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A constructive heuristic for the multi-depot distribution problem of petroleum products in large quantities by a heterogeneous fleet of vehicles

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Abstract- This article gives an approached resolution method based on a constructive heuristic for the multi-depot distribution problem of petroleum products in large quantities by a heterogeneous fleet of vehicles. Test results for this heuristic and for the resolution by CPLEX solver on different problem instances are presented and compared.

Keywords: distribution; petroleum products in large quantities; multi-depot; constructive heuristic; vehicle routing problem.

1. INTRODUCTION

In this work, we address the problem related to the distribution, in large quantities of petroleum products that require the use of tankers fleet, parked in depots. This heterogeneous fleet, made up of vehicles owned by the company or third non-compartmentalized and without volumeters, can serve customers from several removal sites (R.S). The objective is to minimize the total transportation cost while rationalizing the vehicles utilization.

The distribution problem in general is not a recent problem. It started with two classical problems in combinatorial optimization; the Travelling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP).

The basic problem, the TSP, consists of a departure point, to visit a set of customers with one single truck and come back. It consists therefore on planning its tour by finding the sequence of customers with the lowest possible total cost. Historically, [1] is the first work that introduces TSP problem by proposing resolution methods.

The vehicle routing problem addresses the case where each customer has a given request. It consists on determining several tours that all start and end at the depot and where each customer is visited once by a single truck. The first work that addresses the VRP is [2]. The VRP generalizes the travelling salesman problem (TSP). The VRP is much more difficult to solve than the TSP [4].

The vehicle routing problem literature is abundant. At the origin of routing problems, [5] provides a bibliography of 500 studies. Reference [6] presents a VRP literature classification, based on a literature review of about 1,500 documents.

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There are several variants of the VRP; the main ones are presented in [7]. Among the first variant, we find the periodic VRP (PVRP) in which customers are served in a period of time rather than a single day [8], the multiple depot VRP (MDVRP) in which a company may have multiple depots to serve all its customers [9-14], VRP with pickup and delivery (VRPPD) in which vehicles can both load and unload products at the customers [15-17], the VRP with time windows (VRPTW) which defines a time interval within which the customer must be served [18,19], the stochastic VRP (SVRP) where one or more components of the problem are random [20-22] and the VRP with the choice of the heterogeneous fleet of vehicles (FSMVRP) [23-25].

After this literature review summarized in Fig.1, we notice that no research has presented a comprehensive study of the fuel distribution problem as defined above. More specifically, in connection with the problematic issue, two types of problems have been addressed in the literature, the multiple depots VRP (MDVRP) and the fleet size and mix VRP. Indeed, most publications study routing problems variations with inventory management of a single product or multiple products that can be transported in the same compartment of a truck. Furthermore, no publication addresses the problem of distribution with both multiple depots and multiple removal sites.

We note that regardless of the constraints of the VRP problem, especially the variant of the problem we are dealing with, it is still NP-hard, meaning that no known algorithm can guarantee to find in polynomial time the exact solution to these problems.

In our first work [3], we formulated the simplifying assumption that tankers' depots are near the removal sites and as a consequence of this one can assimilate depots to removal sites. Thus the depots did not appear in the model and only displacements between removal sites and customers were considered. Following this work, we raised this assumption to lead to a more complete model including, on the one hand, displacements between depots and removal sites, and on the other hand, displacements between clients and depots. This new model allows giving a more accurate account of the situation of the petroleum products distribution in the real world.

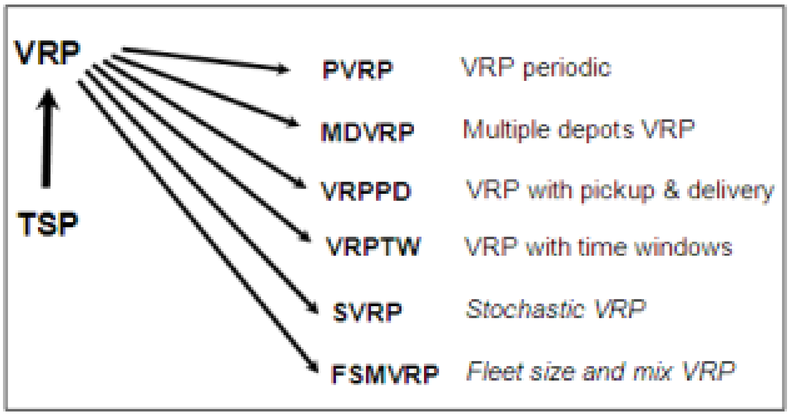

Figure 1: VRP variants.

With the sight of the results obtained after execution of the mathematical model on the CPLEX solver, the character NP-hard of the problem is confirmed, since the execution time deteriorates significantly once we pass to large or even average size instances of the problem.

Given the limitations identified for the exact resolution approach, it is necessary to turn to approximate resolution approaches. In this article, we propose to describe a constructive resolution method based on the principle of neighbourhood for the resolution of the general case of the problem by considering all possible displacements of the vehicles beyond the assumptions of the multi regions case. This method (or heuristic) aims to build a solution of the problem consisting of a set of vehicle tours. According to the principle of neighbourhood, each tour is gradually elaborated according to the various strategies offered for the choice of the transitions. Thus, decision maker can choose between two strategies. The first allows considering only the visit of the vertex having the best cost. The second allows making a probabilistic choice among the vertices having the best costs. Lastly, a battery of heuristic instances functioning in simple or parallel mode makes it possible to launch a great number of heuristics instances and preserves a list of the best solutions obtained. The number of heuristics thus launched is called the number of iterations of the battery.

In this article we describe in details the constructive heuristic.

This article is organized as follows. In Section 2, we describe the problem addressed. Section 3 is dedicated for describing the constructive heuristic. Finally, numerical results are presented in the last section.

2. PROBLEM DESCRIPTION

The problem arises as follows: “Being given:

- A set of customers divided geographically on a vast territory and expressing a strong demand of fuel
- A heterogeneous fleet of vehicles initially parked on a set of depots
- A set of removal sites on which vehicles come to be supplied in order to serve the customers.

The objective is to find vehicle tours of optimal costs that start and end on the same depot and which satisfy the demands of all the customers as well as other operational constraints.”

2.1 Problem characteristics

The fleet of vehicle has the following characteristics:

- It is about a heterogeneous fleet: it consists of vehicles of various capacities.
- It consists of vehicles owned by the company and rented vehicles. Each vehicle has an associated use cost which is definitely less important for a vehicle of the company than for a rented vehicle. Hence, the imperative to give priority to the use company vehicles.
- The vehicles of the fleet are non-compartmentalized, without voltmeters and leave fully a removal site towards a customer to unload the integrality of their capacity.
- The travel time of a vehicle shall not exceed the 8 hours prescribed by the regulations governing the work of drivers.

2.2 Assumptions

In the remainder of this article, we consider the following assumptions:

- H1: The requests relate to only one product;

- H2: The tours' planning horizon is daily;
- H3: Customers demands are linear combinations of the capacities of the vehicles

Note that H3 assumption can be replaced by the less restrictive assumption H'3 which stipulates that:

- H'3: customer demands are expressed with a minimal tolerance and a maximal tolerance.

In other words, with H3 assumption, one admits at the end of the visits of a customer that the final demand balance is exactly equal to 0. On the other hand, with assumption H'3, by noting the minimal and maximal tolerances for a demand respectively by the positive values *toleranceMin* and *toleranceMax*, one admits that the final balance for the demand takes a value between -*toleranceMin* and *toleranceMax*.

2.3 Constraints

- C1: Each vehicle must leave empty its depot towards a removal site for a first loading.
- C2: Each vehicle must leave loaded from a removal site toward a customer and return empty to any removal site or to its depot.
- C3: Each vehicle visits only one customer at a time when it leaves a removal site.
- C4: Compliance with labour's legislation in terms of working hours.

2.4 The objective

The objective is twofold:

- Minimize the total transportation cost.
- Rationalize the vehicles utilization.

3. THE CONSTRUCTIVE HEURISTIC

This section describes the main ideas on which the constructive heuristic is based on.

3.1 Concept of capacity tokens for the balance of a demand

The H_3 assumption states that the demand of a customer must be expressed in such manner so that it is a linear combination of the capacities of the vehicles. This assumption is dictated by the concern of delivering at the customer exactly the quantity which he asks without surplus nor lack. This linear combination not being necessarily unique, it nevertheless limits the possibility of serving a customer by a capacity at a stage of the progress of the heuristic, as well as the number of maximum visits which can be carried out by the means of this capacity. Thus, under penalty of seeing the creation of an unfeasible solution resulting from a random choice of the capacities for the visits of a customer, which constitutes a very probable risk, it is necessary before considering the visit of a customer by a vehicle having a given capacity, to make sure that the current balance of the request can be decreased by this capacity; or in other term that the balance of the demand has at least a token of this capacity. As a definition, the number of tokens of a given capacity for a given balance can be defines as being the maximum of the weights relative to this capacity in all the linear combinations of capacities which can form this balance.

Let us take as example a formed fleet of vehicles of capacities 7, 9 and 18 tons as well as a demand for 34 tons. Initially the balance is equal to the demand; i.e., 34 tons. These 34 tons can be written in the form of the 2 following linear combinations:

34 = 1*7+3*9+0*18 = 1*7+1*9+1*18

Thus, for this balance, the number of tokens for the capacity 7T is equal to 1, that of the capacity 9T is equal to 3 and that of the capacity 18T is equal to 1 ({C07T=1, C09T=3, C18T=1}). That means that at this stage of the satisfaction of demand, the next visit of the customer could be made by the means of any of the 3 capacities. If ever this satisfaction is made by the means of the capacity 7T leading then to a balance of 27T, a recalculation of the tokens ({C07T=0, C09T=3, C18T=1}) gives a number of tokens for the capacity 7T equal to 0. This implies that the customer cannot any more be served by a vehicle of this capacity.

3.2 Tour construction rules

Following the principle of displacements on the neighbourhood, a tour is built progressively by steps called transitions. These ones perform the best local displacements.

At each transition, the heuristic results in a temporary tour that starts and ends at the depot by taking the majority of displacements of the temporary tour resulted from the preceding transition and by adding displacements related with the service of exactly one customer followed by the immediate return towards the depot.

The choice of the next customer to be served is dictated by the exact or probabilistic selection of the shortest path towards the customer to serve. At the end of each transition, the heuristic is positioned on the served customer in order to start from there the next transition.

If at a given transition, it is no longer possible to serve any more customer, the temporary tour which had led the preceding transition is regarded as final and a new tour should be considered to serve the demands not yet fully satisfied. There exist two types of transitions: The transition of the vehicle search and the transition of servicing the next customer.

3.2.1 The transition of the vehicle search

It is the initial transition launched at the beginning of the construction of the tour. It aims to:

- Select randomly a customer among the not yet served customers.
- Create a temporary tour to serve the customer and then return to the depot..
- Assign a vehicle to this tour.

Thus, the tour in question is the path with the 3 displacements Depot → R.S → Customer →Depot where:

- *Depot*: is the depot at which begins and ends the tour whose choice can be done among all the depots of the graph.

- *R.S*: is one of the R.S close to the customer chosen in direction *R.S*→*Customer* (see the section on local data)
- *Customer*: is the randomly selected customer.

Thus, the choice of the temporary tour leads to the selection of a depot and a removal site. This choice corresponds to the best path Depot→R.S→Customer to serve the customer.

Note the points to be considered for this transition:

- The chosen vehicle must have a capacity compatible with the balance of the customer's demand. In other words, this balance must have at least one token for the vehicle capacity's type.
- The tour should be feasible with respect to the C_4 constraint that deals with the maximum duration for the tour of a vehicle.
- Preference is given to vehicles of the company even if it leads to choose a tour less interesting in terms of cost.
- For a randomly chosen capacity among the capacities for which the balance of the demand has at least one token, all the possible path to serve the customer must be evaluated in terms of cost to choose the best path that can be affected to a vehicle. A path can be not feasible if its depot does not have a vehicle for the selected capacity. The fact of considering the customer's supply only from its neighbour's removal sites reduces significantly the computation time.

Fig.2 shows an example of the displacements of the transition of vehicle search.

In Fig.2, client C4 has been randomly chosen, and a minimal temporary tour (in terms of number of vertices visited) is built with the displacements D1→RS3→C4→D1. Note that:

- The consideration of returning to the depot after serving the customer makes it sure to obtain a tour feasible with respect to the constraint C4.
- The displacements D1→RS3 and RS3→C4 are in solid lines to signify that they are final, meaning that they will remain in the final tour.

On the other hand the displacement C4→*D*1 is in dotted line to signify that it's only temporary. Its maintenance will depend on the possibility or not to serve a customer during the following transition. Thus, if a customer is served during the next transition, this displacement will be removed. Otherwise, it will be considered as final

The served customer C4 is represented by a shaded triangle to mean that the heuristic is positioned at the end of this transition on this client in order to initiate the next transition.

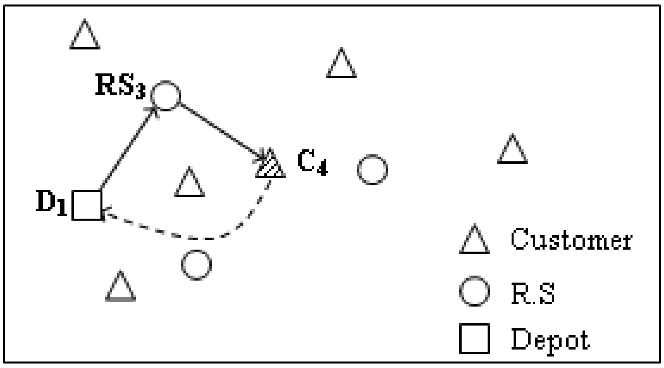

Figure 2: Displacements of the transition of vehicle search.

3.2.2 The transition of servicing the next customer

This transition supposes that a temporary tour is already built and assigned to a vehicle. It aims to find the best way in term of cost relative to:

- The service of exactly one client from the client that has been served during the previous transition.
- Followed-up by an immediate return towards the depot.

Thus, the new displacements to be found are of the type:

LastCustomer→*R.S*→*NewCustomer*→*Depot* where:

- *LastCustomer*: is the last customer served during the preceding transition.
- *R.S*: is one of the R.S close to *LastCustomer* (in the direction Customer→R.S) and to *NewCustomer* (in the direction R.S→Customer).
- *NewCustomer*: is the new customer to be served.
- *Depot*: is the depot of the vehicle affected to the tour.

So the choice of new displacements induces the choice of a R.S and a customer.

In a way similar to the transition of the vehicle search, this choice corresponds to the best path *LastCustomer*→*R.S*→*NewCustomer* to serve the customer.

The achievement of this transition takes into account the following points:

- The new displacement must be chosen so that the tour should be feasible with respect to the C4 constraint concerning the maximum duration for the tour of a vehicle.
- The new customer to serve must have at least one token for the capacity type of the vehicle to which the tour is assigned.

In the case where it is no longer possible to serve any client, the tour is considered completed and the final displacement towards the depot of the previous transition is considered final.

The two graphs in figures below show the extension of the construction of the tour initiated by the transition of the vehicle search of Fig2 through the addition of the displacement of two successive transitions of servicing the next customer:

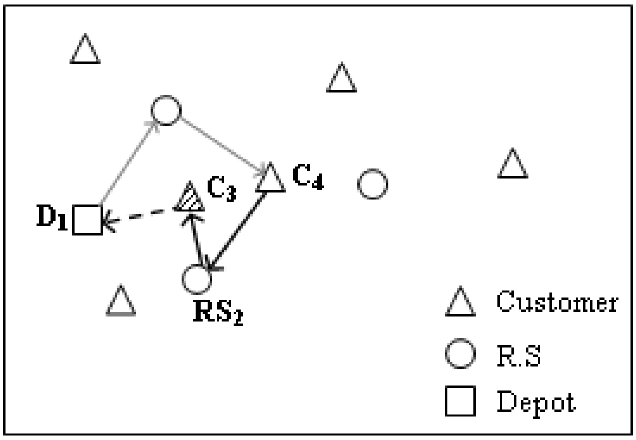

Figure 3: First example of transition's displacements for servicing the next customer.

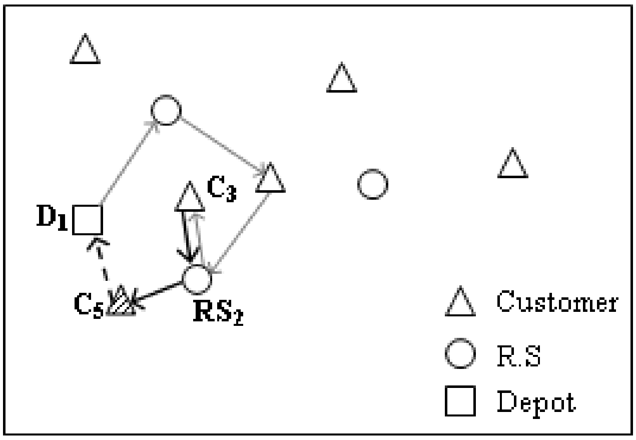

Figure 4: Second example transition's displacements for servicing the next customer

In this figure, final and temporary displacements are represented in the same way as the initial transition. Thus:

- The transition of Fig.4 serves the customer C_3 from the last customer served C_4. It corresponds to the displacements: $C_4 \rightarrow RS_2 \rightarrow C_3 \rightarrow D_1$. The first two displacements are final and the last displacement◊ towards the deposit is temporary. At the end of this transition the heuristic is positioned on C_3.

- The transition of Fig.5 serves the customer C_5 from the last customer served C_3. It corresponds to the displacements: $C_3 \rightarrow RS_2 \rightarrow C_5 \rightarrow D_1$. At the end of this transition the heuristic is positioned on C_5.

3.3 Local data collection

For the construction of the tours, the heuristic follows the principle of neighbourhood by promoting the exact or probabilistic choice of the best path from the customer on which the heuristic positions itself during its execution. To do so, it is necessary to evaluate, in terms of cost, all possible paths, to test feasibility (constraints satisfaction) of the temporary tours induced and thereafter to do an exact or probabilistic choice of a path among the best of these paths. For large instances of the problem, the comprehensive evaluation of all possible paths can significantly alter the overall computation time especially when the heuristic is executed a large number of times within a battery.

Moreover, given a transition characterized by the customer on which the heuristic is positioned, the comprehensive evaluation of all possible paths may consider visits to removal sites and customers that are far away from the current client and thus represent visits of little economic interest or even leading to a tour that violates obviously the constraint C4 of maximal duration for the tours of vehicles.

Thus, in the intention of preserving computing resources, it is wise to consider only the evaluation of paths at the neighbourhood of the customer and in particular in the departure and arrival to a removal site from a customer. Thus, we consider for each customer:

- The list of removal site neighbours to that of customer in the direction Customer→*R.S.*
- The list of the removal site neighbours to that of customer in the direction *R.S*→ Customer.

The need to consider the best neighbours in both directions comes from the asymmetry of cost matrices: a R.S close to a customer for the direction Customer→R.S is not necessarily close in the direction R.S→ Customer.

The removal sites in the neighbourhood of a customer are determined from cost matrices of the instance. The adopted strategy is to find a fixed number of best neighbours on these cost matrices for every customer. This number is a fixed parameter of the solver.

3.4 Selection strategies for displacements of a transition

The two transition types make the choice of a path for the service of exactly one customer among all the possible paths. To be made, two strategies of selection are offered:

3.4.1 Selection according to the best cost

It is the simplest selection strategy that consists on choosing the path of better cost which preserves the feasibility of the current tour. To do so, all possible paths are evaluated and sorted out by ascending order of cost. Then, the first path in this order that preserves the feasibility of the tour is selected. Applied to the two transition types, this strategy makes it possible to obtain solutions of good qualities very early within a battery of heuristics instances.

3.4.2 Probabilistic selection according to the cost

This strategy makes a probabilistic choice among the paths of better costs that also maintain the feasibility of the tour. The more the cost of a path is interesting the greater is its probability of selection.

Denoting by (c_n) the sequence of sorted costs for the paths that lead to feasible tours, the procedure of probabilistic selection is as follows:

- Calculate the threshold cost that should not be exceeded by the paths in order to be kept for the selection. This threshold is: $(1+\frac{r}{100})c_1$ .With r a positive integer parameter.

- Make the selection of the first N costs (c_n) which satisfy the threshold condition $c_n \leq (1+\frac{r}{100})c_1$. With N a positive integer parameter. This results in the effective selection of N**'** costs with $N^{'} \leq N$. If there are enough paths that satisfy the threshold condition then N '= N.
- For each of the N**'** costs, calculate the corresponding selection probability according to the following rule: $P_n = \frac{(2c_{N'} - (c_1 + c_n))^p}{\sum_{i=1}^{N'} (2c_{N'} - (c_1 + c_i))^p}$. The exponent p being a positive integer parameter.
- Make the probabilistic selection of a path according to the calculated probability rule.

The quality of results obtained by applying this strategy to both transitions types within a battery of heuristics instances depends heavily on the values chosen for the parameters N, r and p.

The probabilistic nature of this strategy involves a greater diversity of solutions in terms of paths enclosed than the solutions obtained by the previous strategy.

3.5 Battery of heuristics instances

A battery of heuristics instances will launch the heuristic a significant number of times and maintain a list of the best solutions found. Each launch of a heuristic is called iteration. This battery is designed to be launched in two modes:

- The normal mode where a single process is responsible for running all the iterations of the battery.
- The parallel mode where several independent processes share the iterations of the battery. This mode will be more interesting if it is used in a multi-processor environment.

4. NUMERICAL RESULTS

Two tests are performed based on the same execution environment which consists on an Intel Core Duo 2 PC, with 3 GHz of frequency and 3.25 GB of RAM under the Windows XP operating system.

For the CPLEX approach, the version of the solver used is CPLEX 9.

Table 2 shows the vehicles use cost according to their belonging and their capacities as they are used for all problem instances. It is worth noting that the use cost for third party vehicles is much greater than that of owned vehicles.

Table 1. Vehicles use cost

Vehicles types	7 T	9 T	18 T
Company	350	450	800
Third	700	900	1600

First tests for the constructive heuristic evaluate the quality of the solutions obtained by batteries of constructive heuristics instances over those obtained by the CPLEX solver.

Table 2 summarizes the comparative numerical results obtained. The batteries are run in parallel mode of 4 processes that each perform 25 000 iterations. This makes a total of 100 000 heuristic instances by problem instance. The selection strategy adopted is the probabilistic strategy with parameters (p=9, N=20, r=80).

The results obtained show that the batteries run in a short time (maximum of 1m44s) and in general the solutions obtained by the batteries are of good quality given that the gap between the best solutions of the two approaches is small.

Second tests perform comparison between the normal mode and different configurations of parallel mode for batteries of constructive heuristic instances.

Table 3 summarizes the comparison made for a same problem configuration formed of 3 depots, 4 removal site, 32 customers and 16 vehicles. In this tests batteries are run under the same probabilistic strategy of the previous tests and are all having the same iterations number equal to 1 000 000.

Although these tests are performed in a single processor environment, it is clear that execution time of the parallel modes (number of processes greater than 1) is far better than execution time of the normal mode.

5. CONCLUSION AND PERSPECTIVES

Our study has focused on the multi-depot distribution problem of petroleum products in large quantities using a fleet of heterogeneous tankers. This fleet consists of vehicles owned by the company or rented, non-compartmentalized and without volunteers. We introduced a constructive heuristic based on the principle of neighborhood to gradually build a solution of the problem. Numerical results were introduced to compare results of the constructive heuristic with those of the exact approach.

The main future developments consist in the conception of other approached resolution methods especially via met heuristics adapted to our case study like genetic algorithms family and ant colony algorithms family.

Table 2. Comparison between Numerical results of the constructive heuristic and the exact approach

Depots	R.S	Customers	Available vehicles	CPLEX CPU	CPLEX Best Solution	Heuristic CPU	Heuristic Best Solution	Gap%
1	2	10	7	39	8 987	14	9 112	1,39
1	2	16	8	162	12 906	20	13 115	1,62
1	2	30	15	8 135	19 728	65	20 114	1,96
1	2	40	18	4 995	28 633	96	30 410	6,21
1	2	50	20	21 283	40 024	104	40 104	0,20
2	3	15	10	626	14 498	15	14 739	1,66
2	3	24	12	1 199	16 927	33	17 882	5,64
2	3	30	16	1 451	19 464	55	20 275	4,17
3	4	20	12	210	12 530	24	13 224	5,54
3	4	32	16	13 841	22 761	46	24 769	8,82

Table 3. Comparison between Numerical results of the normal and parallel mode

Number of processes	Number of iterations per process	CPU
1	1 000 000	968
2	500 000	480
3	333 333	451
4	250 000	464

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