right] e^{\frac{-t}{T_d}} \cos(\omega_o t + \lambda) \]  

(18)

Note that the values of \( X_d \) and \( \lambda \) are those identified in the previous section. Using the first four terms of Taylor’s series expansion for the exponential term \( e^{\frac{-t}{T_d}} \) and define \( \frac{1}{(X_d')_T} = \left[ \frac{1}{X_d'} - \frac{1}{X_d} \right] \)

we obtain:

\[ i_{an}(t) = \sqrt{2} E_o \left[ \frac{1}{X_d'} \right] \cos(\omega_o t + \lambda) - \sqrt{2} E_o \left[ \frac{1}{(X_d')_T T_d} \right] \cos(\omega_o t + \lambda) \]

+ \[ \frac{0.5 \sqrt{2} E_o t^2}{(X_d')_T T_d^2} \cos(\omega_o t + \lambda) - \frac{\sqrt{2} E_o t^3}{6 (X_d')_T T_d^3} \cos(\omega_o t + \lambda) \]  

(19)

Define the parameters

\[ y_1 = \left( \frac{1}{(X_d')_T} \right) = \left[ \frac{1}{X_d'} - \frac{1}{X_d} \right] \]

\[ y_2 = \frac{1}{T_d (X_d')_T} \]

\[ y_3 = \frac{1}{T_d^2 (X_d')_T} \]

\[ y_4 = \frac{1}{T_d^3 (X_d')_T} \]

(20)

and the time dependent functions

\[ b_1(t) = \sqrt{2} E_o \cos(\omega_o t + \lambda) \]

\[ b_2(t) = -\sqrt{2} E_o t \cos(\omega_o t + \lambda) \]

\[ b_3(t) = 0.5 \sqrt{2} E_o t^2 \cos(\omega_o t + \lambda) \]

\[ b_4(t) = -\frac{\sqrt{2}}{6} E_o t^3 \cos(\omega_o t + \lambda) \]  

(21)

Then, equation (19) becomes
\[ i_{atr}(t) = b_1(t)y_1 + b_2(t)y_2 + b_3(t)y_3 + b_4(t)y_4 \]

For \( m_1 \) samples of the short-circuit armature current available. Then (22) can be written as:

\[
\begin{bmatrix}
  i_{atr}(t_1) \\
  i_{atr}(t_2) \\
  \vdots \\
  i_{atr}(t_{m_1})
\end{bmatrix}
= 
\begin{bmatrix}
  b_1(t_1) & b_2(t_1) & b_3(t_1) & b_4(t_1) \\
  b_1(t_2) & b_2(t_2) & b_3(t_2) & b_4(t_2) \\
  \vdots & \vdots & \vdots & \vdots \\
  b_1(t_{m_1}) & b_2(t_{m_1}) & b_3(t_{m_1}) & b_4(t_{m_1})
\end{bmatrix}
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_{m_1}
\end{bmatrix}
\]

(23)

which can be written in vector form as:

\[ \mathbf{I}_{atr} = \mathbf{B} \mathbf{Y} + \psi \]

(24)

where \( \mathbf{I}_{atr} \) is \( m_1 \times 1 \) transient and the steady state samples, \( \mathbf{B} \) is \( m_1 \times 4 \) matrix of measurements, \( \mathbf{Y} \) is \( 4 \times 1 \) parameters vector to be estimated and \( \psi \) (is \( m_1 \times 1 \) errors vector to be minimized. The solution to (24) based on least error squares is given by:

\[ \mathbf{Y}^* = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{I}_{atr} \]

(25)

Having identified the parameters vector \( \mathbf{Y}^* \), then the transient parameters \( X_d^* \) and \( T_d^* \) can be computed as:

\[ X_d^* = \frac{1}{y_1 + \frac{1}{X_d}} \]

(26)

\[ T_d^* = \frac{y_1}{y_2} \]

or

\[ T_d^* = \left[ \frac{y_1}{y_3} \right]^{1/2} \]

(27)

or

\[ T_d^* = \left[ \frac{y_1}{y_4} \right]^{1/3} \]
3.3 The subtransient Current

The first few cycles of the armature short-circuit current present the subtransient current superimposed on this current is the d.c component of the armature current. This can be expressed as:

\[ i_{asr}(t) = \sqrt{2}E_o \left[ \frac{1}{X_d^*} - \frac{1}{X_d'} \right] e^{-\frac{t}{T_d}} \cos(\omega_ot + \lambda) + i_{dc} \]  

(28)

From the estimation point of view, \( i_{dc} \) can be considered as a noise superimposed on the subtransient current. Following the steps explained earlier, and replace the exponential term \( e^{-\frac{t}{T_d}} \) by the first four terms of Taylor's series, and define

\[ \frac{1}{(X_d^*)_T} = \left[ \frac{1}{X_d^*} - \frac{1}{X_d'} \right], \text{ we obtain} \]

\[ i_{asr}(t) = \left[ \frac{\sqrt{2}E_o}{(X_d^*)_T} \right] \cos(\omega_ot + \lambda) - \left[ \frac{\sqrt{2}E_o t}{T_d'(X_d^*)_T} \right] \cos(\omega_ot + \lambda) + \]

\[ \frac{0.5\sqrt{2}E_o t^2}{T_d''(X_d^*)_T} \cos(\omega_ot + \lambda) - \frac{\sqrt{2}E_o t^3}{6T_d'''(X_d^*)_T} \cos(\omega_ot + \lambda) \]

(29)

Define the following parameters:

\[ \theta_1 = \frac{1}{(X_d^*)_T} = \left[ \frac{1}{X_d^*} - \frac{1}{X_d'} \right] \]

(30)

\[ \theta_2 = \frac{1}{T_d'(X_d^*)_T} \]

(31)

\[ \theta_3 = \frac{1}{(T_d'')^2(X_d^*)_T} \]

(32)

\[ \theta_4 = \frac{1}{(T_d''')^3(X_d^*)_T} \]

(33)

Then, equation (19) becomes

\[ i_{asr}(t) = b_1(t)\theta_1 + b_2(t)\theta_2 + b_3(t)\theta_3 + b_4(t)\theta_4 \]
if \( m_2 \) samples of the subtransient armature short-circuit current sampled at \( t_1, t_2 = t_1 + \Delta T, \ldots, t_1 + (m_2-1) \Delta T \) are available, then (34) can be written as:

\[
\begin{bmatrix}
  i_{astr}(t_1) \\
  i_{astr}(t_2) \\
  \vdots \\
  i_{astr}(t_{m_2})
\end{bmatrix} = 
\begin{bmatrix}
  b_1(t_1) & b_2(t_1) & b_3(t_1) & b_4(t_1) \\
  b_1(t_2) & b_2(t_2) & b_3(t_2) & b_4(t_2) \\
  \vdots & \vdots & \vdots & \vdots \\
  b_1(t_{m_2}) & b_2(t_{m_2}) & b_3(t_{m_2}) & b_4(t_{m_2})
\end{bmatrix}
\begin{bmatrix}
  \theta_1 \\
  \theta_2 \\
  \theta_3 \\
  \theta_4
\end{bmatrix}
\]

(36)

In vector form, (35) can be written as

\[
I_{astr} = B\theta + \zeta
\]

where \( I_{astr} \) is \( m_2 \times 1 \) current samples available in the period under study, \( B \) is \( m_2 \times 4 \) measurement matrix, \( \theta \) is \( 4 \times 1 \) parameters vector to be estimated and \( \zeta \) is \( m_2 \times 1 \) errors vector to be minimized. This errors vector contains the d.c current. The solution to (36) in the least error squares sense is:

\[
\theta^* = [B^TB]^{-1}B^TI_{astr}
\]

(37)

Having identified the parameters vector \( \theta^* \), then the subtransient period parameters \( X_d^* \) and \( T_d^* \) can be computed as:

\[
X_d^* = \frac{1}{\theta_1 + \frac{1}{X_d'}}
\]

(38)

\[
T_d^* = \frac{\theta_1}{\theta_2} \quad \text{or}
\]

(39a)

\[
T_{d}^{*2} = \begin{bmatrix} \theta_1 \\ \theta_3 \end{bmatrix} \quad \text{or}
\]

(39b)

\[
T_{d}^{*3} = \begin{bmatrix} \theta_1 \\ \theta_4 \end{bmatrix}
\]

(39c)
3.4. The D.C. current

Having identified \( X_d^*, X_q^*, T_d^* \) and \( T_a^* \) then the d.c. component of the short circuit armature current can be obtained as:

\[
i_a(t) - i_{sts}(t) = -E_O \left[ \frac{1}{X_d^*} + \frac{1}{X_q^*} \right] e^{\frac{t}{T_a}} (0.5\sqrt{2} \cos \lambda) \\
+ E_O \left[ \frac{1}{X_d^*} - \frac{1}{X_q^*} \right] e^{\frac{t}{T_a}} [0.5\sqrt{2} \cos (2\omega_o t + \lambda)]
\]  

(40)

where \( i_{sts}(t) \) is the a.c subtransient, transient and steady state armature current calculated at time \( t \) using the parameters estimated earlier in the previous sections. The left hand side of equation (40) is the d.c armature current.

\[
I_{dc}(t) = -E_O \left[ \frac{1}{X_d^*} + \frac{1}{X_q^*} \right] \left[ 1 - \frac{t}{T_a} + \frac{t^2}{2T_a^2} \right] (0.5\sqrt{2} \cos \lambda) \\
+ E_O \left[ \frac{1}{X_d^*} - \frac{1}{X_q^*} \right] \left[ 1 - \frac{t}{T_a} + \frac{t^2}{2T_a^2} \right] [0.5\sqrt{2} \cos (2\omega_o t + \lambda)]
\]  

(41)

\[
\theta_s = \left[ \frac{1}{X_d^*} + \frac{1}{X_q^*} \right]
\]  

(42)

\[
\theta_d = \left[ \frac{1}{X_d^*} - \frac{1}{X_q^*} \right]
\]  

(43)

and define the following parameters

\[
\phi_1 = \theta_s \quad ; \quad \phi_4 = \theta_d \\
\phi_2 = \theta_s T_a \quad ; \quad \phi_5 = \theta_d T_a \\
\phi_3 = \theta_s T_a^2 \quad ; \quad \phi_6 = \theta_d T_a^2
\]  

(44)

\[
c_1(t) = -0.5 \sqrt{2} E_o \cos \lambda \quad ; \quad c_4(t) = -0.5 \sqrt{2} E_o \cos (2\omega_o t + \lambda) \\
c_2(t) = 0.5 \sqrt{2} \omega_o E_o \cos \lambda \quad ; \quad c_5(t) = 0.5 \sqrt{2} \omega_o E_o \cos (2\omega_o t + \lambda) \\
c_3(t) = -0.25 \sqrt{2} \omega_o E_o \cos \lambda \quad ; \quad c_6(t) = -0.25 \sqrt{2} \omega_o E_o \cos (2\omega_o t + \lambda)
\]  

(45)

Then equation (41) becomes

\[
i_{dc}(t) = \phi_1 c_1(t) + \phi_2 c_2(t) + \phi_3 c_3(t) + \phi_4 c_4(t) + \phi_5 c_5(t) + \phi_6 c_6(t)
\]  

(46)

If \( M \) samples of the d.c current are available at \( t_1, t_1 + \Delta T, ..., t_1 + (M-1)\Delta T \). Then equation (46) can be written as:
which can be written in vector form as:

\[
I_{\text{dc}} = C \phi + \nu
\]  

(47)

The solution to (48) in the least error squares sense is

\[
\phi^* = (C^T C)^{-1} C^T I_{\text{dc}}
\]  

(49)

Having identified the parameters vector \(\phi^*\), then the parameters \(X_{\text{d}}^\prime, X_{\text{q}}^\prime\) and \(T_a\) can be computed as

\[
X_{\text{d}}^\prime = 2/(\phi_1 + \phi_4)
\]  

(50)

\[
X_{\text{q}}^\prime = 2/(\phi_1 - \phi_4)
\]  

(51)

\[
T_a = \phi_1 / \phi_2
\]

or

\[
T_a = \phi_4 / \phi_5
\]  

(52)

4. Testing the Algorithm

The proposed algorithm is used to estimate the parameters of a large synchronous machine rated 589 MVA, 22kV with 0.85 power factor lagging at full load. The data for the machine is given as:

\[
X_{\text{d}} = 2.36 \text{ p.u}, \quad X_{\text{d}}' = 0.287 \text{ p.u}, \quad X_{\text{d}}'' = 0.211 \text{ p.u}, \quad X_{\text{q}} = 0.225 \text{ p.u}, \quad T_{\text{d}} = 0.925 \text{ s},
\]

\[
T_{\text{d}}' = 0.0382 \text{ s}, \quad T_{\text{d}}'' = 7.606 \text{ s}, \quad T_{\text{q}}' = 0.052 \text{ s}, \quad T_a = 0, \quad \lambda = 0,
\]

These parameters together with equation (1) are used to generate the samples necessary for each period. In the next sections, we offer the results obtained for each current period, with the effects of the critical parameters, such as the sampling frequency, the number of samples and the location of the initial sampling time, on the accuracy of the estimation.

4.1 Estimation of \(X_{\text{d}}\) and \(\lambda\) (Steady state period)

In this period only the steady state short-circuit current exists, a number of samples equals 10 is used to estimate \(X_{\text{d}}\) and (with a sampling frequency of 1000 Hz (\(\Delta T = 1\) ms), data window size used in this case is 1/2 cycle. The initial sampling time is located after 12 s (600 cycle) from the initial short-circuit time, to reach the steady-state period. The results obtained are:

\[
X_{\text{d}} = 2.359987, \quad \lambda = -0.00147\text{o}
\]

comparing this results with the actual values given above, we conclude that the proposed algorithm produces very accurate estimates for these two parameters.
4.1.1 Effects of sampling frequency

Effects of sampling frequency on the estimated parameters are studied, where we change the sampling frequency from 500 Hz to 2500 Hz with a number of samples equals 10 and the initial sampling time is located at 12s (600 cycle). Table 1 gives the results obtained.

Examining this table reveals the following:

♦ The sampling frequency variation has a little effect on the estimated parameters, especially for sampling frequency less than 2000 Hz, and the maximum error obtained during this range is 1.09 %, but greater than this sampling frequency, the error in the estimated parameter increases as the sampling frequency increases. Indeed from the engineering point of view, the error is still acceptable.

♦ The most suitable sampling frequency, which produces a zero error is 500 Hz, but the other sampling frequencies are also suitable.

♦ The estimated $\lambda$ is almost zero at all the sampling frequencies with almost zero errors.

Table 1. Effects of sampling frequency on the estimates $X_d, \lambda$

<table>
<thead>
<tr>
<th>Sampling Frequency (Hz)</th>
<th>$X_d$ (p.u)</th>
<th>% Error</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>2.36</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>750</td>
<td>2.3599</td>
<td>0.00424</td>
<td>0.0000</td>
</tr>
<tr>
<td>1000</td>
<td>2.35999</td>
<td>0.00424</td>
<td>-0.0015</td>
</tr>
<tr>
<td>1250</td>
<td>2.35947</td>
<td>0.02230</td>
<td>0.00</td>
</tr>
<tr>
<td>1500</td>
<td>2.35700</td>
<td>0.1700</td>
<td>0.00013</td>
</tr>
<tr>
<td>1750</td>
<td>2.34980</td>
<td>0.4320</td>
<td>0.0015</td>
</tr>
<tr>
<td>2000</td>
<td>2.33432</td>
<td>1.0900</td>
<td>0.0027</td>
</tr>
<tr>
<td>2250</td>
<td>2.30811</td>
<td>2.2000</td>
<td>0.0057</td>
</tr>
<tr>
<td>2500</td>
<td>2.26863</td>
<td>3.9000</td>
<td>0.0138</td>
</tr>
</tbody>
</table>

Figure 1. Variation of $X_d$ with Sampling Frequency
4.1.2. Effects of number of samples

The effects of number of samples on the estimated parameters are studied in this section. Here we change the number of samples from 5 samples to 15 samples. Note that the minimum number of samples necessary to estimate $X_d$ and $\lambda$ is two samples, but using two samples may produce poor estimates, since we force the error to be zero. The sampling frequency used in this study is 1000 Hz with initial sampling time located at 12s. It has been shown through extensive rubs that the variation of number of samples has no effect on the estimated parameters $X_d$ and $\lambda$ and the proposed algorithm produces the exact values of these parameters.

4.1.3. Location of the initial sampling time

The effects of the location of the initial sampling time on the estimated parameters are studied in this section. Table 2 gives the results obtained when the sampling frequency is 1000 Hz and the number of samples is 10 samples.

<table>
<thead>
<tr>
<th>$T_0$ (s)</th>
<th>$X_d$ (p.u)</th>
<th>% Error</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.2895</td>
<td>45.4</td>
<td>0.07612</td>
</tr>
<tr>
<td>4</td>
<td>3.1542</td>
<td>8.7</td>
<td>0.01486</td>
</tr>
<tr>
<td>6</td>
<td>2.3343</td>
<td>1.1</td>
<td>1.964</td>
</tr>
<tr>
<td>8</td>
<td>2.357</td>
<td>0.13</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>2.3597</td>
<td>0.013</td>
<td>-0.0013</td>
</tr>
<tr>
<td>12</td>
<td>2.3599</td>
<td>0.00424</td>
<td>0.00135</td>
</tr>
</tbody>
</table>
Examining this table reveals the following:

- The location of the initial sampling time has a great effect on the estimation of $X_d$ and has no appreciable effect on the estimation of $\lambda$.
- As the location of the initial sampling time increases, a good estimate is obtained. Indeed, this is true, as the initial sampling time increases, the armature short-circuit current reaches the steady state value and there is neither transient or subtransient current in the wave.
- The most suitable location for the initial sampling time is after 12s (the error at this location is almost zero).

![Figure 3 Variation of $X_d$ with $T_o$ at $m=10$, Sampling frequency =1000 (Hz)](image)

### 4.2. Estimation of $X'_d$ and $T'_d$ (transient period)

The parameters of the transient period, which occurs after completion of the subtransient period, are determined in this section, where a number of samples equals 10 is used with a sampling frequency of 1000 Hz and the initial sampling time is located at 0.2s (=200 ms = 10 cycles on 50 Hz wave). The parameters are found to be:

$$X'_d = 0.2506 \text{ p.u, and } T'_d = 0.9674 \text{ s}$$

The error in these estimates is calculated using the actual values mentioned above, and has found to be

$$\varepsilon_{X'_d} = 12.7 \% \; \text{ and } \; \varepsilon_{T'_d} = -4.6 \%$$

as we see, the error in $X'_d$ is a little bit high, but the error in $T'_d$ is within the necessary accuracy. However, both estimates are still within the required accuracy for such type of estimates.

#### 4.2.1 Effects of the sampling frequency

The effects of sampling frequency on the estimated parameters at different initial sampling times are studied in this section. Table 3 gives the results obtained when the sampling frequency varies from 100 Hz to 2250 Hz at different initial sampling times.
starting from $T_o = 0.10s$ (5 cycle) to 0.3s (15 cycle), and when the number of samples is $m = 10$. Furthermore, table 4 gives the errors for the estimated parameters at these sampling frequencies. Examining these tables reveals the following:

- The sampling frequency has a great effect on the estimated parameters.
- The initial time has also a great effect on the estimated parameters, for example at $T_o = 0.15s$, the errors in the estimated parameters, at different sampling frequency, are too high. As the initial sampling time increases, these errors are reduced greatly.
- The most acceptable sampling frequency and initial sampling time are 1000 Hz and 0.2 s at which we obtain a minimum error, for both estimates. Meanwhile, at a 100 Hz sampling frequency and initial sampling time equals 0.25 s a good estimate is obtained as well, for the parameters $X'_{d}$ and $T'_{d}$ (the error in $X'_{d}$ is -12.2%, while the error in $T'_{d}$ is -0.1%).

### Table 3. Effects of sampling frequency, at different initial sampling time, on the estimates of $X'_{d}$ and $T'_{d}$

<table>
<thead>
<tr>
<th>$T_o$(s)</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(Hz)</td>
<td>$X'_{d}$</td>
<td>$T'_{d}$</td>
<td>$X'_{d}$</td>
<td>$T'_{d}$</td>
<td>$X'_{d}$</td>
</tr>
<tr>
<td>100</td>
<td>0.2184</td>
<td>0.3055</td>
<td>0.2369</td>
<td>0.5383</td>
<td>0.2451</td>
</tr>
<tr>
<td>250</td>
<td>0.2152</td>
<td>0.2552</td>
<td>0.2287</td>
<td>0.4105</td>
<td>0.2246</td>
</tr>
<tr>
<td>500</td>
<td>0.2087</td>
<td>0.2285</td>
<td>0.2288</td>
<td>0.4113</td>
<td>0.2480</td>
</tr>
<tr>
<td>750</td>
<td>0.2144</td>
<td>0.2734</td>
<td>0.2375</td>
<td>0.5822</td>
<td>0.2474</td>
</tr>
<tr>
<td>1000</td>
<td>0.2042</td>
<td>0.2011</td>
<td>0.2390</td>
<td>0.6311</td>
<td>0.2506</td>
</tr>
<tr>
<td>1250</td>
<td>0.2062</td>
<td>0.2123</td>
<td>0.2387</td>
<td>0.6227</td>
<td>0.2540</td>
</tr>
<tr>
<td>1500</td>
<td>0.2059</td>
<td>0.2106</td>
<td>0.2383</td>
<td>0.6140</td>
<td>0.2459</td>
</tr>
<tr>
<td>1750</td>
<td>0.2074</td>
<td>0.2193</td>
<td>0.2381</td>
<td>0.6150</td>
<td>0.2502</td>
</tr>
<tr>
<td>2000</td>
<td>0.2081</td>
<td>0.2241</td>
<td>0.2380</td>
<td>0.6057</td>
<td>0.2511</td>
</tr>
<tr>
<td>2250</td>
<td>0.2070</td>
<td>0.2170</td>
<td>0.2378</td>
<td>0.018</td>
<td>0.2673</td>
</tr>
</tbody>
</table>

### Table 4. Percentage errors on the estimated $X'_{d}$ and $T'_{d}$ at different sampling frequency (m=10)

<table>
<thead>
<tr>
<th>$T_o$(s)</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(Hz)</td>
<td>$\varepsilon_{X'_{d}}$</td>
<td>$\varepsilon_{T'_{d}}$</td>
<td>$\varepsilon_{X'_{d}}$</td>
<td>$\varepsilon_{T'_{d}}$</td>
<td>$\varepsilon_{X'_{d}}$</td>
</tr>
<tr>
<td>100</td>
<td>-24.0</td>
<td>-66.9</td>
<td>-17.5</td>
<td>-41.8</td>
<td>-14.6</td>
</tr>
<tr>
<td>250</td>
<td>-26</td>
<td>-72.4</td>
<td>-20.3</td>
<td>-55.6</td>
<td>-15.5</td>
</tr>
<tr>
<td>500</td>
<td>-27</td>
<td>-75.3</td>
<td>-20.3</td>
<td>-55.5</td>
<td>-13.6</td>
</tr>
<tr>
<td>750</td>
<td>-25</td>
<td>-70.4</td>
<td>-17.2</td>
<td>-37.1</td>
<td>-13.8</td>
</tr>
<tr>
<td>1000</td>
<td>-29</td>
<td>-78.3</td>
<td>-16.7</td>
<td>-31.8</td>
<td>-12.7</td>
</tr>
<tr>
<td>1250</td>
<td>-28.1</td>
<td>-77.1</td>
<td>-16.8</td>
<td>-32.7</td>
<td>-11.5</td>
</tr>
<tr>
<td>1500</td>
<td>-28.3</td>
<td>-77.2</td>
<td>-16.96</td>
<td>-33.6</td>
<td>-14.3</td>
</tr>
<tr>
<td>1750</td>
<td>-27.7</td>
<td>-76.3</td>
<td>-17.0</td>
<td>-34.1</td>
<td>-12.8</td>
</tr>
<tr>
<td>2000</td>
<td>-27.7</td>
<td>-75.8</td>
<td>-17.10</td>
<td>-34.5</td>
<td>-12.5</td>
</tr>
<tr>
<td>2250</td>
<td>-27.9</td>
<td>-76.5</td>
<td>-17.1</td>
<td>-34.9</td>
<td>-6.9</td>
</tr>
</tbody>
</table>

244
From the above discussions, the choice of sampling frequency and the initial sampling time depends upon the experience of the analyst, as well as the size of the machine under investigation. We recommend that serious consideration be given to the shape of the short-circuit armature to discriminate between different periods of the current.

4.2.2. Effect of number of samples

The effects of the number of samples on the estimated parameters are studied in this section. Here the number of samples is changed from 5 samples to 15 samples, with the initial sampling time located at different initial time starting from 0.2 s to 0.3 s, while the sampling frequency is 1000 Hz. Tables 5 and 6 give the results obtained for $X_d$ and $T_d$ and the errors in these estimates.

Table 5. Effects of the number of samples at different initial sampling time on the estimates of $X_d$ and $T_d$, $Fs = 100$ Hz.

<table>
<thead>
<tr>
<th>$T_d$(s)</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td># of samples</td>
<td>$X_d$</td>
<td>$T_d$</td>
<td>$X_d$</td>
</tr>
<tr>
<td>5</td>
<td>0.2696</td>
<td>-2.647</td>
<td>0.2533</td>
</tr>
<tr>
<td>10</td>
<td>0.2506</td>
<td>0.9674</td>
<td>0.2533</td>
</tr>
<tr>
<td>15</td>
<td>0.2478</td>
<td>0.8242</td>
<td>0.2539</td>
</tr>
</tbody>
</table>

Table 6. The percentage errors on the estimated $X_d$ and $T_d$ at different initial sampling times $Fs = 100$ Hz.

<table>
<thead>
<tr>
<th>$T_d$(s)</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td># of samples</td>
<td>$\epsilon_{X_d}$</td>
<td>$\epsilon_{T_d}$</td>
<td>$\epsilon_{X_d}$</td>
</tr>
<tr>
<td>5</td>
<td>-6.1</td>
<td>-386.2</td>
<td>-11.74</td>
</tr>
<tr>
<td>10</td>
<td>-12.7</td>
<td>4.6</td>
<td>-11.67</td>
</tr>
<tr>
<td>15</td>
<td>13.7</td>
<td>-10.6</td>
<td>-11.53</td>
</tr>
</tbody>
</table>

Examining these tables reveals the following:
- The variation of the number of samples at the same initial time has a great effect on the estimated parameters.
The variation of the initial sampling time at the same number of samples has also a great effect on the estimate of the parameters $X'_d$ and $T'_d$.

The most suitable number of samples which produce a minimum error on the parameters estimate is $m = 10$ ($(X'_d = -12.7\%$ and $(T'_d = 4.6)$ at 1000 Hz sampling frequency.

4.3 Estimating $X''_d$ and $T''_d$

The parameters of the subtransient period are estimated in this section. The number of samples used is 10 samples and the sampling frequency is 1000 Hz, while the initial sampling time is located at 0.5 ms, after 1/40 of the cycle. The parameters estimated are $X''_d = 0.211$ p.u and $T''_d = 0.0382$ s. Therefore, the proposed algorithm estimates exactly the subtransient parameters. Note that $T''_d$ calculated from (39a) is $T''_d = 0.0383$ s, while $T''_d$ calculated from (39b) is $T''_d = 0.0396$ s, which are still good estimates.

4.3.1. Effect of the sampling frequency

The effects of sampling frequency on the estimated parameters are examined in this section. Here the sampling frequency varies from 100 Hz to 1500 Hz with the initial sampling time varies from zero to 1.5 ms, while the number of samples used is 10 samples. Table 7 gives the results obtained.

Examining this table reveals the following:

- When the initial sampling time is located at $T_o = 0$, a poor estimate is obtained at all sampling frequencies.
- When the initial sampling time is located at any other time rather than the zero, the proposed technique produce good estimates in the range of the sampling frequency under consideration.

It can be concluded that if the initial time is located at $T_o = 0.5$ ms, any sampling frequency can be used and good estimates will be obtained.

<table>
<thead>
<tr>
<th>$T_o$(s)</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_s$(Hz)</td>
<td>$X''_d$</td>
<td>$T''_d$</td>
<td>$X''_d$</td>
<td>$T''_d$</td>
</tr>
<tr>
<td>100</td>
<td>14.48</td>
<td>0.0164</td>
<td>0.2111</td>
<td>0.0398</td>
</tr>
<tr>
<td>250</td>
<td>2.27</td>
<td>0.0033</td>
<td>0.2111</td>
<td>0.0384</td>
</tr>
<tr>
<td>500</td>
<td>1.83</td>
<td>0.0016</td>
<td>0.2111</td>
<td>0.0382</td>
</tr>
<tr>
<td>750</td>
<td>2.07</td>
<td>0.0001</td>
<td>0.2111</td>
<td>0.0382</td>
</tr>
<tr>
<td>1000</td>
<td>1.77</td>
<td>0.0008</td>
<td>0.2111</td>
<td>0.0382</td>
</tr>
<tr>
<td>1250</td>
<td>1.45</td>
<td>0.0007</td>
<td>0.2111</td>
<td>0.0382</td>
</tr>
<tr>
<td>1500</td>
<td>1.35</td>
<td>0.0006</td>
<td>0.2111</td>
<td>0.0382</td>
</tr>
</tbody>
</table>

4.3.2. Effect of the number of samples

The effect of the number of samples on the estimated parameters are studied in this section. Here the number of samples is changed from 5 samples to 15 samples, with the initial sampling time located at different initial time starting from 0.2 ms to 2.0 ms, while the sampling frequency is 1000 Hz. Table 8 gives the results obtained.
Table 8. Effect of number of samples on the estimates of $X''_d$ and $T'\_d$ with a sampling frequency $F_s = 1000$ Hz.

<table>
<thead>
<tr>
<th>$T_d$(s)</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>$X''_d$</td>
<td>$T'_d$</td>
<td>$X''_d$</td>
<td>$T'_d$</td>
</tr>
<tr>
<td>5</td>
<td>0.2111</td>
<td>0.0382</td>
<td>0.2111</td>
<td>0.0382</td>
</tr>
<tr>
<td>10</td>
<td>0.2111</td>
<td>0.0382</td>
<td>0.2111</td>
<td>0.0382</td>
</tr>
<tr>
<td>15</td>
<td>0.2111</td>
<td>0.0382</td>
<td>0.2111</td>
<td>0.0382</td>
</tr>
</tbody>
</table>

Looking to this table, we can note that the number of samples has no effect on the estimated parameters, and the estimated parameters are exactly equal to the actual values.

4.4. Estimating $X''_q$ and $T_a$

The equations developed in section 3.4 are used to estimate the parameters $X''_q$ and $T_a$. Equation (40) is used to generate the required samples. For the given parameters, since $T_a = 0$, and the machine parameters estimated in the previous section are almost accurate, we found that the generated samples are almost zero, i.e., the left hand side of equation (48) is almost zero, i.e., the left hand side of equation (48) is almost zero. Therefore, for this machine, it has been found the proposed algorithm can not be used to estimate the parameters $X''_q$ and $T_a$.

5. Conclusions

A new technique is presented, in this paper, for estimating the synchronous machine parameters, from the digitized short circuit armature current. The proposed technique is based on the least error squares algorithm. The problem in this technique is divided into three sub-estimation problems namely, the steady state, the transient and subtransient problems and finally the dc armature current problem. The proposed technique estimates accurately the parameters of each problem with small errors in the transient period. The effects of locating the initial sampling time, the sampling frequency and the number of samples on the estimated parameters are studied in the paper.

References


