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On the Stabilization of Continuous Takagi Sugeno Fuzzy Systems using Discrete Control Approach

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Abstract-This paper deals with the control of the continuous Takagi Sugeno (TS) fuzzy models using discrete control approach. This approach is structured as follows: first, a discrete model is obtained from the discretization of the continuous TS fuzzy model. In this case, the Euler discretization is used for order of approximation superior to one. Second the gains of a non PDC control law ensuring the stabilization of the discrete model are determined. Third by keeping the values of the gains, we check by simulation the stability of the continuous TS fuzzy models through the zero order hold. In this case, the effect of the discretization step on the continuous TS fuzzy models stability results is studied. Simulation example illustrates the effectiveness of this approach.

Keywords: Control, Continuous Takagi Sugeno (TS) fuzzy models, Discrete control approach, Euler Discretization, Non PDC control law

1. INTRODUCTION

During the last twenty years, Takagi–Sugeno (TS) fuzzy models have been used for the stabilization of a large class of non linear systems [1]. In this case the models can be represented means of a collection of linear models which are interconnected by nonlinear functions. Designs of fuzzy controller have been done in the continuous and discrete cases to relax LMIs conditions based on the candidate Lyapunov functions [2-6]. Contrarily to the discrete case, the continuous stability results suffer from the conservatism. Since, in the discrete case, non quadratic Lyapunov function can be used leading to LMI problems, equivalent approach in the continuous case leads to BMI problems. To avoid the BMI problems, it is often required to use hypothesis about the derivative of the membership functions that must be checked a posteriori during experiments or simulations [7-9].

Some works are directed to fuse the continuous and discrete cases through the redesign or the digital control framework where a discrete control law is applied to a continuous model through a zero order hold. In fact, the closed loop is turned into a pure continuous model with the help of delays and Lyapunov- Krasovsky functional [10-11]. But the latter is usually seen as a constraint, since the associated conditions are much more complex. In addition, in this path, in [12], the authors attempted a first solution when they fused the two continuous and discrete cases to obtain new results about the continuous case. In fact, since

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very complex Lyapunov functions can be used only in the discrete case, first, they discretized a continuous model to compute a complex control law associated to a non quadratic Lyapunov function. Then, the stability of the closed loop for the continuous model was checked by keeping the discrete gains determined by ensuring the stabilization of its discretized models. Such models are obtained from the Euler discretization which is limited to first order. The results obtained in [12] are interesting since the authors tried to deal with the stability of the continuous model by means of results obtained from its discrete model. But, the results are restrictive since the Euler approximation is limited to first order.
Therefore, based on the same principle described in [12], the idea in this paper is to increase the order of the Euler approximation (superior than 1). In this case, several models have been obtained for various orders that lead to study the effect of the discretization step, related to the variations of the sampling period and the order of approximation, on the stability results.

This paper is organized as follows. Section II recalls the TS continuous fuzzy models. Section III presents the Euler discretization method applied for continuous fuzzy systems and the discrete stabilization conditions. Section IV illustrates a design example to show the merits of the proposed idea.

2. TAKAGI SUGENO FUZZY MODELS

For any positive scalar functions $h_i(z(\cdot)) \geq 0$, and matrices Y_i with $i \in \{1,\ldots,r\}$, we define the following notations:

$Y_z = \sum_{i=1}^{r} h_i(z(t))Y_i$, or in the discrete case $Y_z = \sum_{i=1}^{r} h_i(z(k))Y_i$

$Y_{z+} = \sum_{i=1}^{r} h_i(z(k+1))Y_i$, $Y_z^{-1} = \left(\sum_{i=1}^{r} h_i(z(k))Y_i\right)^{-1}$

$Y \in \{A, B, K, G, P\}$

A $(*)$ indicates a transpose quantity. For example $Y_z^T P_z(*) - P_z < 0$ stands for $Y_z^T P_z Y_z - P_z < 0$ and $\begin{bmatrix} -P_z & (*) \\ Y_z & -P_z \end{bmatrix} < 0$ for $\begin{bmatrix} -P_z & Y_z^T \\ Y_z & -P_z \end{bmatrix} < 0$.

Using these notations, a continuous TS fuzzy system can be described as a polytopic form with linear models blended together by non linear functions [1]:

$$\dot{x}(t) = A_{z(t)}^{c} x(t) + B_{z(t)}^{c} u(t) \tag{1}$$

Where the index c indicates the continuous case $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $z(t)$ is the vector of premises, r is the number of rules of the fuzzy model and $A_{z(t)}^{c}$ and $B_{z(t)}^{c}$ are matrices of appropriate dimensions, defined in the continuous case. The nonlinear weights h_i satisfy the convex sum property:

$$\sum_{i=1}^{r} h_i(z(t)) = 1 \tag{2}$$

The stabilization of the continuous TS fuzzy system (1) can be assured using the classical PDC law (Parallel Distributed Compensation) which is given by [13]:

$$u(t) = -K^c_{z(t)} x(t) \tag{3}$$

The synthesis of the controller (3) consists in finding the gains K_i^c that ensure the closed loop stabilization of the continuous fuzzy system (1). A quadratic Lyapunov function is mainly used:

$$V(x(t)) = x^T(t) P x(t),\ P = P^T > 0 \tag{4}$$

Theorem 1 [2]: The equilibrium of the continuous fuzzy system (1), based on the controller (3), is globally asymptotically stable if a common matrix $X = X^T > 0$ and matrices M_i satisfying these LMIs can be effectively determined via LMI control toolbox [14]:

$$\begin{cases} \Upsilon_{ii} < 0 & i = 1,2,...,r \\ \dfrac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0 & i,j = 1,2,...,r, \quad i \neq j \end{cases} \tag{5}$$

with: $\Upsilon_{ij} = A_i^c X + X A_i^{c^T} - B_i^c M_j - M_j^T B_i^T,\ \ X = P^{-1},\ \ K_i^c = M_i^{-1} X$

3. EULER DISCRETIZATION AND DISCRETE STABILIZATION CONDITIONS

In this paper, the discrete time model based on the Euler approximation for an order of approximation superior than one is adopted. For $t \in [k\delta,\ (k+1)\delta]$ with δ is the sampling period, the Euler discretization for m order of the TS continuous fuzzy model (1) is given by:

$$x((k+1)\delta) = \sum_{i=1}^{r} h_i(z(k\delta))\left(A_i^d x(k\delta) + B_i^d u(k\delta)\right) \tag{6}$$

With

$$\begin{aligned} A_i^d &= I + A_i^c.\delta + A_i^{c^2}.\frac{\delta^2}{2!} + \cdots + A_i^{c^m}.\frac{\delta^m}{m!}, \\ B_i^d &= B_i^c.\delta + A_i^c.B_i^c.\frac{\delta^2}{2!} + \cdots + A_i^{c^{m-1}}.B_i^c.\frac{\delta^m}{m!} \end{aligned} \tag{7}$$

The index d indicates the discrete case.

The stabilization of the discretized model is assured using the following non quadratic Lyapunov function and the non PDC controller [4]:

$$V(x(k)) = x^T(k) G^T_{z(k)} P_{z(k)} G^{-T}_{z(k)} x(k) \tag{8}$$

$$u(k) = -K^d_{z(k)} G^{-1}_{z(k)} x(k) \tag{9}$$

With $P_{z(k)} = \sum_{i=1}^{r} h_i P_i$ and $P_{z(k)} = P_{z(k)}{}^T$

Theorem 2 [4]: The equilibrium of the closed loop for the discrete TS fuzzy system (6) is globally asymptotically stable if there exist a common symmetric and positive definite matrices P_i, K_i^d and G_i satisfying these LMIs:

$$\begin{cases} \Upsilon_{ii}^{k} < 0 & i,k = 1,2,...,r \\ \dfrac{2}{r-1}\Upsilon_{ii}^{k} + \Upsilon_{ij}^{k} + \Upsilon_{ji}^{k} < 0 & i,j,k = 1,2,...,r, \quad i \neq j \end{cases} \tag{10}$$

with $\Upsilon_{ij}^{k} = \begin{bmatrix} -P_i & (*) \\ A_i^d G_j - B_i^d K_j^d & -G_k - G_k^T + P_k \end{bmatrix}$

The control law (9) is, then, applied to the stabilization of the continuous TS fuzzy system. The proposed method can be summarized by the following steps:

- *Step 1* The Euler discretization is used to obtain the discrete fuzzy model associated to the continuous TS fuzzy model (1). In this case several models will be presented by increasing, each time, the order of the Euler approximation m. Therefore, these various models lead to study the effect of the discretization step, included the variations of the sampling period and the orders of approximation, on the stability results.
- *Step 2* By applying the theorem 2, we determine the discrete gains K_i^d and G_i ensuring the stabilization of the obtained discrete model.
- *Step 3* By considering the gains K_i^d and G_i obtained previously, we cheek by simulation the stability of the continuous TS fuzzy model (1) through the zero order hold.

This idea leads to use non quadratic Lyapunov function for the control of continuous TS fuzzy models without any BMIs conditions or the use of hypothesis about the derivative of the membership functions.

To illustrate the effectiveness of this idea, the following example is presented.

4. SIMULATION EXAMPLE

Consider the continuous TS fuzzy model described by the following matrices [9], [15]:

$$A_1^c = \begin{bmatrix} 3.6 & -1.6 \\ 6.2 & -4.3 \end{bmatrix}, \quad B_1^c = \begin{bmatrix} -0.45 \\ -3 \end{bmatrix}$$
$$A_2^c = \begin{bmatrix} -a & -1.6 \\ 6.2 & -4.3 \end{bmatrix}, \quad B_2^c = \begin{bmatrix} -b \\ -3 \end{bmatrix} \tag{11}$$

This example is considered to verify the stability regions based on the parameters a and b. In fact, solving the LMI conditions given by theorem 1, the stabilization of the continuous model (1) leads to feasible solutions for $a \in [0,25]$ and $b \in [0,1]$. Some works in the literature studied this example for $a \in [0,25]$ and $b \in [0,3]$ to determine the stability region [9], [15].

So, the idea is to apply the steps described previously on the continuous model (11) to expand the feasibility of the solutions ensuring its stability. In fact, for $a \in [0,25]$, the parameter b is adjusted as much as possible to found the widest regions of stabilization that guarantees the stabilization of the discrete fuzzy system (6)-(7). Once the stabilization of the Euler discrete system is guaranteed, it will be checked, also, to its continuous model. In this case, several tests have been accomplished by varying the values of m and b, and it is found that the feasibility of the solutions can been guaranteed for $m \geq 100$.

It is to be noted that the feasibility of the solutions depends on the parameters m and δ. Indeed, the sampling period δ varies by varying the order of approximation m. As can be seen in table 1, δ increases by increasing the order of approximation m. For example, for

$a \in [0,25]$ and $b \in [0,3]$, the following table I presents some results for the evolution of m and δ.

TABLE I. VARIATIONS OF THE PARAMETERS m AND δ

m	δ
4	$[0.08, 0.1]$
10	$[0.1, 0.2]$
35	$[0.2, 0.5]$
50	$[0.2, 0.8]$
70	$[0.2, 1.1]$
100	$[0.2, 1.4]$

In the following and as comparison with the literature [9], [15] and theorem 1, the simulation results based on the parameters a and b are recalled. Figures 1-3 present the stability regions, in the plan a - b, based on the existing results [9], [15] and theorem 1.

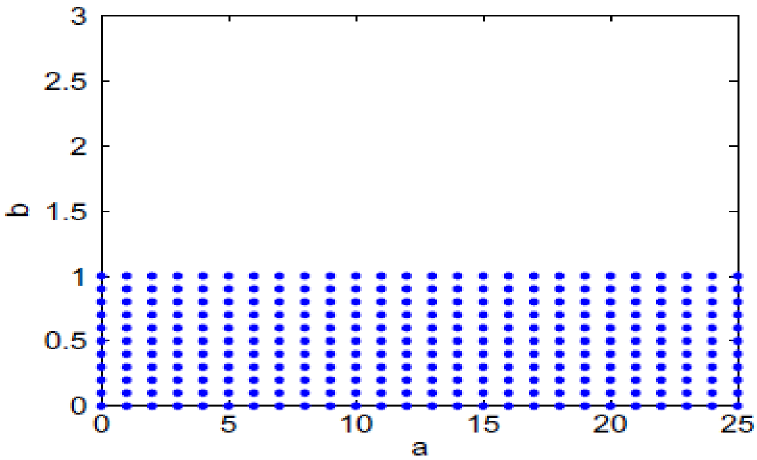

Figure 1 Stability regions found based on Theorem 1.

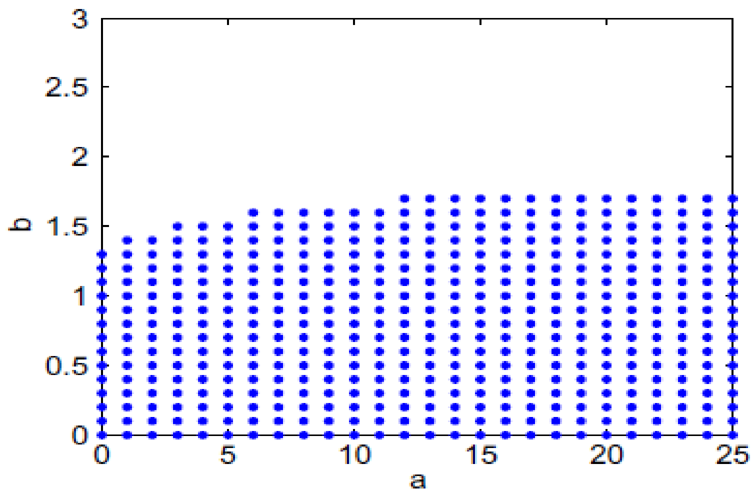

Figure 2 Stability regions found based on Theorem 6 in [15]

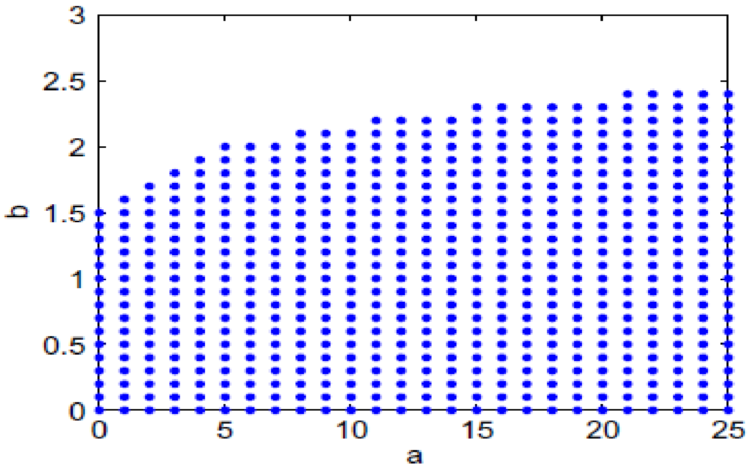

Figure 3 Stability regions found based on Theorem 10 in [9]

Figures 4-8 present the stability regions found based on our proposed idea. These figures are obtained from different values of m and δ to demonstrate the influence of the two parameters on the stability results of the continuous fuzzy systems controlled by the discrete gains.

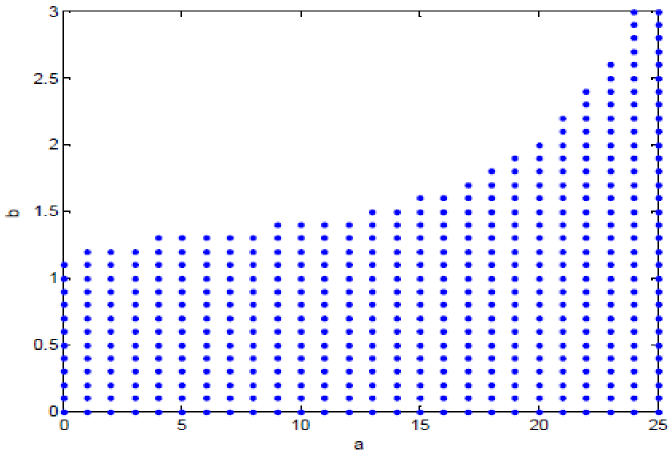

Figure 4 Stability regions for $m = 4$ and $\delta = 0.08\,\text{sec}$

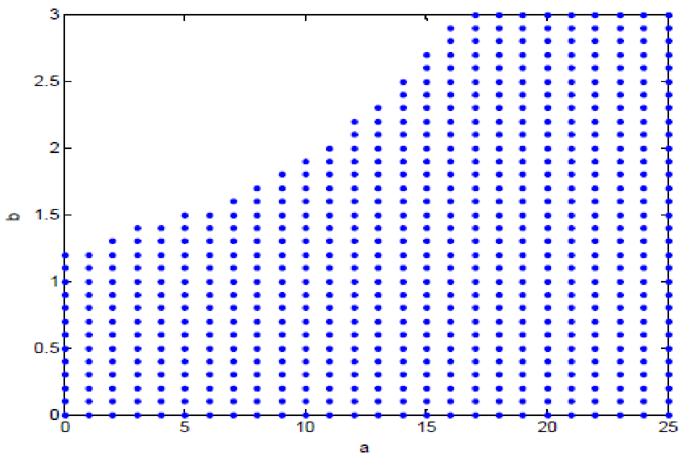

Figure 5 Stability regions for $m = 10$ and $\delta = 0.2\,\text{sec}$

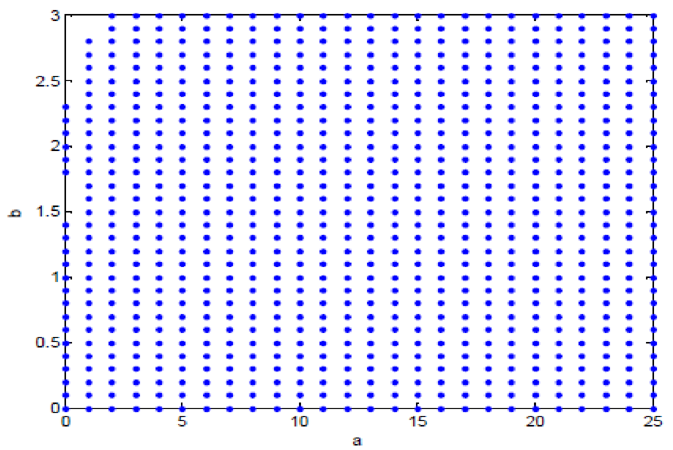

Figure 6 Stability regions for $m = 50$ and $\delta = 0.5\,\text{sec}$

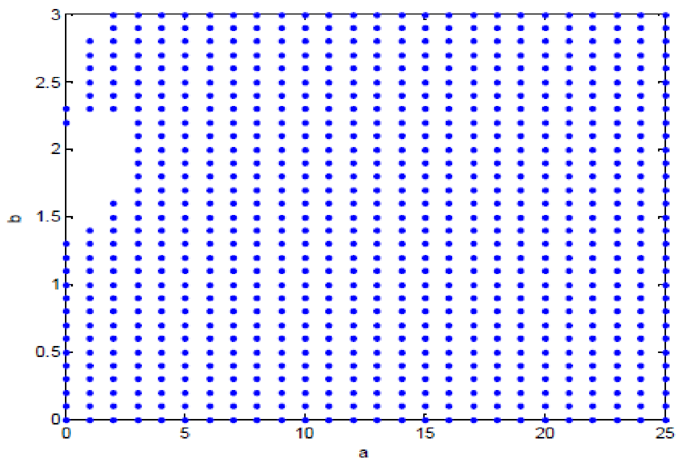

Figure 7 Stability regions for $m = 100$ and $\delta = 0.4\,\text{sec}$

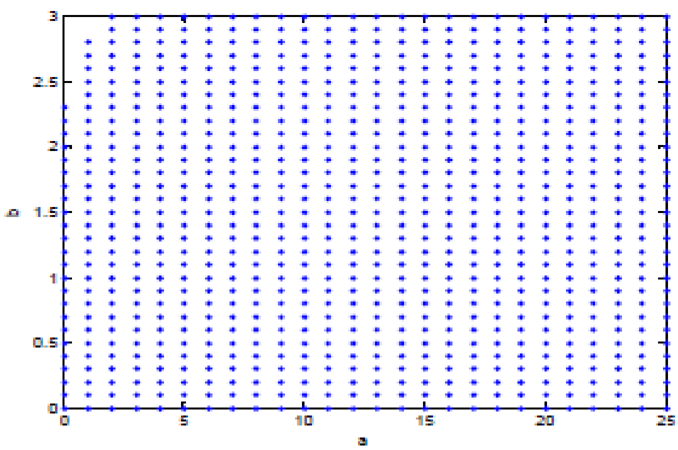

Figure 8 Stability regions for $m = 100$ and $\delta = 1.4\,\text{sec}$

It can be seen from these figures, that the proposed idea offers larger stability regions than those presented in the literature. Also, it should be noted that more we increase the values of m and δ , more the feasibility region of the solutions becomes widest.

In the following, the simulation results will be presented. For the simulation, the following premise is used: $z(t)=\dfrac{\sin(x_1(t))}{x_1(t)}$ with $-\dfrac{\pi}{3}\le x_1(t)\le\dfrac{\pi}{3}$

With the initial conditions given by $x(0)=[0.5 \quad 0.5]$, the following figure illustrates the closed-loop results for the continuous TS fuzzy system stabilized by the discrete gains ensuring the stabilization of its associated discrete model.

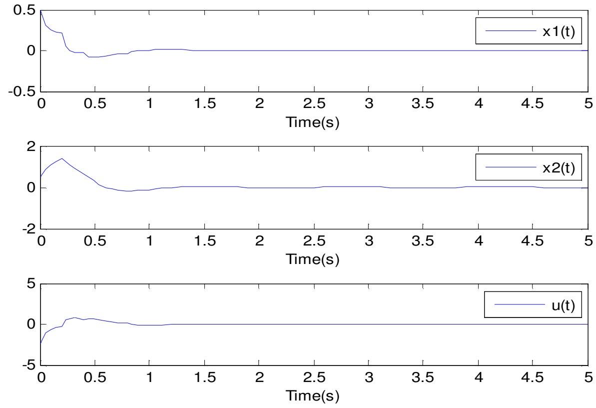

Figure 9 Evolutions of the state variable $x_1(t)$, $x_2(t)$ and the control signal $u(t)$ ($a=25$, $b=3$, $m=10$ and $\delta=0.2$ sec)

As can be seen in Figure 9, the evolutions of the state response $x_1(t)$, $x_2(t)$ and the control signal $u(t)$ leads to interesting results regarding fast stabilization of the continuous TS fuzzy system (11). Thus, it can be concluded that the state response of the closed loop continuous system is asymptotically stabilized by the gains of the non PDC controller ensuring the stabilization of its associated discrete model.

For the same membership functions and the same initial conditions, the following simulation results are presented:

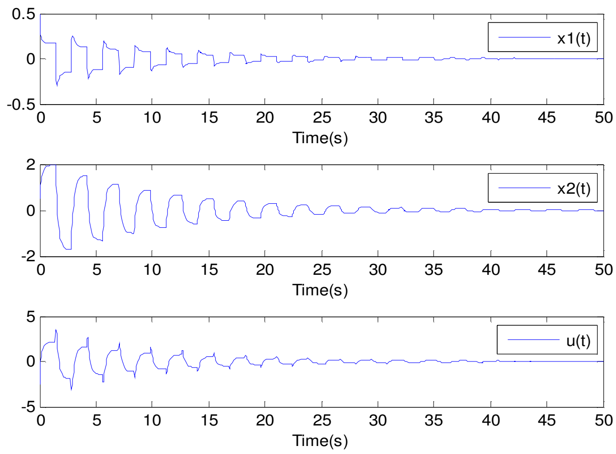

Figure 10 Evolutions of the state variable $x_1(t)$, $x_2(t)$ and the control signal $u(t)$ ($a=25$, $b=3$, $m=100$ and $\delta=1.4$ sec)

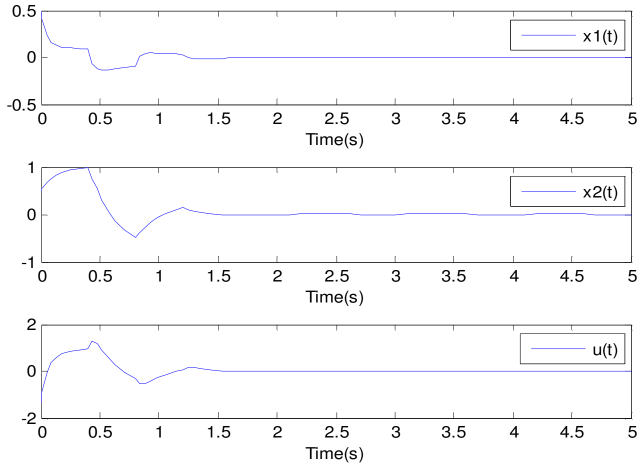

Figure 11 Evolutions of the state variable $x_1(t), x_2(t)$ and the control signal $u(t)$ ($a = 25$, $b = 3$, $m = 100$ and $\delta = 0.4$ sec)

The evolution of the state variable $x_1(t), x_2(t)$ and the control signal $u(t)$ demonstrate the stability of the continuous model (11) through the discrete gains. It should be noted, that for the same value of the order m , the stabilization needs more time for $\delta = 1.4$ in comparison with $\delta = 0.4$. So, we can conclude that the variation of the sampling period δ affects the stability of TS fuzzy models as showed by the figure 11.

5. CONCLUSIONS

In this paper, it has been used to control continuous fuzzy model by a zero-order hold discrete control law computed for its discretized fuzzy model. In this case, this control approach allows the use of non quadratic Lyapunov function for continuous TS fuzzy models. The discretized fuzzy models have been obtained from the Euler approximation which has been used for order of approximation superior to one. This way leads to study the effect of the variations of the order of the Euler approximation and the sampling period on the continuous fuzzy system stability results. In fact, the simulation example demonstrates the advantage of this method, since more we increase the order of the approximation more the sampling period increases and the regions of the feasibility solutions become larger.

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