

## Joint optimization of maintenance and inventory with double age ordering policy

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*Abstract: In this paper, we study joint optimization of maintenance and inventory. We consider a system containing a single machine subject to random failure, and have two kinds of failures, major and minor ones. If a major failure does not occur until a scheduled replacement time  $T$ , the system is replaced at  $T$ . An order for a spare is placed at a scheduled ordering time  $t_0$  or unscheduled ordering time on failure before to  $t_e$  (expedited order). The objective is to determine jointly the three values  $(t_e, t_0, T)$  so as to minimize the expected cost rate. This cost includes the replacement, repair, inventory holding and shortage costs. To explain this policy a numerical example is also included.*

Keywords: Maintenance cost, Major failure, Minor failure, Ordering policy, Spare.

### 1 Introduction

Most of the maintenance models assume that the spare part is immediately available ([12];[7];[8]). However, in practice, this was not always the case. We should study jointly a spare ordering policy and a preventive maintenance policy to ensure the availability of spare part at preventive maintenance time. Therefore, in the last decades, the studies have proposed the joint optimization of spare parts inventory and maintenance policies ([1];[3];[4];[6];[9]). But they have assumed that the lead times and delivery costs of scheduled order and unscheduled order (expedited order) are equal. Normally, the administrative costs of expedited order are more expensive since the procedures of ordering and delivery of the spare part are faster.

This paper presents a spare ordering policy for preventive age replacement. This policy is specified by three ages  $(t_e, t_0, T)$  can be discussed as follows. If the component fails before expedited order  $t_e$ , an unscheduled order is placed immediately and is delivered after a lead time  $L_e$ . If a major failure occurs after  $t_e$  but before  $t_0$ , a scheduled order is placed at  $t_0$  and is delivered after a lead time  $L(L_e < L)$ . If there has been no major failure by  $t_0$  then place a scheduled order at that time. The component is replaced at preventive replacement  $T$  if a major failure has not occurred by then.

We consider that the cost of the unscheduled order is greater than the cost of scheduled order. The objective is to determine jointly the three values  $(t_e, t_0, T)$  so as to minimize the expected cost rate.

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## 2 Joint optimization of maintenance and inventory policy

### 2.1 Model description

We consider a system containing one component subject to random failure and one spare in stock. A system has two kinds of failures, major and minor ones. A minor failure is always rectified by a minimal repair but a major failure does incur the breakage penalty and can only be rectified by replacing the failed component with a new one.

In this work, the ordering policy is specified by the two ages  $(t_e, t_0)$ . If a major failure occurs before  $t_e$ , an unscheduled order is placed immediately. If the component fails after  $t_e$  and before  $t_0$  then a scheduled order is placed at  $t_0$ . If not an order for a spare is placed at a scheduled ordering time  $t_0$ . The component is replaced as soon as a spare is available. Preventive replacement will occur at time  $T$  if failure has not occurred by then.

### 2.2 Notations

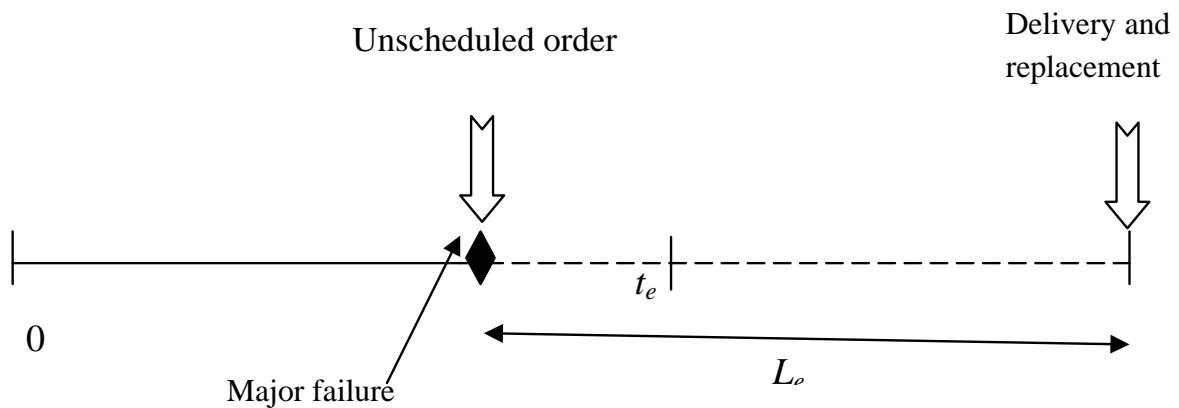
$f(x)$ :	probability density function
$F(x)$ :	cumulative distribution function
$\overline{F}(x)$ :	survivor function of time to failure
$g(x)$ :	probability density function of time to major failure
$G(x)$ :	cumulative distribution function of time to major failure
$\overline{G}(x)$ :	survivor function of time to major failure
$p$ :	probability that a failure is a minor one
$M(x)$ :	number of minimal repairs in $(0, x)$
$t_0$ :	scheduled ordering time
$t_e$ :	unscheduled ordering time
$L$ :	lead time between scheduled order and receipt of a spare part
$L_e$ :	lead time between un scheduled order and receipt of a spare machine in urgency case ( $L_e < L$ )
$T$ :	scheduled time for preventive replacement ( $T \geq t_0 + L$ )
$c_p$ :	preventive replacement cost
$c_r$ :	corrective replacement cost ( $c_r \geq c_p$ )
$c_h$ :	holding cost of a spare per unit time
$c_s$ :	downtime cost per unit time due to spare shortage
$c_f$ :	average cost of a minimal repair
$c_e$ :	delivery cost of unscheduled order
$c_0$ :	delivery cost of scheduled order
$C(t_e, t_0, T)$ :	expected cost rate for an infinite time span

### 2.3 Maintenance Cost

Since each replacement is a regeneration point, the time between successive replacements can be regarded as one cycle. The expected cost per cycle is the sum of the replacement, holding, shortage and delivery costs.

There exist the following four mutually exclusive and exhaustive possibilities in every cycle. For each scenario  $i$ , we evaluate the total average cost  $N_i(t_e, t_0, T)$  and the average duration of the cycle corresponding  $D_i(t_e, t_0, T)$ .

**Scenario 1:** The component fails before the unscheduled ordering time  $t_e$ .

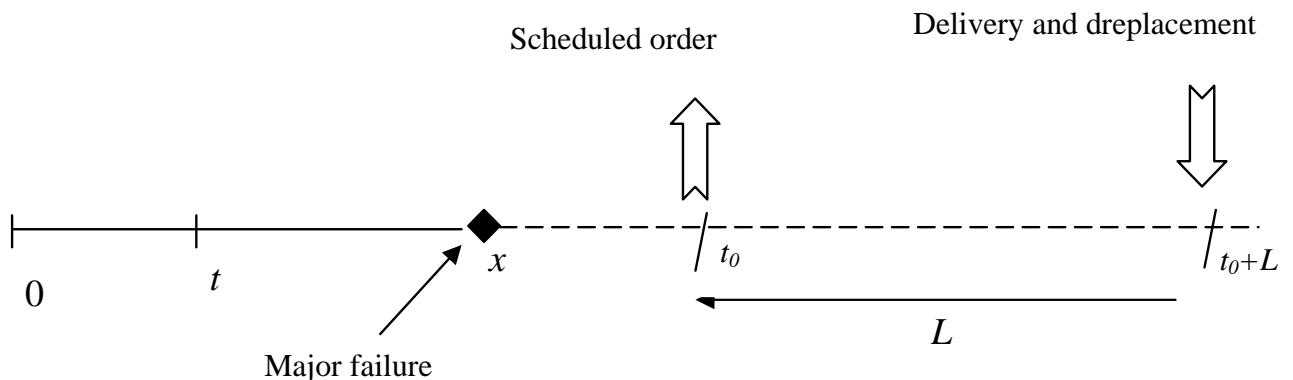


The order for a spare is placed immediately and the spare is delivered after a lead time  $L_e$ . Thus it is necessary to assume the corrective replacement cost, the shortage cost and the delivery cost of unscheduled order.

$$N_1(t_e, t_0, T) = (c_r + c_e + c_s \cdot L_e) \int_0^{t_e} g(x) dx + c_f \cdot E[M(t_e)] \quad (1)$$

$$D_1(t_e, t_0, T) = \int_0^{t_e} (x + L_e) g(x) dx \quad (2)$$

**Scenario 2:** The component fails between the unscheduled ordering time  $t_e$  and the scheduled ordering time  $t_0$ .

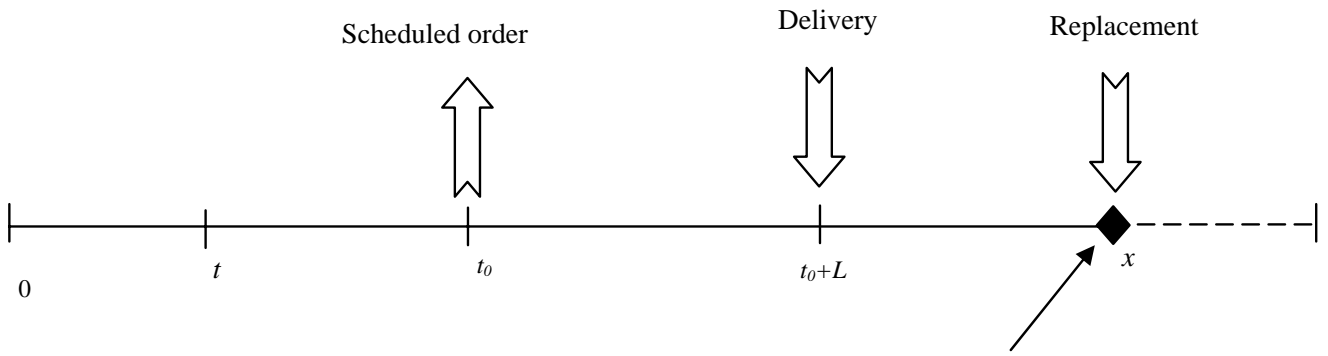


The regular order for a spare is placed at time  $t_0$  and the spare is delivered after a lead time  $L$ . We must assume the corrective replacement cost, the shortage cost and the delivery cost of scheduled order.

$$\begin{aligned}
 N_2(t_e, t_0, T) &= (c_r + c_0) \int_{t_e}^{t_0+L} g(x) dx \\
 &+ c_s \int_{t_e}^{t_0+L} (t_0 + L - x) g(x) dx \\
 &+ c_f (E[M(t_0 + L)] - E[M(t_e)])
 \end{aligned} \tag{3}$$

$$D_2(t_e, t_0, T) = (t_0 + L) \int_{t_e}^{t_0+L} g(x) dx \tag{4}$$

**Scenario 3:** The operating unit fails between  $t_0 + L$  and the scheduled preventive replacement time  $T$ .

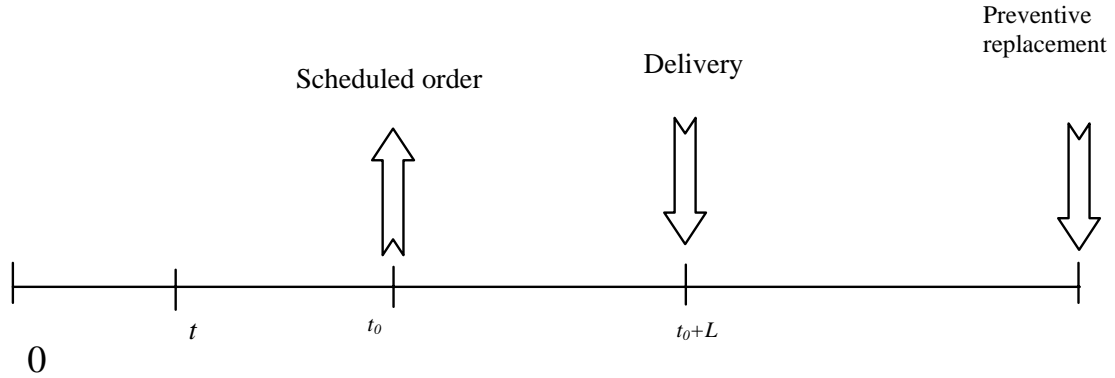


The part is delivered and stored. So, we should assume the cost of holding the spare, the holding cost and the delivery cost of scheduled order.

$$\begin{aligned}
 N_3(t_e, t_0, T) &= (c_r + c_0) \int_{t_0+L}^T g(x) dx \\
 &+ c_h \int_{t_0+L}^T (x - t_0 - L) g(x) dx \\
 &+ c_f (E[M(T)] - E[M(t_0 + L)])
 \end{aligned} \tag{5}$$

$$D_3(t_e, t_0, T) = \int_{t_0+L}^T x g(x) dx \tag{6}$$

**Scenario 4:** The operating unit survive until time  $T$ .



In this case, we must assume the preventive replacement cost, the holding cost and the delivery cost of scheduled order.

$$N_4(t_e, t_0, T) = (c_p + c_0) \int_T^\infty g(x) dx + c_h \int_T^\infty (T - t_0 - L) g(x) dx \tag{7}$$

$$D_4(t_e, t_0, T) = T \int_T^\infty g(x) dx \tag{8}$$

Since the cost per cycle is the sum of (1),(3), (5) and (7), the expected cost per cycle,  $N(t_e, t_0, T)$ , is

$$N(t_e, t_0, T) = c_0 + (c_e - c_0)G(t_e) + c_f E[M(T)] + c_p + (c_r - c_p)G(T) + c_h \int_{t_0+L}^T \bar{G}(x) dx + c_s \cdot A(t_e, t_0) \tag{9}$$

where,  $A(t_e, t_0) = \int_{t_e}^{t_0+L} G(x) dx - G(t_e)[(t_0 + L) - (t_e + L_e)]$  (10)

$$G(x) = 1 - F(x)^{1-p} \tag{11}$$

The expected number of minimal repairs,  $E[M(x)]$ , is

$$E[M(x)] = \frac{p}{1-p} G(x) \tag{12}$$

Thus the expected cycle length,  $D(t_e, t_0, T)$ , is computed as follows.

$$D(t_e, t_0, T) = A(t_e, t_0) + \int_0^T \bar{G}(x) dx \tag{13}$$

From the renewal reward theorem, the expected cost rate for an infinite time span is the

expected cost per cycle divided by the expected cycle length.

Hence the expected cost rate, denoted  $C(t_e, t_0, T)$ , is

$$C(t_e, t_0, T) = \frac{c_0 + c_p + (c_e - c_0)G(t_e) + B.G(T) + c_s.A(t_e, t_0) + c_h \int_{t_0+L}^T \bar{G}(x)dx}{A(t_e, t_0) + \int_0^T \bar{G}(x)dx} \quad (14)$$

Where,  $B = c_r - c_p + c_f \frac{P}{1-p}$

The derived cost model is more generalized and the special cases of this model reduce to some previous works:

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**case 1:**  $L = L_e = 0, t_0 = t_e = T, p = 0$

It reduces to the classical age replacement (Policy I of [Barlow and Hunter(1960)]).

•

**case 2:**  $L = L_e = 0, t_0 = t_e = T, p = 1$

It reduces to the periodic replacement with minimal repair (Policy II of [Barlow and Hunter(1960)]).

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**case 3:**  $L = L_e = 0, t_0 = t_e = T, 0 < p < 1$

It reduces to model developed by [Cleroux and al.(1979)].

•

**case 4:**  $L = L_e, t_0 = t_e, 0 < p < 1$

It reduces to policy of [Park and Sun(2009)].

## 2.4 Optimization Procedure

The problem is to determine jointly the values of  $t_e$ ,  $t_0$  and  $T$ , so as to minimize the expected cost rate.

We formulate our problem as a constrained nonlinear optimization : Find values of  $(t_e, t_0, T)$  that minimize  $C(t_e, t_0, T)$ , as follows.

$$\begin{cases} \min_{(t_e, t_0, T)} C(t_e, t_0, T) \\ t_e \geq 0 \\ t_0 \geq t_e \\ T \geq t_0 + L \\ (t_e, t_0, T) = (0, 0, 0) \end{cases} \quad (15)$$

The optimal values of  $t_e$ ,  $t_0$  and  $T$ , can be obtained by Matlab software using the function `fmincon`.

This function finds a constrained minimum of a scalar function of several variables starting at an initial estimate.

### 3 . Numerical example

In order to illustrate the advantage of this policy, we use the numerical example. In this example, we suppose that the maintenance costs can be identified and the lifetime distribution of component can be determined.

We consider three components. The lifetime of each components is supposed to be a Weibull distribution, then the reliability of component was expressed a

$$R(t) = \bar{F}(t) = e^{-\left(\frac{t}{\theta}\right)^\beta} \tag{16}$$

Where  $(\theta, \beta)$  denote the scale and the shape parameters of the Weibull distribution, respectively. The supposed related parameters  $(\theta, \beta)$  and the maintenance costs in the example are listed in Table 0. The probability of minor failure:  $p = 0.6$  and the length of lead time:  $L = 80$  and  $L_e = 40$ .

**Table 1:** The supposed parameters of the components in the example

Components	$\theta$	$\beta$	$c_p$	$c_r$	$c_f$	$c_h$	$c_s$	$c_e$	$c_0$
1	1800	1.8	800	1400	480	150	360	30	10
2	2600	2.5	1600	3000	960	650	900	80	30
3	3200	3	1800	3400	1080	800	1200	120	70

According to the given parameters, we use the Matlab software to find the optimal values  $t_e^*, t_0^*$  and  $T^*$  so as to minimize the expected cost rate  $C(t_e, t_0, T)$ .

The numerical results are reported in table 1. We note that  $T^*$  is less than *MTTF* and the reliability component at  $t = T^*$  is greater than 0.8. Therefore, the proposed policy allows to minimize the maintenance cost while guaranteeing the availability of system.

**Table 2:** The numerical results of joint optimization with double age ordering policy

Components	$t_e^*(h)$	$t_0^*(h)$	$T^*(h)$	Cost	<i>MTTF</i> (h)	$R(T^*)$
1	564	579	659	\$28.53	1600	0.8
2	797	812	892	\$30.7	2306	0.9
3	1042	1057	1137	\$24.8	2857	0.9

If we consider that  $t_e = t_0$  (a single age ordering policy) and  $L = L_e$ . The optimal ordering time, replacement time and the corresponding cost rate are reported in table 2.

**Table 3:** The numerical results of joint optimization with single age ordering policy

Components	$t_e^* = t_0^*(h)$	$T^*(h)$	Cost
1	367	447	\$36.62
2	620	700	\$36.86
3	846	926	\$28.68

According to these results, the joint optimization of maintenance and inventory with double age ordering policy is less expensive than the single age ordering policy. This shows that the use of double age ordering policy is more convenient and economical.

#### 4. Conclusion

In the present paper, the joint optimization of maintenance and inventory with double age ordering policy is proposed. This policy is more economical than the single age ordering policy.

Moreover, in practice, there is a difference in the lead times and delivery costs between scheduled order (placed at  $t_0$ ) and unscheduled order (i.e., placed on failure before  $t_e$ ). Therefore, this policy is realistic for industrial world.

In the future research, we study the problem of jointly optimizing preventive maintenance (PM) and production policies.

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