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Unknown inputs observers for a class of non triangular nonlinear systems

Moncef TRIKI[1,2], Mondher farza[3] and Yassine KOUBAA[1,2]

Abstract- In the present paper, we suggest a high gain observer for a class of multi-output nonlinear systems with nonlinearly parameterized unknown inputs. This observer should enable to estimate both the whole state and the unknown inputs, simultaneously. Not only does the gain of the observer not require the resolution of any dynamical system, but it is also explicitly given. For the sake of simplicity, the observer tuning is reduced to the choice of a single design parameter. The paper also reports on the simulation results for the sake of highlighting the performance of the proposed observer.

Keywords: Nonlinear system, High gain observer, non triangular, Unknown inputs

1. INTRODUCTION

Over the past few years, the design problem of the unknown input observers for nonlinear systems has received much attention from researchers and triggered a lot of studies. Multiple results are widely available for linear systems and can be found in [1, 4, 8, 9, 10]. Our objective was to estimate the non-measured state variables without the adoption of any assumption on the part of the unknown inputs. A necessary and sufficient condition is derived to design such observers for linear systems. In the literature, many formulations of such a condition could be found [1, 4, 10]. In a relatively recent work [7], the authors provide a necessary and sufficient condition to design an unknown input observer for bilinear systems. This means that the unknown input interferes in a bilinear manner. In case of the unknown inputs interfering nonlinearly, the target objective is seldom attained and the new objective consequently consists in estimating either an input subset or all unknown inputs and/or the missing state, simultaneously (see e.g. [2, 3, 5, 11, 12]).

The present work investigates a class of uniformly observable MIMO systems involving nonlinear inputs that intervene in a nonlinear manner. It then proceeds to suggest a full-order, high gain observer for the simultaneous estimation of both non-measured states and unknown inputs. The proposed approach does not necessitate the output differentiation. Nor does it only assume the dynamics of these inputs are bounded without making any hypothesis on how these inputs vary.

The present paper contains four sections. Together with the first introductory section, the second section outlines the class of nonlinear systems at the basis of the proposed observer design. While the third section comes in the form of an observer synthesis, the fourth section is a simulation example with the aim of highlighting the performance of the proposed observer.

1. Laboratory of Sciences and Techniques of Automatic control and computer engineering (Lab-STA)
2.National School of Engineering of Sfax, University of Sfax Postal Box 1173, 3038 Sfax, Tunisia
Email: moncef_trik1@yahoo.fr
3. GREYC, UMR 6072 CNRS, Université de Caen, ENSICAEN 6 Bd Maréchal Juin, 14050 Caen Cedex, France

2. PROBLEM FORMULATION

Consider the following class of MIMO nonlinear systems:

$$\begin{cases} \dot{x} = f(u,x) + g(u,x,v) \\ y = x^1 \end{cases} \tag{1}$$

with

$$x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix}, \quad f(u,x) = \begin{pmatrix} f^1(u,x^1,x^2) \\ f^2(u,x^1,x^2,x^3) \\ \vdots \\ f^{q-1}(u,x) \\ f^q(u,x) \end{pmatrix}, \quad g(u,x,v) = \begin{pmatrix} g^1(u,x^1,v) \\ g^2(u,x^1,x^2,v) \\ \vdots \\ g^{q-1}(u,x^1,\dots,x^{q-1},v) \\ g^q(u,x,v) \end{pmatrix}$$

where $x \in n$ the state vector $x^k \in n_k$, k=1,….,q et $p = n_1 \geq n_2 \geq \cdots \geq n_q$, $\sum_{k=1}^{q} n_k = n;$

the unknown input $v = \begin{pmatrix} v^1 \\ v^2 \\ \vdots \\ v^r \end{pmatrix} \in m$ $v^j \in m_j$, $j = 1,\dots,r$ with $\sum_{j=1}^{r} m_j = m$

the known input the known input $u(t) \in \bigcup$ the set of absolutely continuous functions with bounded time derivatives from $\mathbb{R}^+ \in \bigcup$ a compact subset of $\mathbb{R}^\mu$; $f(u,x) \in \mathbb{R}^n$ and $f^k(u,x) \in \mathbb{R}^{n_k}$. One defines the characteristic indices, ρ_k, associated to the unknown inputs as follows. One assumes that there exist r integers $1 \leq \rho_1 \leq \rho_2 \leq \cdots \leq \rho_r \leq q$ such that

$$\forall i < \rho_j : \frac{\partial g^i}{\partial v^i}(u,x,v) \equiv 0 \text{ et } \frac{\partial g^{\rho j}}{\partial v^j}(u,x,v) \neq 0 \tag{2}$$

Due to the lack of place, we only deal with the case where $\rho_k < q$. Nevertheless, the case where $\rho_k = q$ can be similarly treated and it explained elsewhere. For the same reason, we also suppose that $\rho_1 > 1$. Moreover, the case where $\rho_1 = 1$ can also be treated in a similar manner.

For notation convenience, one sets for k=1, ….., q,

$$\varphi^k(u,x,v) = f^k(u,x) + g^k(u,x,v) \tag{3}$$

The observer synthesis requires the adoption of some hypotheses that shall be stated at due time. At this juncture, we assume the following two hypotheses:

H1: For 1≤ k ≤q_{nk+1}, the map $x^{k+1} \to f^k(u,x^1, \dots, x^k, x^{k+1})$ from $\mathbb{R}^{n_{k+1}} \to \mathbb{R}^{n_k}$ is one to one for all $(u,x^1, \dots, x^k)$. Moreover, one assumes that $\exists$ $\alpha_1, \beta_1 > 0$ such that for all $k \in \{1,\dots,q-1\}, \forall x \in \mathbb{R}^n, \forall u \in \bigcup$,

$$0 < \alpha_1^2 I_{n_k+1} \leq \left(\frac{\partial f^k}{\partial x^{k+1}}(u,x)\right)^T \frac{\partial f^k}{\partial x^{k+1}}(u,x) \leq \beta_1^2 I_{n_k+1}$$

where I_{n_k+1} is the identity matrix $(n_{k+1})\times(n_{k+1})$.

H2: For $1,....,r$, the map $\begin{pmatrix} x^{\rho_j+1} \\ v^j \end{pmatrix} \rightarrow \varphi^{\rho_j}\left(u, x^1, \ ...\ , x^{\rho_j}, x^{\rho_j+1}, v^1, \ ...\ , v^j\right)$

from $\mathbb{R}^{n_{\rho_{j+1}}+m_j} \rightarrow \mathbb{R}^{n_{\rho_j}}$ from is one to one for all $\left(u, x^1, \ ...\ , x^{\rho_j}, v^1, \ ...\ , v^{j-1}\right)$. Moreover, one assumes that $\exists\ \alpha_2, \beta_2 > 0$ such that, $\forall x \in \mathbb{R}^n$, $\forall u \in U$, $\forall v \in \mathbb{R}^m$

$$0 < \alpha_2^2 I_{n_{\rho_j}+1+m_j} \leq \left(F^{\rho_j}(u,x,v)\right)^T F^{\rho_j}(u,x,v) \leq \beta_2^2 I_{n_{\rho_j}+1+m_j} \tag{4}$$

where

$$F^{\rho_j}(u,x,v) = \frac{\partial \varphi^{\rho_j}}{\partial x^{\rho_j+1}}(u,x,v) \frac{\partial \varphi^{\rho_j}}{\partial v^j}(u,x,v) \tag{5}$$

Before detailing the observer synthesis, one notices that hypothesis (H2) implies that m≥p-1. Indeed, according to this hypothesis, the rank of the matrix

$F^{\rho_j}(u,x,v)$ is equal to $n_{\rho_{j+1}} + m_j$. Since this matrix is rectangular with n_{ρ_j} rows and $n_{\rho_{j+1}} + m_j$ column, condition (4) then implies that:

$$n_{\rho_{j+1}} + m_j \leq n_{\rho_j} \ , \ j = 1, ..., r \tag{6}$$

or equivalently

$$m_j \leq n_{\rho_j} - n_{\rho_{j+1}} \ , \ j = 1, ..., r \tag{7}$$

Taking the sum from *1 to r*, one gets:

$$\begin{aligned} m &\leq \sum_{j=1}^{r}\left(n_{\rho_j} - n_{\rho_{j+1}}\right) \\ &= n_{\rho 1} - n_{\rho_r+1} \\ &\leq n_1 - n_q \\ &\leq p - 1 \end{aligned}$$

The last two equalities result from the facts that $p = n_1 \geq n_2 \geq ..... \geq n_q$.

In the sequel, we are to immerse the original system (1) in an augmented system with the same outputs and that has some structural properties that shall be put forward. Then, we introduce a state transformation that puts the augmented system into a form characterizing a class of systems that are observable for any input. This form shall then be used to design an observer for the simultaneous estimation of the states and the unknown inputs. For purposes of clarity and due to the limited space available, the design shall be detailed in the case where *r=2*. Nevertheless, the case where *r ≥ 1* can be dealt with in a similar manner.

3. OBSERVER DESIGN

Since ***r=2***, the unknown input ***v*** contains two components

$$v = \begin{pmatrix} v^1 \\ v^2 \end{pmatrix} \in \mathbb{R}^m, \ v^j \in \mathbb{R}^{m_j}, \ j = 1,2 \text{ with } m = m_1 + m_2$$

For clarity purposes, one uses the notation v^1, v^2 to refer to the unknown input. As stated above, one shall assume that $1<\rho_2<q$. To summarize, system (1) can be written as follows:

$$\begin{cases} \dot{x}^1 = f^1\left(u,x^1,x^2\right)+g^1\left(u,x^1\right) \\ \vdots \\ \dot{x}^{\rho_1-1} = f^{\rho_1-1}\left(u,x^1,\cdots,x^{\rho_1}\right)+g^{\rho_1-1}\left(u,x^1,\cdots,x^{\rho_1-1}\right) \\ \dot{x}^{\rho_1} = f^{\rho_1}\left(u,x^1,\cdots,x^{\rho_1+1}\right)+g^{\rho_1}\left(u,x^1,\cdots,x^{\rho_1},v^1\right) \\ \vdots \\ \dot{x}^{\rho_2-1} = f^{\rho_2-1}\left(u,x^1,\cdots,x^{\rho_2}\right)+g^{\rho_2-1}\left(u,x^1,\cdots,x^{\rho_2-1},v^1\right) \\ \dot{x}^{\rho_2} = f^{\rho_2}\left(u,x^1,\cdots,x^{\rho_2+1}\right)+g^{\rho_2}\left(u,x^1,\cdots,x^{\rho_2},v^1,v^2\right) \\ \vdots \\ \dot{x}^{\rho_r} = f^{\rho_r}\left(u,x^1,\cdots,x^{\rho_r+1}\right)+g^{\rho_r}\left(u,x^1,\cdots,x^{\rho_r},v^1,v^2,\ldots,v^r\right) \\ \vdots \\ \dot{x}^{q-1} = f^{q-1}\left(u,x^1,\cdots,x^{q}\right)+g^{q-1}\left(u,x^1,\cdots,x^{q-1},v\right) \\ \dot{x}^{q} = f^{q}\left(u,x^1,\cdots,x^{q}\right)+g^{q}\left(u,x^1,\cdots,x^{q},v\right) \\ y = x^1 \end{cases} \tag{8}$$

a) Augmented system

We shall immerse the system (8) in an augmented system. More precisely, the augmented system is composed by three cascade blocs and the outputs of each these blocs is equal to those of the original system, i.e. system (8).The first system is composed of the ρ_1+1 first blocks of the system (8) and the unknown input v^1. The second system is composed of the ρ_2+1 first blocks of the system (8) and the unknown input v^2.

Those associated with the third bloc are a copy of the original system state variables, i.e. $x^1 \ldots\ldots x^q$. Therefore, the map that immerses the original system into the augmented one can be described by the following one-to-one map:

$$\begin{cases} \dot{x}^1 = f^1\left(u,x^1,x^2\right)+g^1\left(u,x^1\right) \\ \vdots \\ \dot{x}^{\rho_1-1} = f^{\rho_1-1}\left(u,x^1,\cdots,x^{\rho_1}\right)+g^{\rho_1-1}\left(u,x^1,\cdots,x^{\rho_1-1}\right) \\ \dot{x}^{\rho_1} = f^{\rho_1}\left(u,x^1,\cdots,x^{\rho_1+1}\right)+g^{\rho_1}\left(u,x^1,\cdots,x^{\rho_1},v^1\right) \\ \dot{x}^{\rho_1+1} = f^{\rho_1+1}\left(u,x^1,\cdots,x^{\rho_1+2}\right)+g^{\rho_1+1}\left(u,x^1,\cdots,x^{\rho_1+1},v^1\right) \\ \dot{v}^1(t) = \varepsilon^1(t) \\ y = x^1 \end{cases} \tag{9}$$

$$\begin{cases} \dot{x}^1 = f^1\left(u,x^1,x^2\right)+g^1\left(u,x^1\right) \\ \vdots \\ \dot{x}^{\rho_1-1} = f^{\rho_1-1}\left(u,x^1,\cdots,x^{\rho_1}\right)+g^{\rho_1-1}\left(u,x^1,\cdots,x^{\rho_1-1}\right) \\ \dot{x}^{\rho_1} = f^{\rho_1}\left(u,x^1,\cdots,x^{\rho_1+1}\right)+g^{\rho_1}\left(u,x^1,\cdots,x^{\rho_1},v^1\right) \\ \vdots \\ \dot{x}^{\rho_2-1} = f^{\rho_2-1}\left(u,x^1,\cdots,x^{\rho_2}\right)+g^{\rho_2-1}\left(u,x^1,\cdots,x^{\rho_2-1},v^1\right) \\ \dot{x}^{\rho_2} = f^{\rho_2}\left(u,x^1,\cdots,x^{\rho_2+1}\right)+g^{\rho_2}\left(u,x^1,\cdots,x^{\rho_2},v^1,v^2\right) \\ \dot{x}^{\rho_2+1} = f^{\rho_2+1}\left(u,x^1,\cdots,x^{\rho_2+2}\right)+g^{\rho_2+1}\left(u,x^1,\cdots,x^{\rho_2+1},v^1,v^2\right) \\ \dot{v}^2(t) = \varepsilon^2(t) \\ y = x^1 \end{cases} \tag{10}$$

$$\begin{cases} \dot{x}^1 = f^1\left(u, x^1, x^2\right) + g^1\left(u, x^1\right) \\ \vdots \\ \dot{x}^{\rho_1 - 1} = f^{\rho_1 - 1}\left(u, x^1, \cdots, x^{\rho_1}\right) + g^{\rho_1 - 1}\left(u, x^1, \cdots, x^{\rho_1 - 1}\right) \\ \dot{x}^{\rho_1} = f^{\rho_1}\left(u, x^1, \cdots, x^{\rho_1 + 1}\right) + g^{\rho_1}\left(u, x^1, \cdots, x^{\rho_1}, v^1\right) \\ \vdots \\ \dot{x}^{\rho_2 - 1} = f^{\rho_2 - 1}\left(u, x^1, \cdots, x^{\rho_2}\right) + g^{\rho_2 - 1}\left(u, x^1, \cdots, x^{\rho_2 - 1}, v^1\right) \\ \dot{x}^{\rho_2} = f^{\rho_2}\left(u, x^1, \cdots, x^{\rho_2 + 1}\right) + g^{\rho_2}\left(u, x^1, \cdots, x^{\rho_2}, v^1, v^2\right) \\ \vdots \\ \dot{x}^{\rho_r} = f^{\rho_r}\left(u, x^1, \cdots, x^{\rho_r + 1}\right) + g^{\rho_r}\left(u, x^1, \cdots, x^{\rho_r}, v^1, v^2, \ldots, v^r\right) \\ \vdots \\ \dot{x}^{q-1} = f^{q-1}\left(u, x^1, \cdots, x^q\right) + g^{q-1}\left(u, x^1, \cdots, x^{q-1}, v\right) \\ \dot{x}^q = f^q\left(u, x^1, \cdots, x^q\right) + g^q\left(u, x^1, \cdots, x^q, v\right) \\ y = x^1 \end{cases} \tag{11}$$

b) Observer equations in the original coordinates}

Considering a similar transformation to (12), we can see [6, 7]

$$\Psi = \mathbb{R}^n \to \mathbb{R}^{n_0 q}$$

$$x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix} \to z = \Psi(u, x) = \begin{pmatrix} z^1 \\ z^2 \\ \vdots \\ z^q \end{pmatrix}$$

with

$$\begin{cases} \dot{z}^1 = f^0\left(u, x^1\right) \\ \dot{z}^1 = \dfrac{\partial f^0}{\partial x^1}\left(u, x^1\right) f^1\left(u, x^1, x^2\right) \\ \dot{z}^3 = \dfrac{\partial f^0}{\partial x^1}\left(u, x^1\right) \dfrac{\partial f^1}{\partial x^2}\left(u, x^1, x^2\right) f^2\left(u, x^1, x^2, x^3\right) \\ \vdots \\ \dot{z}^q = \left(\displaystyle\prod_{k=0}^{q-2} \dfrac{\partial f^k}{\partial x^{k+1}}(u, x) \right) f^{q-1}(u, x) \end{cases} \tag{12}$$

where $z^k \in \mathbb{R}^{n_0}, k = 1, \ldots, q$

We can put the whole system formed by the ***r+1*** subsystems as (13) for which a type of observer (14) will be synthesized.

$$\begin{cases} \dot{x} = Ax + \varphi(u, x) + \bar{\varepsilon}(t) \\ y = Cx \end{cases} \tag{13}$$

$$x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix} \in \mathbb{R}^n \quad \text{with} \quad x^k = \begin{pmatrix} x_1^k \\ x_2^k \\ \vdots \\ x_{\lambda_k}^k \end{pmatrix} \in \mathbb{R}^{n_k} \quad \text{where} \quad x_i^k = \begin{pmatrix} x_{i,1}^k \\ x_{i,2}^k \\ \vdots \\ x_{i,p_k}^k \end{pmatrix} \in \mathbb{R}^{p_k} \quad \text{with } x_{i,j}^k \in \mathbb{R}$$

for $k=1,.....,q,\ \ i=1,...,\lambda_k,\ \ j=1,...,p_k$

with $\sum_{k=1}^{q} n_k = \sum_{k=1}^{q} p_k \lambda_k = n$; $p_k \geq 1$ et $\lambda_k \geq 2$

$$\dot{x}^k = A_k \hat{x}^k + \varphi^k(u,\hat{x}) - \theta^{\delta_k} \Delta_k^{-1}(\theta) S_k^{-1} C_k^T C_k e^k \quad (14)$$

In fact, the global observer is constituted by ***r+1*** subsystems cascade. The first subsystem serves to provide an estimation of v^1, the second is to estimate of v^2. The last subsystem allows the estimation of all the states.

We give in follows the ***r+1*** diagonal matrices by block, $\Lambda_k(u,x,v)$, which are in fact on the diagonal of the jacobienne of the transformation that puts the system (8) under the shape(13).

Before giving the expressions of $\Lambda_k(u,x,v)$, we define the following matrices:

$$F_j(u,x,v) = \left[\frac{\partial f^{\rho_j}}{\partial x^{\rho_j+1}}(u,x) \quad \frac{\partial g^{\rho_j}}{\partial v^j}(u,x,v) \right] \text{ for } j=1,...,r \quad (15)$$

The matrices $\Lambda_k(u,x,v)$ are defines us following :

$$\Lambda_1(u,x,v) = diag\left(I_p, \frac{\partial f^1}{\partial x^2}(u,x),......, \prod_{k=1}^{\rho_1-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x), \prod_{k=1}^{\rho_1-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x) F_1(u,x,v) \right)$$

$$\Lambda_2(u,x,v) = diag\left(I_p, \frac{\partial f^1}{\partial x^2}(u,x),......, \prod_{k=1}^{\rho_2-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x), \prod_{k=1}^{\rho_2-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x) F_2(u,x,v) \right)$$

$$\vdots \quad (16)$$

$$\Lambda_r(u,x,v) = diag\left(I_p, \frac{\partial f^1}{\partial x^2}(u,x),......, \prod_{k=1}^{\rho_r-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x), \prod_{k=1}^{\rho_r-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x) F_r(u,x,v) \right)$$

$$\Lambda_{r+1}(u,x,v) = diag\left(I_p, \frac{\partial f^1}{\partial x^2}(u,x),......, \prod_{k=1}^{q-1} \frac{\partial f^k}{\partial x^{k+1}}(u,x) \right)$$

Before giving the equations of the observer, we will rewrite the equations of systems by allocating to every block a variable of state. Indeed, each block ***k*** for ***k=1, …. , r***, We associate the state variable

$$\bar{z}^k = \begin{pmatrix} z^k \\ v^k \end{pmatrix} \quad \text{where} \quad z^k = \begin{pmatrix} z_1^k \\ \vdots \\ z_{\rho_k+1}^k \end{pmatrix}$$

With these notations, the previous system can be rewritten as follows:

$$\begin{cases} \dot{z}_1^1 = f^1\left(u, z_1^1, z_2^1\right) + g^1\left(u, z_1^1\right) \\ \vdots \\ \dot{z}_{\rho_1-1}^1 = f^{\rho_1-1}\left(u, z_1^1, \ldots, z_{\rho_1}^1\right) + g^{\rho_1-1}\left(u, z_1^1, \ldots, z_{\rho_1-1}^1\right) \\ \dot{z}_{\rho_1}^1 = f^{\rho_1}\left(u, z_1^1, \ldots, z_{\rho_1+1,1}^1\right) + g^{\rho_1}\left(u, z_1^1, \ldots, z_{\rho_1}^1, z_{\rho_1+1,2}^1\right) \\ \begin{pmatrix} \dot{z}_{\rho_1+1}^1 \\ \dot{v}^1 \end{pmatrix} = \begin{pmatrix} f^{\rho_1+1}\left(u, z_1^1, \ldots, z_{\rho_1+2}^1\right) + g^{\rho_1+1}\left(u, z_1^1, \ldots, z_{\rho_1}^1, z_{\rho_1+1,1}^1, z_{\rho_1+1,2}^1\right) \\ \varepsilon^1(t) \end{pmatrix} \\ y = z_1^1 \end{cases} \tag{17}$$

$$\begin{cases} \dot{z}_1^2 = f^1\left(u, z_1^2, z_2^2\right) + g^1\left(u, z_1^2\right) \\ \vdots \\ \dot{z}_{\rho_1-1}^2 = f^{\rho_1-1}\left(u, z_1^2, \ldots, z_{\rho_1}^2\right) + g^{\rho_1-1}\left(u, z_1^2, \ldots, z_{\rho_1-1}^2\right) \\ \dot{z}_{\rho_1}^2 = f^{\rho_1}\left(u, z_1^2, \ldots, z_{\rho_1+1,1}^2\right) + g^{\rho_1}\left(u, z_1^2, \ldots, z_{\rho_1}^2, z_{\rho_1+1,2}^2\right) \\ \vdots \\ \dot{z}_{\rho_2-1}^2 = f^{\rho_2-1}\left(u, z_1^2, \ldots, z_{\rho_2}^2\right) + g^{\rho_2-1}\left(u, z_1^2, \ldots, z_{\rho_2-1}^2, z_{\rho_1+1,2}^2\right) \\ \dot{z}_{\rho_2}^2 = f^{\rho_2}\left(u, z_1^2, \ldots, z_{\rho_2+1,1}^2\right) + g^{\rho_2}\left(u, z_1^2, \ldots, z_{\rho_2}^2, z_{\rho_1+1,2}^2, z_{\rho_2+1,2}^2\right) \\ \dot{z}_{\rho_2+1}^2 = \begin{pmatrix} f^{\rho_2+1}\left(u, z_1^2, \ldots, z_{\rho_2+2}^2\right) + g^{\rho_2+1}\left(u, z_1^2, \ldots, z_{\rho_2}^2, z_{\rho_2+1,1}^2, z_{\rho_2+1,2}^2\right) \\ \varepsilon^2(t) \end{pmatrix} \\ y = z_1^2 \end{cases} \tag{18}$$

$$\begin{cases} \dot{z}_1^r = f^1\left(u, z_1^r, z_2^r\right) + g^1\left(u, z_1^r\right) \\ \vdots \\ \dot{z}_{\rho_1-1}^r = f^{\rho_1-1}\left(u, z_1^r, \ldots, z_{\rho_1}^r\right) + g^{\rho_1-1}\left(u, z_1^r, \ldots, z_{\rho_1-1}^r\right) \\ \dot{z}_{\rho_1}^r = f^{\rho_1}\left(u, z_1^r, \ldots, z_{\rho_1+1,1}^r\right) + g^{\rho_1}\left(u, z_1^r, \ldots, z_{\rho_1}^r, z_{\rho_1+1,2}^r\right) \\ \vdots \\ \dot{z}_{\rho_2-1}^r = f^{\rho_2-1}\left(u, z_1^r, \ldots, z_{\rho_2}^r\right) + g^{\rho_2-1}\left(u, z_1^r, \ldots, z_{\rho_2-1}^r, z_{\rho_1+1,2}^r\right) \\ \dot{z}_{\rho_2}^r = f^{\rho_2}\left(u, z_1^r, \ldots, z_{\rho_2+1,1}^r\right) + g^{\rho_2}\left(u, z_1^r, \ldots, z_{\rho_2}^r, z_{\rho_1+1,2}^r, z_{\rho_2+1,2}^r\right) \\ \vdots \\ \dot{z}_{\rho_r-1}^r = f^{\rho_r-1}\left(u, z_1^r, \ldots, z_{\rho_r}^r\right) + g^{\rho_r-1}\left(u, z_1^r, \ldots, z_{\rho_r-1}^r, z_{\rho_1+1,2}^r, \ldots. z_{\rho_{r-1}+1,2}^r\right) \\ \dot{z}_{\rho_r}^r = f^{\rho_r}\left(u, z_1^r, \ldots, z_{\rho_r+1}^r\right) + g^{\rho_r}\left(u, z_1^r, \ldots, z_{\rho_r}^r, z_{\rho_1+1,2}^r, \ldots, z_{\rho_r+1,2}^r\right) \\ \dot{z}_{\rho_r+1}^r = \begin{pmatrix} f^{\rho_r+1}\left(u, z_1^r, \ldots, z_{\rho_r+2}^r\right) + g^{\rho_r+1}\left(u, z_1^r, \ldots, z_{\rho_r}^2, z_{\rho_1+1,1}^2, z_{\rho_2+1,2}^r, \ldots, z_{\rho_r+1,2}^r\right) \\ \varepsilon^r(t) \end{pmatrix} \\ y = z_1^r \end{cases} \tag{19}$$

$$
\begin{cases}
\dot{z}^1 = f^1\left(u, z^1, z^2\right) + g^1\left(u, z^1\right) \\
\vdots \\
\dot{z}^{\rho_1-1} = f^{\rho_1-1}\left(u, z^1, \cdots, z^{\rho_1}\right) + g^{\rho_1-1}\left(u, z^1, \cdots, z^{\rho_1-1}\right) \\
\dot{z}^{\rho_1} = f^{\rho_1}\left(u, z^1, \cdots, z^{\rho_1+1}\right) + g^{\rho_1}\left(u, z^1, \cdots, z^{\rho_1}, z^1_{\rho_1+1,2}\right) \\
\vdots \\
\dot{z}^{\rho_2-1} = f^{\rho_2-1}\left(u, z^1, \cdots, z^{\rho_2}\right) + g^{\rho_2-1}\left(u, z^1, \cdots, z^{\rho_2-1}, z^1_{\rho_1+1,2} v^1\right) \\
\dot{z}^{\rho_2} = f^{\rho_2}\left(u, z^1, \cdots, z^{\rho_2+1}\right) + g^{\rho_2}\left(u, z^1, \cdots, z^{\rho_2}, z^1_{\rho_1+1,2}, z^2_{\rho_2+1,2}\right) \\
\vdots \\
\dot{z}^{\rho_r} = f^{\rho_r}\left(u, z^1, \cdots, z^{\rho_r+1}\right) + g^{\rho_r}\left(u, z^1, \cdots, z^{\rho_r}, z^1_{\rho_1+1,2}, z^2_{\rho_2+1,2}, \ldots, z^r_{\rho_r+1,2}\right) \\
\vdots \\
\dot{z}^{q-1} = f^{q-1}\left(u, z^1, \cdots, z^q\right) + g^{q-1}\left(u, z^1, \cdots, z^{q-1}, v\right) \\
\dot{z}^q = f^q\left(u, z^1, \cdots, z^q\right) + g^q\left(u, z^1, \cdots, z^q, v\right) \\
y = z^1
\end{cases}
\tag{20}
$$

The Observer equations are:

$$
\begin{cases}
\dot{\hat{z}}^1_1 = f^1\left(u, z^1_1, \hat{z}^1_2\right) + g^1\left(u, z^1_1\right) - C^1_{\rho_1+1}\theta^{\delta_1}\Lambda^+_{1,1}\left(u, \hat{z}^1\right)\left(\hat{z}^1_1 - x^1\right) \\
\vdots \\
\dot{\hat{z}}^1_{\rho_1-1} = f^{\rho_1-1}\left(u, z^1_1, \ldots, \hat{z}^1_{\rho_1}\right) + g^{\rho_1-1}\left(u, z^1_1, \hat{z}^1_2, \ldots, \hat{z}^1_{\rho_1-1}\right) \\
\qquad - C^{\rho_1-1}_{\rho_1+1}\theta^{(\rho_1-1)\delta_1}\Lambda^+_{1,\rho_1-1}\left(u, \hat{z}^1\right)\left(\hat{z}^1_1 - x^1\right) \\
\dot{\hat{z}}^1_{\rho_1} = f^{\rho_1}\left(u, z^1_1, \ldots, \hat{z}^1_{\rho_1+1}\right) + g^{\rho_1}\left(u, z^1_1, \ldots, \hat{z}^1_{\rho_1}, \hat{v}^1\right) \\
\qquad - C^{\rho_1}_{\rho_1+1}\theta^{\rho_1\delta_1}\Lambda^+_{1,\rho_1}\left(u, \hat{z}^1\right)\left(\hat{z}^1_1 - x^1\right) \\
\begin{pmatrix} \dot{\hat{z}}^1_{\rho_1+1} \\ \dot{\hat{v}}^1 \end{pmatrix} = \begin{pmatrix} f^{\rho_1+1}\left(u, z^1_1, \ldots, \hat{z}^1_{\rho_1+2}\right) + g^{\rho_1+1}\left(u, z^1_1, \ldots, \hat{z}^1_{\rho_1}, \hat{z}^1_{\rho_1+1}, \hat{v}^1\right) \\ 0 \end{pmatrix} \\
\qquad - C^{\rho_1+1}_{\rho_1+1}\theta^{(\rho_1+1)\delta_1}\Lambda^+_{1,\rho_1+1}\left(u, \hat{z}^1, \hat{z}^1_{\rho_1+1,2}\right)\left(\hat{z}^1_1 - x^1\right)
\end{cases}
\tag{21}
$$

$$
\begin{cases}
\dot{\hat{z}}^2_1 = f^1\left(u, z^2_1, \hat{z}^2_2\right) + g^1\left(u, z^2_1\right) - C^1_{\rho_2+1}\theta^{\delta_2}\Lambda^+_{2,1}\left(u, \hat{z}^2\right)\left(\hat{z}^2_1 - \mathrm{x}^1\right) \\
\vdots \\
\dot{\hat{z}}^2_{\rho_1-1} = f^{\rho_1-1}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_1}\right) + g^{\rho_1-1}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_1-1}\right) - C^{\rho_1-1}_{\rho_1+1}\theta^{(\rho_1-1)\delta_2}\Lambda^+_{2,\rho_1-1}\left(u, \hat{z}^2\right)\left(\hat{z}^2_1 - \mathrm{x}^1\right) \\
\dot{z}^2_{\rho_1} = f^{\rho_1}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_1+1}\right) + g^{\rho_1}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_1}, \hat{v}^1\right) - C^{\rho_1}_{\rho_1+1}\theta^{\rho_1\delta_1}\Lambda^+_{1,\rho_1}\left(u, \hat{z}^2\right)\left(\hat{z}^2_1 - \mathrm{x}^1\right) \\
\vdots \\
\dot{\hat{z}}^2_{\rho_2-1} = f^{\rho_2-1}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_2}\right) + g^{\rho_2-1}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_2-1}, \hat{v}^1\right) - C^{\rho_2-1}_{\rho_2+1}\theta^{(\rho_2-1)\delta_2}\Lambda^+_{2,\rho_2-1}\left(u, \hat{z}^2\right)\left(\hat{z}^2_1 - \mathrm{x}^1\right) \\
\dot{\hat{z}}^2_{\rho_2} = f^{\rho_2}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_2+1}\right) + g^{\rho_2}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_2}, \hat{v}^1, \hat{v}^2\right) - C^{\rho_2}_{\rho_2+1}\theta^{\rho_2\delta_2}\Lambda^+_{2,\rho_2}\left(u, \hat{z}^2\right)\left(\hat{z}^2_1 - \mathrm{x}^1\right) \\
\begin{pmatrix} \dot{\hat{z}}^2_{\rho_2+1} \\ \dot{\hat{v}}^2 \end{pmatrix} = \begin{pmatrix} f^{\rho_2+1}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_2+1}, \hat{z}^2_{\rho_2+2}\right) + g^{\rho_2+1}\left(u, z^2_1, \ldots, \hat{z}^2_{\rho_2}, \hat{z}^2_{\rho_2+1}, \hat{v}^1, \hat{v}^2\right) \\ 0 \end{pmatrix} \\
\qquad - C^{\rho_2+1}_{\rho_2+1}\theta^{(\rho_2+1)\delta_2}\Lambda^+_{2,\rho_2+1}\left(u, \hat{z}^2, \hat{z}^1_{\rho_1+1,2}, \hat{z}^2_{\rho_2+1,2}\right)\left(\hat{z}^2_1 - \mathrm{x}^1\right)
\end{cases}
\tag{22}
$$

$$
\left\{
\begin{array}{l}
\dot{\hat{z}}_1^r = f^1\left(u, z_1^r, \hat{z}_2^r\right) + g^1\left(u, z_1^r\right) - C_{\rho_1+1}^1 \theta^{\delta_r} \Lambda_{r,1}^+\left(u, \hat{z}^r\right)\left(\hat{z}_1^r - x^1\right) \\
\vdots \\
\dot{\hat{z}}_{\rho_1-1}^r = f^{\rho_1-1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_1}^r\right) + g^{\rho_1-1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_1-1}^r\right) - C_{\rho_r+1}^{\rho_1-1} \theta^{(\rho_1-1)\delta_r} \Lambda_{r,\rho_1-1}^+\left(u, \hat{z}^r\right)\left(\hat{z}_1^r - x^1\right) \\
\dot{\hat{z}}_{\rho_1}^r = f^{\rho_1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_1+1}^r\right) + g^{\rho_1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_1}^r, \hat{v}^1\right) - C_{\rho_r+1}^{\rho_1} \theta^{\rho_1\delta_r} \Lambda_{r,\rho_1}^+\left(u, \hat{z}^r\right)\left(\hat{z}_1^3 - x^1\right) \\
\vdots \\
\dot{\hat{z}}_{\rho_2-1}^r = f^{\rho_2-1}\left(u, z_1^r, \hat{z}_2^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_2}^r\right) + g^{\rho_2-1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_2-1}^r, \hat{v}^1\right) - C_{\rho_r+1}^{\rho_2-1} \theta^{(\rho_2-1)\delta_r} \Lambda_{r,\rho_2-1}^+\left(u, \hat{z}^r\right)\left(\hat{z}_1^r - x^1\right) \\
\dot{\hat{z}}_{\rho_2}^r = f^{\rho_2}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_2+1}^r\right) + g^{\rho_2}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_2}^r, \hat{v}^1, \hat{v}^2\right) \\
\quad - C_{\rho_r+1}^{\rho_2} \theta^{\rho_2\delta_r} \Lambda_{r,\rho_2}^+\left(u, \hat{z}^r\right)\left(\hat{z}_1^r - x^1\right) \\
\vdots \\
\dot{\hat{z}}_{\rho_r-1}^r = f^{\rho_r-1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_r}^r\right) + g^{\rho_r-1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_r-1}^r, \hat{v}^1, \ldots, \hat{v}^{r-1}\right) \\
\quad - C_{\rho_r+1}^{\rho_r-1} \theta^{(\rho_r-1)\delta_r} \Lambda_{r,\rho_r-1}^+\left(u, \hat{z}^r\right)\left(\hat{z}_1^r - x^1\right) \\
\dot{\hat{z}}_{\rho_r}^r = f^{\rho_r}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_r+1}^r\right) + g^{\rho_r}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_r}^r, \hat{v}^1, \ldots, \hat{v}^r\right) \\
\quad - C_{\rho_r+1}^{\rho_r} \theta^{\rho_r\delta_r} \Lambda_{r,\rho_r}^+\left(u, \hat{z}^r\right)\left(\hat{z}_1^r - x^1\right) \\
\begin{pmatrix} \dot{\hat{z}}_{\rho r+1}^r \\ \hat{v}^r \end{pmatrix} = \begin{pmatrix} f^{\rho_r+1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_r+1}^r, \hat{z}_{\rho_r+2}^r\right) + g^{\rho_r+1}\left(u, z_1^r, \hat{z}_2^r, \ldots, \hat{z}_{\rho_r}^r, \hat{v}^1, \ldots, \hat{v}^r\right) \\ 0 \end{pmatrix} \\
\quad - C_{\rho_r+1}^{\rho r+1} \theta^{(\rho_r+1)\delta_r} \Lambda_{r,\rho_r+1}^+\left(u, \hat{z}^r\right)\left(\hat{z}_1^r - x^1\right)
\end{array}
\right. \tag{23}
$$

where δ_k, for $k = 1 \ldots., r$ are given by:

$$
\begin{aligned}
\delta_k &= 2^{r+1-k}\left(q - \frac{3}{2}\right)\prod_{i=k+1}^{r}\left(\rho_i + 1 - \frac{3}{2}\right) \\
&= 2^{r+1-k}\left(q - \frac{3}{2}\right)\prod_{i=k+1}^{r}\left(\rho_i - \frac{1}{2}\right)
\end{aligned} \tag{24}
$$

The expressions of δ_k, given by (24) are valid when $\rho_r + 1 < q$ with $\rho_r \neq q - 1$.

$$
\begin{aligned}
\delta_k &= 2^{r-k}\left(q - \frac{3}{2}\right)\prod_{i=k+1}^{r-1}\left(\rho_i + 1 - \frac{3}{2}\right) \\
&= 2^{r-k}\left(q - \frac{3}{2}\right)\prod_{i=k+1}^{r-1}\left(\rho_i - \frac{1}{2}\right)
\end{aligned} \tag{25}
$$

For the last block, we have $\delta_r = 1$

$$\begin{cases}
\dot{\hat{x}}^1 = f^1\left(u,x^1,\hat{x}^2\right)+g^1\left(u,x^1\right)-C_q^1\theta\Lambda_{r+1,1}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)\\
\vdots\\
\dot{\hat{x}}^{\rho_1-1} = f^{\rho_1-1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_1}\right)+g^{\rho_1-1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_1-1}\right)-C_q^{\rho_1-1}\theta^{(\rho_1-1)}\Lambda_{r+1,\rho_1-1}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)\\
\dot{\hat{x}}^{\rho_1} = f^{\rho_1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_1+1}\right)+g^{\rho_1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_1},\hat{v}^1\right)-C_q^{\rho_1}\theta^{\rho_1}\Lambda_{r+1,\rho_1}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)\\
\vdots\\
\dot{\hat{x}}^{\rho_2-1} = f^{\rho_2-1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_2}\right)+g^{\rho_2-1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_2-1},\hat{v}^1\right)-C_q^{\rho_2-1}\theta^{(\rho_2-1)}\Lambda_{r+1,\rho_2-1}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)\\
\dot{\hat{x}}^{\rho_2} = f^{\rho_2}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_2+1}\right)+g^{\rho_2}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_2},\hat{v}^1,\hat{v}^2\right)-C_q^{\rho_2}\theta^{\rho_2}\Lambda_{r+1,\rho_2}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)\\
\vdots\\
\dot{\hat{x}}^{\rho_r-1} = f^{\rho_r-1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_r}\right)+g^{\rho_r-1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_r-1},\hat{v}^1,\ldots,\hat{v}^{r-1}\right)-C_q^{\rho_r-1}\theta^{(\rho_r-1)}\Lambda_{r+1,\rho_r-1}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)\\
\dot{\hat{x}}^{\rho_r} = f^{\rho_r}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_r+1}\right)+g^{\rho_r}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{\rho_r},\hat{v}^1,\ldots,\hat{v}^{r}\right)-C_q^{\rho_r}\theta^{\rho_r}\Lambda_{r+1,\rho_r}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)\\
\vdots\\
\dot{\hat{x}}^{q-1} = f^{q-1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{q}\right)+g^{q-1}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{q},\hat{v}\right)-C_q^{q-1}\theta^{q-1}\Lambda_{r+1,q-1}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)\\
\dot{\hat{x}}^{q} = f^{q}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{q}\right)+g^{q}\left(u,x^1,\hat{x}^2,\ldots,\hat{x}^{q},\hat{v}\right)-C_q^{q}\theta^{q}\Lambda_{r+1,q}^{+}\left(u,\hat{x}\right)\left(\hat{x}^1-x^1\right)
\end{cases} \tag{26}$$

4. EXAMPLE

Consider the following dynamical system

$$\begin{cases}
\dot{x}_1 = \cos(t)x_4 - \sin(t)x_5\\
\dot{x}_2 = \sin(t)x_4 + \cos(t)x_5\\
\dot{x}_3 = x_4^3 + x_5^5 + \sin(x_3) + v_1^3 + v_1\\
\dot{x}_4 = x_3x_6 - (x_3-a)\left(v_2^3+v_2\right) - v_1\\
\dot{x}_5 = (x_3-a)x_6 + x_6v_2 - x_5 - \sin(v_1)\\
\dot{x}_6 = x_7^3 + x_7 - 3v_1\sin^2(v_2) - x_5 - v_1v_2\\
\dot{x}_7 = x_6 - 3x_5\cos(v_1) + 3x_5\sin(v_2)\\
y = \begin{pmatrix} x_1\\ x_2\\ x_3 \end{pmatrix}
\end{cases} \tag{27}$$

For simulation purposes, the following expression (unknown by the observer) has been considered for the unknown input:

$$v_1(t) = 0.5\cos(2\pi t) \tag{28}$$

$$v_2(t) = 0.3\cos(3\pi t) \tag{29}$$

we have here q=4, $\rho_1 = 1$ and $\rho_2 = 2$. The observer is composed by three cascade subsystems with $\delta_1 = 2^{3-1}\left(4-\frac{3}{2}\right)\left(3-\frac{3}{2}\right) = 15$ and $\delta_2 = 2^{2-1}\left(4-\frac{3}{2}\right)$=5 and $\delta_3 = 1$

according to (24). The employed values $\theta = 5$ and $a = 0.3$. The initial conditions for the model and the observer are:

$$x(0) = [1, 0.5, 0.9, 0.25, 0.9, -0.2, 0.9]^T$$
$$\hat{x}(0) = [1, 0.5, 0.9, -0.25, 0.4, -0.3, 0]^T$$
$$\hat{v}_1(0) = 0 \; ; \; \hat{v}_2(0) = -0.5$$

The observer is written as follows:

$$\begin{cases} \begin{pmatrix} \dot{\hat{z}}_1^1 \\ \dot{\hat{z}}_2^1 \\ \dot{\hat{z}}_3^1 \end{pmatrix} = \begin{pmatrix} \cos(t)\,\hat{z}_4^1 - \sin(t)\,\hat{z}_5^1 \\ \sin(t)\,\hat{z}_4^1 + \cos(t)\,\hat{z}_5^1 \\ \left(\hat{z}_4^1\right)^3 + \left(\hat{z}_5^1\right)^5 + \left(\hat{v}_1\right)^3 + \hat{v}_1 \end{pmatrix} - 2\theta^{\delta_1} \begin{pmatrix} \hat{z}_1^1 - x_1 \\ \hat{z}_2^1 - x_2 \\ \hat{z}_3^1 - x_3 \end{pmatrix} \\ \begin{pmatrix} \dot{\hat{z}}_4^1 \\ \dot{\hat{z}}_5^1 \\ \dot{\hat{v}}_1 \end{pmatrix} = \begin{pmatrix} x_3\hat{z}_6^2 - (x_3 - a)\left(\hat{v}_2^3 + \hat{v}_2\right) - \hat{v}_1 \\ (x_3 - a)\,\hat{z}_6^2 + x_3\hat{v}_2 - \hat{z}_5^1 - \sin(\hat{v}_1) \\ 0 \end{pmatrix} - \theta^{2\delta_1} \begin{pmatrix} \cos(t) & \sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 3\left(\hat{z}_4^1\right)^2 & 5\left(\hat{z}_5^1\right)^4 & 3\left(\hat{v}_1\right)^2 + 1 \end{pmatrix}^{-1} \begin{pmatrix} \hat{z}_1^1 - x_1 \\ \hat{z}_2^1 - x_2 \\ \hat{z}_3^1 - x_3 \end{pmatrix} \end{cases} \tag{30}$$

$$\begin{cases} \begin{pmatrix} \dot{\hat{z}}_1^2 \\ \dot{\hat{z}}_2^1 \\ \dot{\hat{z}}_3^2 \end{pmatrix} = \begin{pmatrix} \cos(t)\,\hat{z}_4^2 - \sin(t)\,\hat{z}_5^2 \\ \sin(t)\,\hat{z}_4^2 + \cos(t)\,\hat{z}_5^2 \\ \left(\hat{z}_4^2\right)^3 + \left(\hat{z}_5^2\right)^5 + \left(\hat{v}_1\right)^3 + \hat{v}_1 \end{pmatrix} - 3\theta^{\delta_2} \begin{pmatrix} \hat{z}_1^2 - x_1 \\ \hat{z}_2^2 - x_2 \\ \hat{z}_3^2 - x_3 \end{pmatrix} \\ \begin{pmatrix} \dot{\hat{z}}_4^2 \\ \dot{\hat{z}}_5^2 \end{pmatrix} = \begin{pmatrix} x_3\hat{z}_6^2 - (x_3 - a)\left(\hat{v}_2^3 + \hat{v}_2\right) - \hat{v}_1 \\ (x_3 - a)\,\hat{z}_6^2 + x_3\hat{v}_2 - \hat{z}_5^2 - \sin(\hat{v}_1) \end{pmatrix} - 3\theta^{2\delta_2} \begin{pmatrix} \cos(t) & \text{-}\sin(t) \\ \sin(t) & \cos(t) \\ 3\left(\hat{z}_4^2\right)^2 & 5\left(\hat{z}_5^2\right)^4 \end{pmatrix}^{+} \begin{pmatrix} \hat{z}_1^2 - x_1 \\ \hat{z}_2^2 - x_2 \\ \hat{z}_3^2 - x_3 \end{pmatrix} \\ \begin{pmatrix} \dot{\hat{z}}_6^2 \\ \dot{\hat{v}}_1 \end{pmatrix} = \begin{pmatrix} \left(\hat{z}_7^2\right)^3 + \hat{z}_7^2 - 3\hat{v}_1 \sin^2(\hat{v}_2) - \hat{v}_1\hat{v}_2 \\ 0 \end{pmatrix} - \theta^{3\delta_2} \left\{ \begin{pmatrix} \cos(t) & \text{-}\sin(t) \\ \sin(t) & \cos(t) \\ 3\left(\hat{z}_4^2\right)^2 & 5\left(\hat{z}_5^2\right)^4 \end{pmatrix} \begin{pmatrix} x_3 & (x_3 - a)\left(3\hat{v}_2^2 + 1\right) \\ (x_3 - a) & x_3 \end{pmatrix} \right\}^{+} \begin{pmatrix} \hat{z}_1^2 - x_1 \\ \hat{z}_2^2 - x_2 \\ \hat{z}_3^2 - x_3 \end{pmatrix} \end{cases} \tag{31}$$

$$\left\{\begin{array}{l}
\begin{pmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\\ \dot{\hat{x}}_3\end{pmatrix}=\begin{pmatrix}\cos(t)\,\hat{x}_4-\sin(t)\,\hat{x}_5\\ \sin(t)\,\hat{x}_4+\cos(t)\,\hat{x}_5\\ (\hat{x}_4)^3+(\hat{x}_5)^5+(\hat{v}_1)^3+\hat{v}_1\end{pmatrix}-4\theta\begin{pmatrix}\hat{x}_1-x_1\\ \hat{x}_2-x_2\\ \hat{x}_3-x_3\end{pmatrix}\\[2ex]
\begin{pmatrix}\dot{\hat{x}}_4\\ \dot{\hat{x}}_5\end{pmatrix}=\begin{pmatrix}x_3\hat{x}_6-(x_3-a)\left(\hat{v}_2^3+\hat{v}_2\right)-\hat{v}_1\\ (x_3-a)\,\hat{x}_6+x_3\hat{v}_2-\hat{x}_5-\sin(\hat{v}_1)\end{pmatrix}-6\theta^2\begin{pmatrix}\cos(t) & -\sin(t)\\ \sin(t) & \cos(t)\\ 3(\hat{x}_4)^2 & 5(\hat{x}_5)^4\end{pmatrix}^{+}\begin{pmatrix}\hat{x}_1-x_1\\ \hat{x}_2-x_2\\ \hat{x}_3-x_3\end{pmatrix}\\[2ex]
\left(\dot{\hat{x}}_6\right)=(\hat{x}_7)^3+\hat{x}_7-3\hat{v}_1\sin^2(\hat{v}_2)-\hat{x}_5+\hat{v}_1\hat{v}_2-4\theta^3\left\{\begin{pmatrix}\cos(t) & -\sin(t)\\ \sin(t) & \cos(t)\\ 3(\hat{x}_4)^2 & 5(\hat{x}_5)^4\end{pmatrix}\begin{pmatrix}x_3\\ (x_3-a)\end{pmatrix}\right\}^{+}\begin{pmatrix}\hat{x}_1-x_1\\ \hat{x}_2-x_2\\ \hat{x}_3-x_3\end{pmatrix}\\[2ex]
\dot{\hat{x}}_7=-\hat{x}_6-3\hat{x}_5\cos(\hat{v}_1)+3\hat{x}_5\sin(\hat{v}_2)-\theta^4\left\{\begin{pmatrix}\cos(t) & -\sin(t)\\ \sin(t) & \cos(t)\\ 3(\hat{x}_4)^2 & 5(\hat{x}_5)^4\end{pmatrix}\begin{pmatrix}x_3\\ (x_3-a)\end{pmatrix}\right\}^{+}\begin{pmatrix}\hat{x}_1-x_1\\ \hat{x}_2-x_2\\ \hat{x}_3-x_3\end{pmatrix}
\end{array}\right. \tag{32}$$

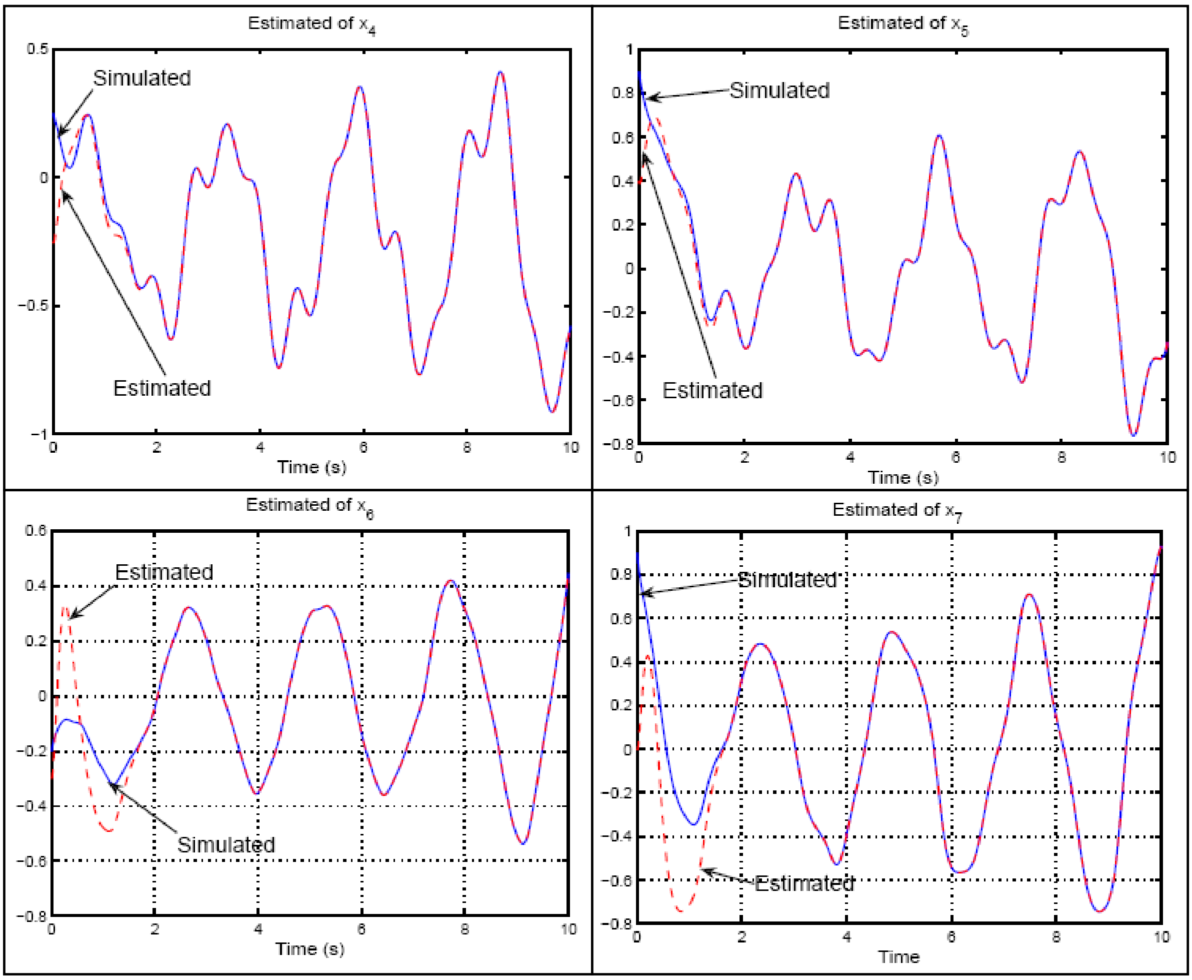

Figure 1 : Estimated of x_4, x_5, x_6 and x_7

Observer (30), (31) and (32) has been simulated using data issued from simulation. In figure1, the true time evolutions of x_4, x_5, x_6 and x_7 (issued from model simulation) are compared with their respective estimates provided by the observer.
The comparison between estimated and simulated unknown inputs is provided in figure 2. The obtained results clearly show the good agreement between the estimated and simulated variables. Recall that the expression of the unknown input (28) and (29) introduced for simulation purposes is ignored by the observer. The employed values of θ =5.

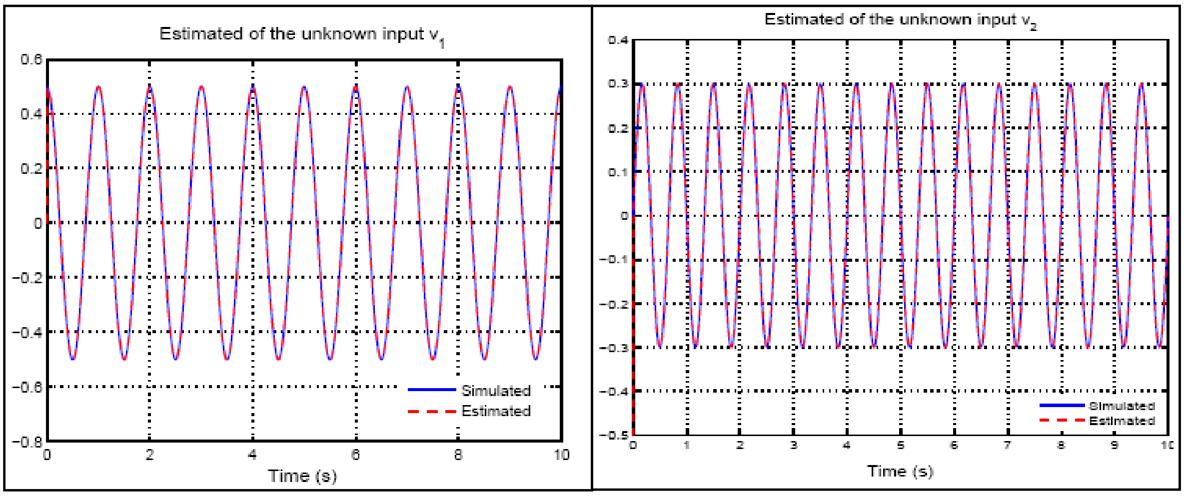

Figure 2 : Estimated of unknown inputs v_1 and v_2

5. CONCLUSION

An observer design is proposed for a class of nonlinear systems involving unknown inputs. The proposed observers allows the simultaneous estimation of the state and the unknown inputs. Simulation results have been provided in order to highlighted the performances of the proposed observers.

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