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Comparative Study of Robust Exponential Higher Order Sliding Mode Control with Standard Techniques of HOSMC

Abdelhak Msaddek, Abderraouf Gaaloul and Faouzi M'sahli

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***Abstract-** The standard sliding mode control (SMC) lads, generally, to the appearing of an undesirable chattering phenomenon to solve this problem we develop a novel approach of higher order sliding mode control (HOSMC). The proposed procedure provides an exponential stability on the sliding surface and guarantees the robustness against uncertainties and external matched disturbances. In this paper, and to show the good performances of the new design scheme we present a comparative study between three well known techniques of HOSMC in literature with our approach. Numerical simulations are developed to show the effectiveness of the proposed approach.*

Keywords: Nonlinear systems, Sliding mode control, Higher-order, Robustness.

1. INTRODUCTION

The dynamical systems are usually affected by external matched disturbances, parametric variations, modeling uncertainties and nonlinearities (hysteresis, friction, etc.). The sliding mode control (SMC) is a well known solution used to solve these problems. Indeed, SMC is able to overcome these barriers in regulation and tracking. Such control technique has amply demonstrated its effectiveness through theoretical and practical studies in main areas of applications including robotics, chemical reactors, mobile manipulators and electrical machinery [5], [15], [20] and [23]. The robustness property of SMC is achieved by using a high frequency switching to steer the states of a system into the sliding surface [24].

The high-frequency switching leads, generally, to the appearing of an undesirable chattering phenomenon at outputs of the system and on the control input. This leads to the dissipation of a large quantity of energy in electric actuators and a rapid wear of mechanical actuators. This limits seriously the implementation of SMC in real time. To overcome this difficulty, several solutions have been proposed in the literature. Among them, the "$sign$" function is replaced by a smooth similar function such as the saturation function or the sigmoid function. Furthermore, some non conventional techniques such as fuzzy logic and neural networks were combined together with SMC [6], [7], [21] and [22].

The most interesting way to get rid of the chattering phenomenon consists of enforcing a higher-order sliding mode (HOSM). The main objective of SMC of order ρ (called ρ-SMC) is to obtain a finite time convergence onto the manifold $S^{\rho} = \left\{ s = \dot{s} = \ldots = s^{(\rho-1)} = 0 \right\}$, where s is the sliding variable. So, the philosophy of HOSMC consists of enforcing the

National School of Engineers of Monastir, University of Monastir, Avenue Ibn Jazzar 5019, Monastir, Tunisia
msaddekabdelhak@hotmail.fr, abderraouf.gaaloul@enim.rnu.tn, faouzi.msahli@enim.rnu.tn.

sliding variable and its ρ -1 first time derivatives to zero in finite time [10]. In [11], author proposed the so-called quasi-continuous SMC where the control input is chattering free except on the sliding surface.

Laghrouche et al. [8] developed another approach of HOSMC based on the minimization of a quadratic criterion using the concept of SMC with integral action. This allows stabilizing in finite time a system of high order on the sliding surface. Besides, it permits to choose in advance the convergence time to the sliding surface. Although these algorithms are general, a priori accurate knowledge of the initial conditions of the system limit seriously the applicability of this approach. In [2], [3] and [4], authors proposed a finite time HOSMC for a class of multivariable nonlinear systems. In such work, the drawbacks devoted in [8] are removed and a simple method for adjusting the synthesis parameters of the control law is presented. However, this controller contains necessarily a discontinuous part to reject the effect of disturbances and requires the knowledge of the maximum amplitudes of the disturbances.

Mondal and Mahanta [16] proposed a second order sliding mode controller based on a nonlinear sliding surface to control uncertain linear systems with matched uncertainty. Likewise another approach of HOSMC presented in [14] for uncertain nonlinear systems with relative degree three. The stability on the sliding surface is guaranteed and the chattering is reduced using these controllers. Nevertheless, these two approaches are applicable only for uncertain systems with relative degree two and three, respectively.

In this paper, we present a new technique of HOSMC for uncertain nonlinear systems of any relative degree [19], and we demonstrated its exponential stability and robustness against external matched disturbances, parametric variations and modeling uncertainties. Furthermore, the convergence to the sliding surface is independent of the initial conditions. The proposed controller is compared with three most important works dealing with higher order sliding mode controller proposed by Levant [11], Laghrouche et al. [8] and Defoort et al. [4]. Such algorithms are applied to solve a tracking problem of the trajectory of a car. Furthermore, we evaluate the robustness of such controllers against parameters uncertainties and external disturbances. Simulations results developed in this work show the effectiveness of our proposed HOSMC over the other approaches. The proposed approach is applied to a model of a car in order to ensure a robust tracking of a prescribed reference trajectory.

This paper is organized as follows. The next section is devoted to the presentation of the standard techniques of higher order sliding mode controllers. In section 3, we present the new higher order sliding mode control. The comparative study, the tests and the simulations results are given in section 4. Conclusions are reported in the last section of the paper.

2. SUMMARY OF THE PAPER

2. STANDARD HIGHER ORDER SLIDING MODE CONTROLLERS

Consider a nonlinear dynamical system described by

$$\begin{aligned} \dot{x} &= f(x,t) + g(x,t)u \\ s &= s(x,t) \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in X$ is the state variable of the system with X an open set of $\Re^n$ and $u \in U$ the control input is a feature possibly discontinuous and bounded, depending on time and the system state, with U is an open set $\Re$, $f(x,t)$ and $g(x,t)$ are sufficiently differentiable vector fields.

Assumption 1. System (1) admits a $\rho \in N$ constant and known relative degree with respect to the sliding variable $s(x,t)$.

2.1 Quasi-Continuous Sliding Mode Controller

To eliminate the effect of chattering appeared, generally, using the standard sliding mode control, Levant [12], proposed a robust homogenous higher order sliding mode controller. This control technique allows to stabilize the system after a finite transient time on the sliding surface defined by $S^{\rho} = \{s = \dot{s} = \ldots = s^{(\rho-1)} = 0\}$. Such quasi-continuous higher order sliding mode controller is described as follows for $\rho \le 4$

$$u = -\alpha sign(s) \qquad \text{for } \rho = 1 \tag{2}$$

$$u = -\alpha\left(\dot{s} + |s|^{1/2} sign(s)\right) \Big/ \left(|\dot{s}| + |s|^{1/2}\right) \qquad \text{for } \rho = 2 \tag{3}$$

$$u = -\alpha \frac{\left[\ddot{s} + 2\left(|\dot{s}| + |s|^{2/3}\right)^{-1/2}\left(\dot{s} + |s|^{2/3} sign(s)\right)\right]}{\left[|\ddot{s}| + 2\left(|\dot{s}| + |s|^{2/3}\right)^{1/2}\right]} \qquad \text{for } \rho = 3 \tag{4}$$

$$\varphi_{3,4} = \dddot{s} + 3\left[\ddot{s} + \left(|\dot{s}| + 0.5|s|^{3/4}\right)^{-1/3}\left(\dot{s} + 0.5|s|^{3/4} sign(s)\right)\right] \times \left[|\ddot{s}| + \left(|\dot{s}| + 0.5|s|^{3/4}\right)^{2/3}\right]^{-1/2}$$

$$N_{3,4} = |\dddot{s}| + 3\left[|\ddot{s}| + \left(|\dot{s}| + 0.5|s|^{3/4}\right)^{2/3}\right]^{1/2}$$

$$u = -\alpha\, \varphi_{3,4} / N_{3,4} \qquad \text{for } \rho = 4 \tag{5}$$

where $\alpha > 0$

2.2 Integral Sliding Mode Control

Laghrouche et al. [8] proposed a sliding mode control with integral action. This control strategy allows choosing in advance the convergence time to the sliding surface.

The system (1) can be written as follows:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \quad \vdots \\ \dot{z}_{\rho-1} = z_{\rho} \\ \dot{z}_{\rho} = \phi(x,t) + \varphi(x,t)u \end{cases} \tag{6}$$

with

$$\begin{cases} \phi = \bar{\phi} + \delta_{\phi} \\ \varphi = \bar{\varphi} + \delta_{\varphi} \end{cases}$$

where $z = \begin{bmatrix} z_1 & z_2 & \cdots & z_{\rho} \end{bmatrix}^T = \begin{bmatrix} s & \dot{s} & \cdots & s^{(\rho-1)} \end{bmatrix}^T$, $\bar{\phi}$ and $\bar{\varphi}$ are nominal known parts, δ_{φ} and δ_{ϕ} are unknown parts, including disturbances and uncertainties.

Assumption 2. The nominal part $\overline{\varphi}$ is assumed invertible.
The control is given by [8]

$$u = \overline{\varphi}^{-1}\left(\left(w_0 + w_1\right) - \overline{\phi}\right) \tag{7}$$

with

$$w_0 = \begin{cases} -B^T M z(t) + B^T \delta(t) & for \;\; 0 \le t \le t_F \\ -B^T M z(t) & for \;\; t > t_F \end{cases}$$

and

$$w_1 = -\alpha sign(\sigma) \tag{8}$$

where $\delta(t)$ and M are the solution of the following equations

$$\begin{aligned} \dot{\delta} &= -\left(A^T - MBB^T\right)\delta \\ 0 &= MA + A^T M - MBB^T M + Q \end{aligned} \tag{9}$$

where the matrix Q is symmetric positive definite and the matrices A and B are given, respectively, by

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \cdots & 1 \\ 0 & \ddots & \ddots & \ddots & 0 \end{bmatrix}_{\rho\times\rho} , \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{\rho\times 1}$$

The sliding variable is given by

$$\sigma = z_\rho + \xi$$

with

$$\dot{\xi} = -w_0, \quad \xi(0) = -z_\rho(0)$$

The gain of the discontinuous part satisfies the following property

$$\alpha > \frac{C_0 + \left(K_M + 1\right)w_{0M} + \eta}{K_m} \tag{10}$$

with $|\phi| \le C_0$, $K_m \le \varphi \le K_M$, $\eta > 0$ and $|w_0| < w_{0M}$
where K_m and K_M are two positive constants.

2.3 Higher Order Integral Sliding Mode Control

Consider system (6) which can be rewritten as follows:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \quad\vdots \\ \dot{z}_{\rho-1} = z_\rho \\ \dot{z}_\rho = \nu(x,t) + \left(1 + \varsigma(x,t)\right)w \end{cases} \tag{11}$$

with

$$u = \overline{\varphi}^{-1}\left(w - \overline{\phi}\right) \quad \text{and} \quad \begin{cases} \nu = \delta_\phi - \delta_\varphi \overline{\varphi}^{-1}\overline{\phi} \\ \varsigma = \delta_\varphi \overline{\varphi}^{-1} \end{cases}$$

The functions $\nu(x,t)$ and $\varsigma(x,t)$ may include the uncertainties of the system.

Assumption 3. The functions $\nu(x,t)$ and $\varsigma(x,t)$ are bounded. In addition, there is a positive function $a(x)$ and b a positive constant $0 < b \le 1$, such that:

$$\begin{cases} |\nu(x,t)| & \le & a(x) \\ |\varsigma(x,t)| & \le & 1-b \end{cases} \tag{12}$$

To stabilize the uncertain system (11) in finite time, the control law proposed by Defoort et al. is given by [1]

$$\begin{cases} w(z) & = & w_{nom}(z) + w_{disc}(z, z_{aux}) \\ \dot{z}_{aux} & = & -w_{nom}(z) \end{cases} \tag{13}$$

with

$$w_{nom}(z) = -k_1 sign(z_1)|z_1|^{v_1} - \cdots - k_\rho sign(z_\rho)|z_\rho|^{v_\rho} \tag{14}$$

where $k_1 \; \cdots \; k_\rho$ are positive constants chosen such that the polynomial $p^\rho + k_\rho p^{\rho-1} + \cdots + k_2 p + k_1$ is Hurwitz, and $v_1 \; \cdots \; v_\rho$ satisfy

$$v_{i-1} = \frac{v_i v_{i+1}}{2v_{i+1} - v_i}, \quad i = 2 \; \cdots \; \rho \tag{15}$$

with $v_\rho = v$ and $v_{\rho+1} = 1$, $v \in (1-\varepsilon, 1)$, $\varepsilon \in (0, 1)$.

The discontinuous part is given by

$$w_{disc}(z, z_{aux}) = -G(z) sign(\sigma) \tag{16}$$

with

$$\sigma(z,t) = z_p(t) + z_{aux}(t) \tag{17}$$

The gain $G(z)$ is a positive function which satisfies the fallowing property

$$G(z) \ge \frac{(1-b)|w_{nom}(z)| + a(x) + \eta}{b}, \quad \eta > 0 \tag{18}$$

Theorem I: [2], [3], [4]. Let the uncertain nonlinear system (1) of relative degree ρ with respect to the sliding variable $s(x,t)$. The control law

$$u = \bar{\varphi}^{-1}\left(w_{nom}(z) + w_{disc}(z, z_{aux}) - \bar{\phi}\right) \tag{19}$$

allows to stabilize the system (1) in finite time at the sliding surface $S^\rho = \{s = \dot{s} = \cdots = s^{(\rho-1)} = 0\}$. Therefore, a sliding mode of order ρ is established with respect to the sliding variable $s(x,t)$, provided that Assumption (2) and (3) are verified.

3. NEW HOSM-CONTROL DESIGN

3.1 Robust stabilization of a chain of integrators

Consider a chain of integrators, defined by

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_3 \\ \quad \vdots \\ \dot{z}_{\rho-1} = z_\rho \\ \dot{z}_\rho = z_{\rho+1} = w(z) \end{cases} \tag{20}$$

Theorem II: [18], [19]. To stabilize exponentially the system (20) in presence of uncertainties on S^ρ, we propose the following control law.

$$w(z) = -\alpha \frac{\left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right)}{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)} \tag{21}$$

with $\alpha > 0, a_i > 0\ (1 \le i \le \rho)$, $b_i > 0\ (1 \le i \le \rho)$ and the polynomial $P(x) = a_1 + a_2 x + \cdots + a_\rho x^{\rho-1}$ is Hurwitz.

Proof.

Equation (21) can be written as follows

$$\left(a_1 s + a_2 \dot{s} + \cdots + a_\rho s^{(\rho-1)}\right) = f(t) \tag{22}$$

with

$$f(t) = -\dot{z}_\rho \frac{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)}{\alpha} \tag{23}$$

The Laplace Transform applied to (22) gives

$$S(p)\left(a_1 + a_2 p + \cdots + a_\rho p^{\rho-1}\right) = F(p) \tag{24}$$

As $P(p) = a_1 + a_2 p + \cdots + a_\rho p^{\rho-1}$ is Hurwitz, the solution of (22) is stable.

proof of the convergence of $z = \begin{bmatrix} z_1 & z_2 & \cdots & z_\rho \end{bmatrix}^T$ to the zero vector of $\Re^\rho$. Assume that the states of system are not in the manifold S^ρ. One has

$$z_{\rho+1} = w(z) = -\alpha \frac{\left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right)}{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)} \tag{25}$$

In other words, one obtains

$$\beta z_{\rho+1} = a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho \tag{26}$$

with

$$\beta = -\frac{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)}{\alpha} < 0\ ; \forall t$$

So, (26) is assumed a linear differential equation with second member. The equation without second member is given by

$$a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho = 0 \tag{27}$$

The roots of the polynomial $P(p)$ have a strictly negative real part. So, the polynomial $P(p)$ which is given by

$$P(p) = a_1 + a_2 p + \cdots + a_\rho p^{\rho-1} \tag{28}$$

can be rewritten as follows

$$P(p) = \prod_{i=1}^{r} (p - \lambda_i)^{\eta_i}\ ,\ \sum_{i=1}^{r} \eta_i = \rho - 1\ ,\ \mathrm{Rel}(\lambda_i) < 0$$

with λ_i are the roots of the polynomial $P(p)$ with multiplicity degree η_i.
Therefore, the homogeneous solution of (26) is of the form

$$z_1(t) = \sum_{i=1}^{r} q_i(t) e^{\lambda_i t} \quad (29)$$

with $q_i(t)$ are polynomials of degree $\eta_i - 1$.
Also, one notes that the null function $z_1 = 0$ is a solution of the equation (26). Indeed if $z_1 = 0 \; ; \forall t$, $\beta = 0 \; ; \forall t$. So, one can take it as a particular solution of (26). Consequently, the general solution of (26) is given by

$$z_1(t) = \sum_{i=1}^{r} q_i(t) e^{\lambda_i t} \quad (30)$$

The coefficients of $q_i(t)$ are chosen using initial conditions. So, $z_i = z_1^{(i-1)} \; (1 \le i \le \rho)$ converge exponentially to zero of $\Re^{\rho}$ whatever the initial conditions are chosen.

Proof of robustness

Suppose that the control is affected by the disturbances as follows

$$p_1(t) z_{\rho+1} + p_2(t) = -\alpha \frac{\left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right)}{\left(b_1 |z_1| + b_2 |z_2| + \cdots + b_\rho |z_\rho|\right)} \quad (31)$$

where $z_{\rho+1} = \dot{z}_\rho$, $p_1(t)$ and $p_2(t)$ are two bounded disturbances.
Equation (31) can be rewritten as follows

$$\beta\left(p_1(t) z_{\rho+1} + p_2(t)\right) = \left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right) \quad (32)$$

The homogeneous solution of the equation (15) takes the form

$$z_1(t) = \sum_{i=1}^{r} q_i(t) e^{\lambda_i t}$$

and $z_1 = 0$ is a particular solution of (32). So the general solution of (32) is given by

$$z_1(t) = \sum_{i=1}^{r} q_i(t) e^{\lambda_i t}$$

Thus, $z_i = z_1^{(i-1)} \; (1 \le i \le \rho)$ converge exponentially to zero of $\Re^{\rho}$. Therefore, the proposed controller ensures the robustness against bounded disturbances.

Example: Consider a chain of integrators described by (20). Using the controller (21) to stabilize the chain of integrators, one has

$$w(z) = -5 \frac{(z_1 + z_2)}{(|z_1| + |z_2|)} \quad \text{for} \quad \rho = 2 \quad (33)$$

$$w(z) = -15 \frac{(z_1 + z_2 + z_3)}{(|z_1| + |z_2| + |z_3|)} \quad \text{for} \quad \rho = 3 \quad (34)$$

$$w(z) = -100 \frac{(z_1 + 4z_2 + 8z_3 + 3z_4)}{(|z_1| + 4|z_2| + 8|z_3| + 3|z_4|)} \quad \text{for} \quad \rho = 4 \quad (35)$$

$$w(z) = -100 \frac{(z_1 + 5z_2 + 8z_3 + 8z_4 + 3z_5)}{(|z_1| + 5|z_2| + 8|z_3| + 8|z_4| + 3|z_5|)} \quad \text{for} \quad \rho = 5 \quad (36)$$

where ρ is the number of integrators.

First, we consider that the integrator chains are not affected by noises. Simulation results plotted on figure 1 show that control objective is fulfilled using controllers (33)-(36).

Besides, the proposed controller can stabilize exponentially the states of different chain of integrators in finite time.

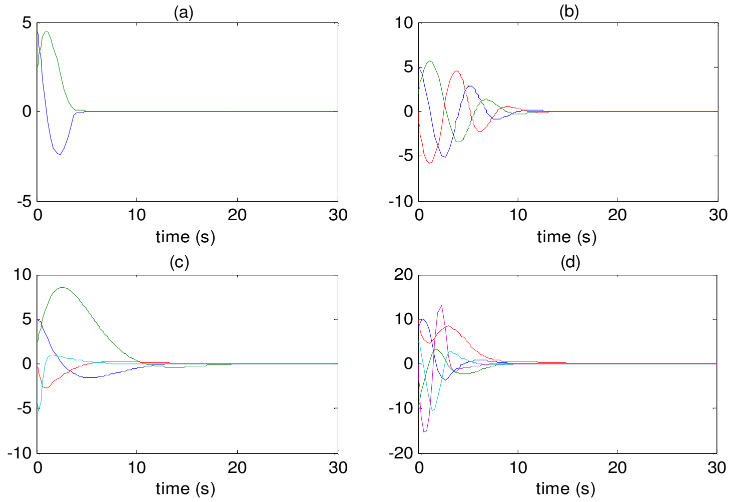

Figure 1 Tracking performances z_i $1 \leq i \leq \rho$ without noises, (a) for $\rho = 2$, (b) for $\rho = 3$, (c) for $\rho = 4$, (d) for $\rho = 5$.

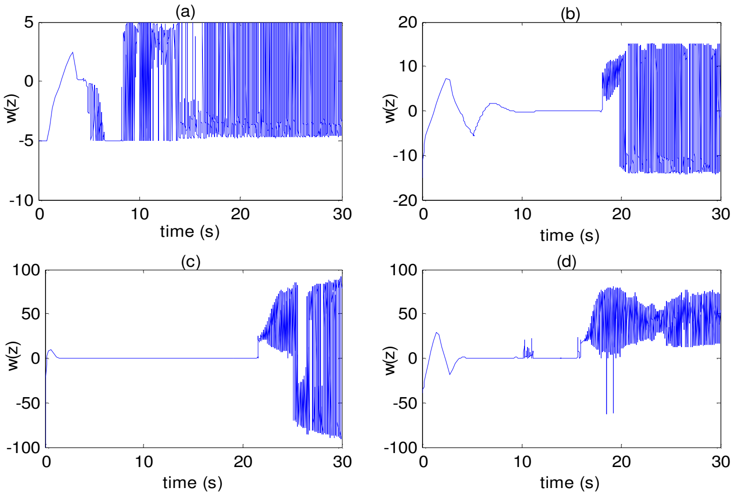

Figure 2 Controls signals $w(z)$ without noises, (a) for $\rho = 2$, (b) for $\rho = 3$, (c) for $\rho = 4$, (d) for $\rho = 5$.

Now, one assumes that the chain of integrators is perturbed. So, the input $w(z)$ is affected by a noise of expression v(t)=2sin(t), t is the time.

Simulations results depicted on figure 3 show that the proposed controller is robust against bounded disturbances.

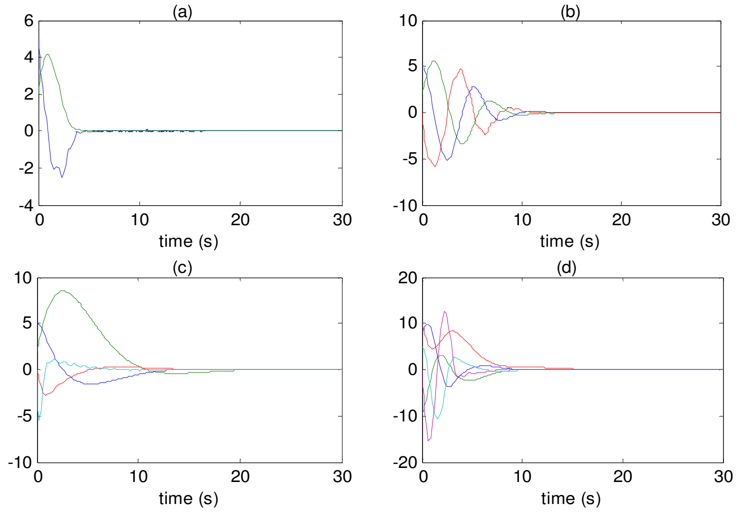

Figure 3 Tracking performances $z_i \ 1 \le i \le \rho$ under disturbances, (a) for $\rho = 2$, (b) for $\rho = 3$, (c) for $\rho = 4$, (d) for $\rho = 5$.

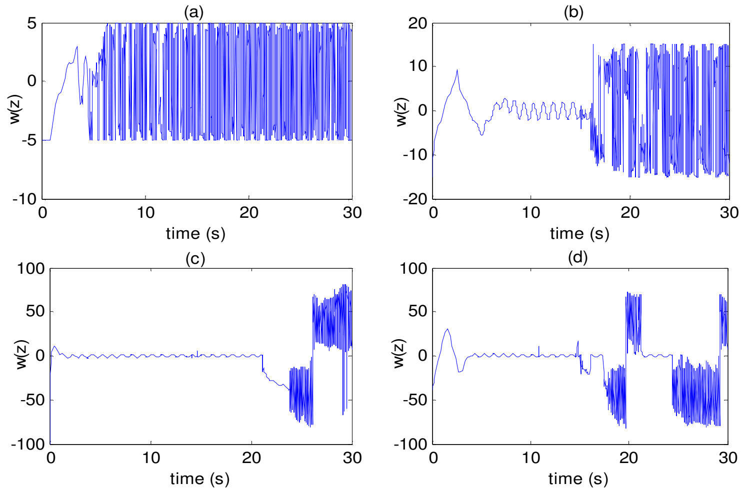

Figure 4 Controls signals $w(z)$ under disturbances, (a) for $\rho = 2$, (b) for $\rho = 3$, (c) for $\rho = 4$, (d) for $\rho = 5$.

3.2 Higher Order Sliding Mode Controller

Consider system (11) and consider the following control law:

$$u = \overline{\varphi}^{-1}\left(w(z) - \overline{\phi}\right) \tag{37}$$

where $\overline{\phi}$ and $\overline{\varphi}$ are obtained according to (6) and $w(z)$ is given by (21).

Theorem III: [18], [19]. The controller (37) ensures a sliding mode of order ρ with respect to $s(x,t)$ provided that assumptions (2) and (3) are verified.

Proof

Using system (20), one has

$$\dot{z}_\rho = \nu(x,t) + \left(1 + \varsigma(x,t)\right)w(z) \tag{38}$$

If Assumption (3) is verified, one can write (23) as follows

$$w(z) = \left(1+\varsigma(x,t)\right)^{-1}\left(\dot{z}_\rho - \nu(x,t)\right)$$
$$= \left(1+\varsigma(x,t)\right)^{-1}\dot{z}_\rho - \left(1+\varsigma(x,t)\right)^{-1}\nu(x,t) \quad (39)$$

Now, applying equation (14) to (24) one obtains

$$p_1(t)z_{\rho+1} + p_2(t) = -\alpha\frac{\left(a_1 z_1 + a_2 z_2 + \cdots + a_\rho z_\rho\right)}{\left(b_1|z_1| + b_2|z_2| + \cdots + b_\rho|z_\rho|\right)} \quad (40)$$

with

$$p_1(t) = \left(1+\varsigma(x,t)\right)^{-1} \quad (41)$$

$$p_2(t) = -\left(1+\varsigma(x,t)\right)^{-1}\nu(x,t) \quad (42)$$

Now, using the proof of robustness presented in the previous section, one concludes that the control law (37) allows stabilizing exponentially the uncertain system (1) on the sliding surface.

Remark: To implement the controllers (2)-(5), (7), (19) and (37) a finite time differentiator [11] is used to estimate the successive derivatives $\left(\dot{s}, \ddot{s}, \cdots, s^{(\rho-1)}\right)$ of the sliding variable (s) [10].

4. SIMULATION EXAMPLE

To make the comparison between the different controls techniques presented previously we use the model of a car to develop the numerical simulations.

4.1 Mathematical Model of the Car

Consider a simple kinematic model of a car given by

$$\begin{cases} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= v\dfrac{\tan\psi}{L} \\ \dot{\psi} &= u \end{cases} \quad (43)$$

where x and y are Cartesian coordinates of the rearaxle middle point, θ is the orientation angle, v is the longitudinal velocity, L is the length between the two axles and ψ is the steering angle (figure 5).

The task is to steer the car to the trajectory $y = y_{ref}$, y is assumed to be measured in real time. Note that the actual control here is ψ and $\dot{\psi} = u$ is used as a new control in order to avoid discontinuities of ψ.

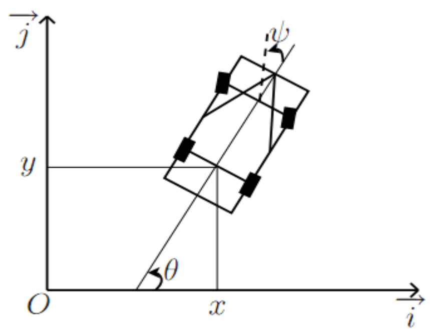

Figure 5 Kinematic car model [17].

$L = 5m$, $x = y = \psi = \theta = 0,$ at $t = 0$. Define the sliding variable $s = y - y_{ref}$. The relative degree of the system is 3. Indeed, one has

$$s^{(3)} = \left(\frac{v^2}{L}\left(1+\tan^2(\psi)\right)\cos(\theta)\right)u - \left(\frac{v^3}{L^2}\tan^2(\psi)\sin(\theta) + y_{ref}^{(3)}\right) \tag{44}$$

Consequently, a 3rd order SMC is designed.

First, we assume that the system is not noisy. So, we consider that $v = \text{const} = 10\ m/s$ and y is measured by a sensor perfectly non-noisy.

In the second part, we suppose that the system is uncertain. Indeed, we suppose that the longitudinal velocity is variable, i.e. $v(t) = 10 + 0.5\sin(t)\ m/s$ and the output y is measured by a sensor affected by disturbances of the form $0.5\sin(2t)$, t is the time variable.

4.2 Controllers Synthesis

Applied to the model of the car, the quasi-continuous sliding mode control [9] is given by

$$u = -\frac{\left[z_3 + 2\left(|z_2| + |z_1|^{2/3}\right)^{-1/2}\left(z_2 + |z_1|^{2/3} sign(z_1)\right)\right]}{\left[|z_3| + 2\left(|z_2| + |z_1|^{2/3}\right)^{1/2}\right]} \tag{45}$$

Now, the algorithm proposed by Laghrouche et al [8] is given by (7) and (8). The components of the controller are

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 2.4142 & 2.4142 & 1.0000 \\ 2.4142 & 2.4142 & 2.4142 \\ 1.0000 & 2.4142 & 2.4142 \end{bmatrix}$$

$$C_0 = 49.62, K_m = 6.38, K_M = 46.77, t_F = 5s$$

$$z(0) = [-2 \quad -2 \quad 0]^T, \ \delta_1(0) = 0.7615, \ \delta_2(0) = 1.5907, \ \delta_3(0) = 0.9399$$

Right now, the integral sliding mode control proposed by Defoort et al. [4] is applied to the model of the car. It's given by

$$u = \bar{\varphi}^{-1}\left(w(z) - \bar{\phi}\right) \tag{46}$$

with

$$\begin{aligned} w(z) &= w_{nom}(z) + w_{disc}(z, z_{aux}) \\ \dot{z}_{aux} &= -w_{nom}(z) \\ w_{nom}(z) &= -sign(z_1)|z_1|^{\frac{1}{2}} - 1.5 sign(z_2)|z_2|^{\frac{3}{5}} - 1.5 sign(z_3)|z_3|^{\frac{3}{4}} \\ w_{disc}(z, z_{aux}) &= -10\left(\frac{2}{\pi}\arctan\left(\frac{z_3 + z_{aux}}{0.001}\right)\right) \end{aligned} \tag{47}$$

Finally, in this section, we apply our proposed higher order sliding mode control (37) to the model of the car. So, the control law is given by

$$u = \bar{\varphi}^{-1}\left(w(z) - \bar{\phi}\right) \tag{48}$$

with

$$w\left(z\right)=-9\frac{\left(z_1+5z_2+2z_3\right)}{\left(\left|z_1\right|+5\left|z_2\right|+2\left|z_3\right|\right)} \tag{49}$$

and

$$\dot{z}_1=v_0\ ;\ v_0=-14.7361\left|z_1-s\right|^{2/3}sign\left(z_1-s\right)+z_2$$

$$\dot{z}_2=v_1\ ;\ v_1=-30\left|z_2-v_0\right|^{1/2}sign\left(z_2-v_0\right)+z_3$$

$$\dot{z}_3=-440sign\left(z_3-v_1\right)$$

where z_1, z_2 and z_3 are the estimation of s, $\dot{s}$ and $\ddot{s}$, respectively, obtained using the robust finite time differentiator [13].

If $y_{ref}^{(3)}$ is independent of u, the expressions of $\bar{\varphi}$ and $\bar{\phi}$ are given by

$$\bar{\varphi}=\left(\frac{v^2}{L}\left(1+\tan^2\left(\psi\right)\right)\cos\left(\theta\right)\right),\quad \bar{\phi}=\left(\frac{v^3}{L^2}\tan^2\left(\psi\right)\sin\left(\theta\right)+y_{ref}^{(3)}\right)$$

The control objective of en forcing the car trajectory to track a reference trajectory given by:

$$y_{ref}=10\sin(0.05x)+5 \tag{50}$$

So, one has

$$\bar{\phi}=\left[\frac{1}{800}\cos\left(\frac{x}{20}\right)\left(\cos\left(\theta\right)\right)^2\right]v^3\cos\left(\theta\right)\tan\left(\psi\right)-\left[\frac{1}{40L}\sin\left(\frac{x}{20}\right)\sin\left(\theta\right)\tan\left(\psi\right)\right]v^3\cos\left(\theta\right)\tan\left(\psi\right)+$$

$$\left[-\frac{1}{20}\sin\left(\frac{x}{20}\right)\cos\left(\theta\right)\sin\left(\theta\right)\right]v^3\frac{\tan\left(\psi\right)}{L}+\left[\frac{\left(\frac{1}{2}\cos\left(\frac{x}{20}\right)\cos\left(\theta\right)-\sin\left(\theta\right)\right)\tan\left(\psi\right)}{L}\right]v^3\frac{\tan\left(\psi\right)}{L}$$

$$\bar{\varphi}=\frac{v^2}{L}\left[\frac{1}{2}\cos\left(\frac{x}{20}\right)\sin\left(\theta\right)+\cos\left(\theta\right)\right]\left[1+\tan^2\left(\psi\right)\right] \tag{51}$$

In this part, we apply the different controls techniques seen above to the model of car (43) and we compare their rapidity of convergence to the sliding surface, their amplitudes, oscillations in signal controls and their robustness against parametric variation and external disturbances. Also, we will show the most efficient algorithm. First, we consider the case where noises are absents. Simulation results plotted on figure 6 show that control objective is fulfilled using different algorithms. Morover, one can show our controller (37) converges fastest of other strategies of HOSMC. Besides, we note our proposed control law has very low amplitude compared with other techniques of HOSMC.

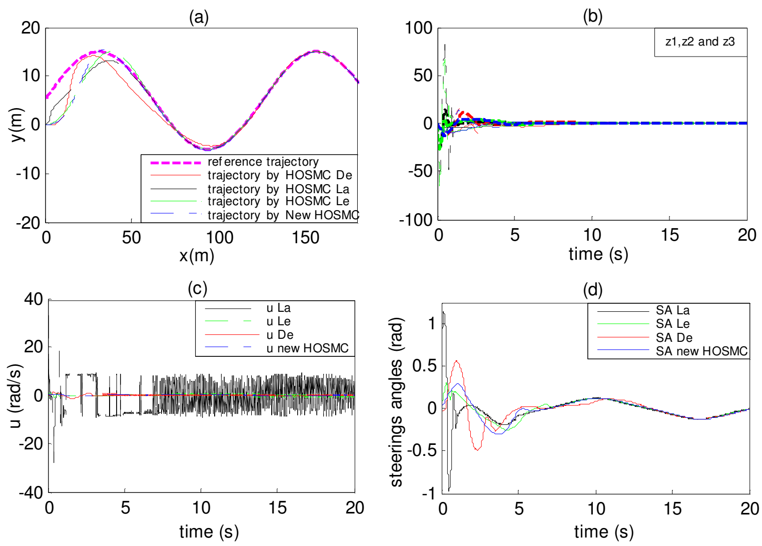

Figure 6 Tracking performances without noises, (a) y and y_{ref}, (b) z_i $0 \le i \le 2$, (c) Controls signals, (d) Steering angle.

Now, the measurement is assumed noisy. So, the output y is affected by a noise of expression 0.5sin(2t), t is the time, and the longitudinal velocity is assumed variable in time according to the law v(t)=10+0.5sin(t).

Simulations results depicted on figure 7 show the good robustness of the different approaches of HOSMC and in particular our proposed control against parametric variation and external disturbances.

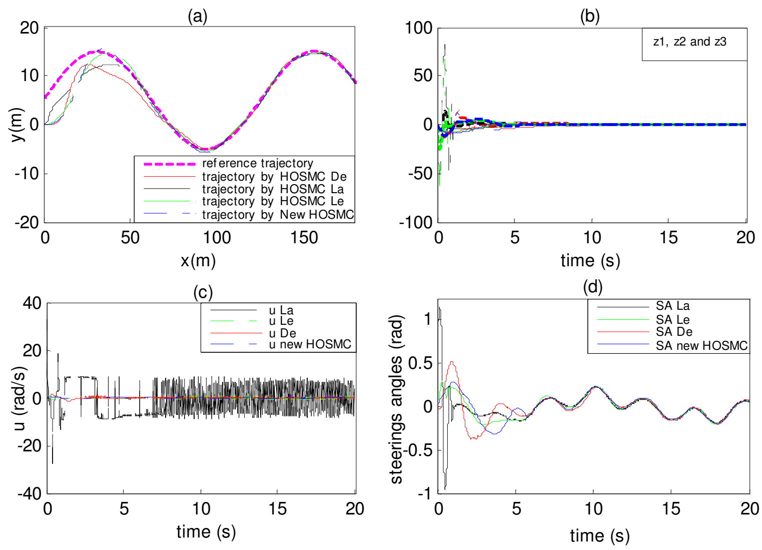

Figure 7 Tracking performances under disturbances, (a) y and yref, (b) z_i $0 \le i \le 2$, (c) Controls signals, (d) Steering angle.

5. CONCLUSION

In this work, we have presented a new design of a robust high-order sliding mode controller for a class of nonlinear uncertain systems. The proposed approach can stabilize exponentially the higher-order systems with respect to the sliding variable in the sliding surface. Furthermore, we have shown the robustness of our proposed controller against parametric uncertainties and output noises. In addition, we have compared three techniques of high-order sliding mode control with our proposed HOSMC. Applied to the model of a car, the simulation results show the novel technique of HOSMC is the most efficient algorithm.

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