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An analysis of recursive parametric estimation methods for large-scale time-varying systems

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In this paper, three recursive parametric estimation methods are analysed to solve the estimate problem for a class of large-scale time-varying systems, which can be decomposed into several interconnected single-input single-output systems. These methods, which are based upon the recursive least squares techniques, incorporate weighted factors into the adaptation gain matrices of their recursive parametric estimation algorithms. It is about the Recursive Least Squares method with Forgetting Factor (RLSFF), the Recursive Least Squares method with Weighted Parameters (RLSWP) and the Recursive Least Squares method using Kalman Filter (RLSKF). Recursive parametric estimation algorithms are consequently developed that permit to follow the parametric variations of each interconnected system in order to ensure a good quality of the estimate. The techniques of the practical implementation of the developed parametric estimation algorithms are given. Furthermore, simulated results are presented illustrating the performance and the effectiveness of these algorithms.

Keywords: Large-scale systems; Time-varying parameter systems; Input-output mathematical models; Interconnected systems; Recursive parametric estimation methods; Parametric estimation algorithms.

1. INTRODUCTION

The objective in this paper is to analyse parametric estimation methods for large-scale time-varying systems, which can be decomposed into several interconnected Single-Input Single-Output (SISO) systems. Research on parametric estimation of large-scale systems, notably with time-invariant parameters, has been done for the three past decades [1-4]. Every large-scale system can be envisaged as system consisting of a large number of interacting interconnected systems [5]. Since such system normally comprises several interconnected systems (power network, articulated mechanical systems, set of coupled tanks, etc.), the formulation problem of their parametric estimation or their control is intricate. On the other hand, there exist numerous interconnected systems that are close to being hierarchical (see, e.g., [6]).

Many real-world plants can be modelled as large-cale systems, which are composed of several interconnected systems [7-9]. Let us note that different works treating various topics (identification, modelling, estimation, decentralisation, control, etc.), which concern the large-scale systems that are composed of several interconnected systems, were published in the engineering literature (see, e.g., [4-7, 10, 11]). In addition, the majority of the published works related to the large-scale systems, which can be described by mathematical models with known or unknown time-invariant parameters. However, very few results were published concerning the large-scale systems being able to be described by mathematical models with unknown time-varying parameters.

The synthesis of a control law (notably, by using an adaptive scheme), which can be applied to a class of large-scale time-varying systems, constitutes a crucial problem,

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because the importance which occupies this class of systems in several industrial fields and the limited number of publications on the subject. In fact, the practical implementation (e.g., in real-time control) of any adaptive control scheme (explicit or implicit) is based on a recursive parametric estimation algorithm (see [4, 12]).

The wished performance indices (precision, stability, rapidity, etc.), which can be obtained during the on-line adaptive control of a large-scale time-varying system, depend on the structure of the used recursive parametric estimation algorithm, particularly the computation procedure of its adaptation gain matrix. In this context, the use of an ordinary parametric estimation algorithm, where the trace of its adaptation gain matrix decreases rapidly towards a very small value, leads to a failure. Indeed, this type of algorithm is not quite capable to follow the time-varying parameters of the considered large-scale system. This motivates the development of recursive parametric estimation algorithms using an adequate computation procedure of their adaptation gain matrix, which can be applied easily to the large-scale time-varying systems that are constituted by several interconnected systems in order to ensure a good quality of the estimate.

Let us note that the most of the estimate or adaptive control schemes of the large-scale systems with unknown and/or time-varying parameters, which are published in the engineering literature, are based primarily on continuous state-space mathematical models. However, the number of works relating to the discrete-time mathematical models, and more particularly the input-output mathematical models, is limited enough. Thus, this paper will be devoted only to the large-scale time-varying systems being able to be described by discrete-time input-output mathematical models. It is assumed that all signal variables of these large-scale systems are accessible for the measurement at every discrete-time.

Our objective is to develop recursive parametric estimation algorithms in order to estimate the parameters of large-scale time-varying systems that are composed of several interconnected SISO systems, which can be described by input-output mathematical models. The formulation of the parametric estimation problem is made on the basis of the prediction error method and the least squares techniques, by using several measured values (inputs, outputs) resulting from the interconnected systems. In this case, we propose to include weighted factors in the computation procedure of the adaptation gain matrix of the recursive parametric estimation algorithms.

The rest of this paper is organized as follows. In Section 2, the description of large-scale time-varying systems by discrete-time input-output mathematical models is treated. Some basic assumptions about the considered class of large-scale time-varying systems are given. Section 3 is devoted to the formulation problem of the parametric estimation for such class of large-scale systems. The main results and developments are given in Section 4. Thus, recursive parametric estimation algorithms are developed by including weighted functions in the computation procedure of their adaptation gain matrices. In Section 5, an illustrative numerical example dealing with the parametric estimation of two interconnected time-varying systems is given. Finally, we conclude in Section 6.

2. PRELIMINARIES AND LARGE-SCALE SYSTEMS DESCRIPTION

All real large-scale systems composed of several interconnected systems are subjected to uncertainty due principally to unmodelled dynamics, unknown parameters, unknown structure variables (delays, orders), disturbances and process changes. Robustness estimation or control methods must to be developed for this class of large-scale systems (see [13]). In certain case of large-scale systems, which are composed of geographically distributed interconnected systems (e.g., power network), the outputs of certain interconnected systems are not easily accessible for the measurement to some other interconnected systems, in the sense such access requires costly data transmission. Decentralisation techniques can be used in this situation of interconnected systems in order to formulate estimation or control schemes (see [10, 11, 14]).

In this work, we consider only the class of large-scale systems that are constituted by several interconnected SISO systems with unknown time-varying parameters, know structure variables and which are subjected to weak disturbances. All outputs signals of the interconnected systems are supposed to be accessible for the measurement. Each interconnected is described by a discrete-time input-output mathematical model, which is often used in several industrial applications. Thus, the formulation of an input-output mathematical model allowing the description of an interconnected system, while being based on an experimental analysis, rests to the knowledge of several measured values (inputs, outputs) resulting from this interconnected system and the other interconnected systems and by using a suitable identification method. In a real industrial application, the measured values of the interconnected systems are attached with errors, which correspond to random disturbances.

In is important to note that two types of discrete-time input-output mathematical models are often used in the description of interconnected systems. The first discrete-time input-output mathematical model, which is known in the engineering literature ″Deterministic AutoRegressive Moving Average (DARMA)″, describes an interconnected system operating in a deterministic environment, where a small noise (which can be negligee or not) acts this system. The second discrete-time input-output mathematical model, which is known in the engineering literature ″Autoregressive Moving Average with eXogenous input (ARMAX)″, describes an interconnected system operating in a stochastic environment, where the noise that acts this system is described by a disturbance mathematical model.

Only interconnected systems operating in a deterministic environment, where small noises having weak variances act the outputs of these interconnected systems, are considered in this paper.

2.1. IDARMA mathematical model

Let us consider a large-scale time-varying system $\mathcal{S}$, which can be decomposed into N interconnected systems $\mathcal{S}_1,\cdots,\mathcal{S}_N$. We suppose that each interconnected system $\mathcal{S}_i$, $1 \le i \le N$, is described by an input-output mathematical model of the type IDARMA (Interconnected Deterministic AutoRegressive Moving Average), SISO, linear and with unknown time-varying parameters.

The considered interconnected system $\mathcal{S}_i$, $1 \le i \le N$, is described by the following IDARMA mathematical model:

$$A_i(q^{-1},k)y_i(k) = B_i(q^{-1},k)u_i(k) + \sum_{j=1,j\neq i}^{N} B_{ij}(q^{-1},k)u_j(k) + \sum_{j=1,j\neq i}^{N} A_{ij}(q^{-1},k)y_j(k) + e_i(k) \quad (1)$$

where $u_i(k)$ and $y_i(k)$ represent the input and the output of the interconnected system $\mathcal{S}_i$ at the discrete-time k, respectively, d_i indicates the noise (together disturbances) which acts on this interconnected system $\mathcal{S}_i$, $u_j(k)$ and $y_j(k)$ are respectively the inputs and the outputs resulting from the other interconnected systems $\mathcal{S}_j$, $j = 1,\cdots,N$, $j \neq i$, and

$A_i(q^{-1},k)$, $B_i(q^{-1},k)$, $B_{ij}(q^{-1},k)$ and $A_{ij}(q^{-1},k)$ are polynomials with unknown time-varying parameters, which are described by:

$$A_i(q^{-1},k) = 1 + a_{i,1}(k)q^{-1} + \cdots + a_{i,nA_i}(k)q^{-nA_i} \quad (2)$$

$$B_i(q^{-1},k) = b_{i,1}(k)q^{-1} + \cdots + b_{i,nB_i}(k)q^{-nB_i} \quad (3)$$

$$B_{ij}(q^{-1},k) = b_{ij,1}(k)q^{-1} + \cdots + b_{ij,nB_{ij}}(k)q^{-nB_{ij}} \quad (4)$$

and

$$A_{ij}(q^{-1},k) = a_{ij,1}(k)q^{-1} + \cdots + a_{ij,nA_{ij}}(k)q^{-nA_{ij}} \quad (5)$$

Let us note that each interconnected system S_i, $1 \le i \le N$, is coupled with the inputs and the outputs of the other interconnected systems S_j, $j = 1,\cdots,N$, $j \ne i$, via the two polynomials $B_{ij}(q^{-1},k)$ and $A_{ij}(q^{-1},k)$.

Our objective is to estimate the parameters intervening in the IDARMA mathematical model (1). For the formulation of this parametric estimation problem, we retain some assumptions concerning each interconnected system S_i, $1 \le i \le N$, which are given in the following subsection.

2.2 Retained assumptions

We suppose that each interconnected system S_i, $1 \le i \le N$, works only in open loop. It is quite obvious that the use of such an operation of the considered interconnected system allows a varied choice of the input signal $u_i(k)$. In order to obtain a mathematical model allowing a better description of the dynamic of the interconnected system S_i, we can apply to him an input signal sufficiently exciting, which can to be able to excite all the modes of this interconnected system.

For reasons of simplicity and without loss of general information, we suppose that the polynomials $A_i(q^{-1},k)$, $B_i(q^{-1},k)$, $B_{ij}(q^{-1},k)$ and $A_{ij}(q^{-1},k)$ intervening in the mathematical model (1) have the same order n_i (i.e., $n_{Ai} = n_{Bi} = n_{Bij} = n_{Aij} = n_i$).

Also, we retain the following assumptions for each interconnected system S_i, $1 \le i \le N$, being able to be described by the IDARMA mathematical model (1):

1. The parameters intervening in the polynomials $A_i(q^{-1},k)$, $B_i(q^{-1},k)$, $B_{ij}(q^{-1},k)$ and $A_{ij}(q^{-1},k)$ are unknown time-varying parameters;
2. The input signal $u_i(k)$ applied to the interconnected system S_i is bounded and sufficiently exciting;
3. The noise sequence $\{e_i(k)\}$ consists of a sequence of an independent random variables with zero mean and variance σ_i^2. We suppose that the value of this variance σ_i^2 is weak enough (e.g., $\sigma_i^2 = 0.005$). This assumption enables us to formulate the parametric problem of estimate posed in a deterministic context. In fact, this noise $e_i(k)$

does not have an influence on the retained deterministic parametric estimation method, and consequently, on the quality of the obtained estimate;

4. The isolated system $\mathcal{S}_i$ is stable in open loop (i.e., the roots (in q) of the polynomial $A_i(q^{-1}, k)$ are inside the unity circle);
5. The number of the measured values M_i is sufficiently large ($M_i >> n_i$), in order to ensure a good convergence of the recursive parametric estimation algorithms. These measured values correspond to the various inputs and outputs resulting from the interconnected systems $\mathcal{S}_i$, $i = 1, \cdots, N$;
6. The signal variables of the considered interconnected systems are accessible, i.e., these are measurable at every discrete-time k. Thus, the information sequence $I_i(k)$ resulting from the interconnected system $\mathcal{S}_i$ and the other interconnected system $\mathcal{S}_j$, is defined by: $I_i(k) = \{u_i(k), y_i(k), u_j(k), y_j(k); k = 1, \cdots, M_i\}$;
7. The noise sequence $\{e_i(k)\}$ and the information sequence $I_i(k)$ are independent.

3. ESTIMATE STRUCTURE AND PROBLEM FORMULATION

The posed parametric estimation problem of each interconnected system $\mathcal{S}_i$, $1 \leq i \leq N$, can be stated as follows: Estimate, starting from the knowledge of several experimental measurements (inputs, outputs) characterizing the dynamic of the interconnected system $\mathcal{S}_i$ and the other interconnected systems $\mathcal{S}_j$, $j = 1, \cdots, N$, $j \neq i$, the values of the parameters intervening in the corresponding IDARMA mathematical model (1). This parametric estimation problem will be formulated on the basis of an adequate estimate structure. In this work, we propose to use an estimate structure on the basis of the prediction error method, which has been very successful in several industrial applications.

Let us note that the estimate structure of the prediction error method is based upon the adjustable model and by using the least squares techniques. Thus, the formulation of the parametric estimation problem can be treated, in order to obtain an optimal estimated parameter vector, by minimizing a certain quadratic criterion. In fact, this quadratic criterion is related to the prediction error, which corresponds to the error between the output signal and the predicted output signal from the adjustable model.

We can express the output $y_i(k)$ of the considered interconnected system $\mathcal{S}_i$, $1 \leq i \leq N$, which is defined by the IDARMA mathematical model (1), by the following expression:

$$\begin{aligned} y_i(k) = &-a_{i,1}(k)y_i(k-1) - \cdots - a_{i,n_i}(k)y_i(k-n_i) + b_{i,1}(k)u_i(k-1) + \cdots + b_{i,n_i}(k)u_i(k-n_i) \\ &+b_{ij,1}(k)u_j(k-1) + \cdots + b_{ij,n_i}(k)u_j(k-n_i) + a_{ij,1}(k)y_j(k-1) + \cdots + a_{ij,n_i}(k)y_j(k-n_i) \\ &+e_i(k) \end{aligned} \tag{6}$$

with $j = 1, \cdots, N$, $j \neq i$.

The output $y_i(k)$ of the interconnected system $\mathcal{S}_i$, such defined by the equation (6), can be rewritten in the following matrix form:

$$y_i(k) = \theta_i^T(k)\psi_i(k) + e_i(k) \tag{7}$$

in which $\theta_i(k)$ is the true parameter vector and $\psi_i(k)$ is the observation vector. These are given by:

$$\theta_i^T(k) = [a_{i,1}(k) \cdots a_{i,n_i}(k)\; b_{i,1}(k) \cdots b_{i,n_i}(k)\; b_{ij,1}(k) \cdots b_{ij,n_i}(k)\; a_{ij,1}(k) \cdots a_{ij,n_i}(k)] \tag{8}$$

and

$$\begin{aligned}\psi_i^T(k) = [&-y_i(k-1) - \cdots - y_i(k-n_i)\; u_i(k-1) \cdots\; u_i(k-n_i) \\ &u_j(k-1) \cdots u_j(k-n_i)\; y_j(k-1) \cdots y_i(k-n_i)]\end{aligned} \tag{9}$$

The problem arising here consists of the formulation of a recursive parametric estimation algorithm making to estimate the unknown time-varying parameters intervening in the parameter vector $\theta_i(k)$, which is defined by (8), and this, starting from the knowledge of several measured values (inputs, outputs) resulting from the interconnected systems S_i, $1 \le i \le N$, and the other interconnected systems S_j, $j = 1, \cdots, N$, $j \neq i$.

The estimate problem of the parameter vector $\theta_i(k)$ can be formulated by using the prediction error method, which is often used in the formulation of parametric estimate schemes of dynamical systems (see, e.g., [15]).

We define the *a priori* predicted output $\hat{y}_i(k)$ as follows:

$$\begin{aligned}\hat{y}_i(k) = &-\hat{a}_{i,1}(k-1)y_i(k-1) - \cdots - \hat{a}_{i,n_i}(k-1)y_i(k-n_i) \\ &+ \hat{b}_{i,1}(k-1)u_i(k-1) + \cdots + \hat{b}_{i,n_i}(k-1)u_i(k-n_i) \\ &+ \hat{b}_{ij,1}(k-1)u_j(k-1) + \cdots + \hat{b}_{ij,n_i}(k-1)u_j(k-n_i) \\ &+ \hat{a}_{ij,1}(k-1)y_j(k-1) + \cdots + \hat{a}_{ij,n_i}(k-1)y_j(k-n_i)\end{aligned} \tag{10}$$

which can be rewritten in the compact form:

$$\hat{y}_i(k) = \hat{\theta}_i^T(k-1)\psi_i(k) \tag{11}$$

where $\hat{\theta}_i(k-1)$ denotes the estimated parameter vector at the discrete-time $k-1$ and $\psi_i(k)$ is the observation vector, as given by (9).

The estimated parameter vector $\hat{\theta}_i(k)$ at the discrete-time k can be given by:

$$\hat{\theta}_i^T(k) = [\hat{a}_{i,1}(k) \cdots \hat{a}_{i,n_i}(k)\; \hat{b}_{i,1}(k) \cdots \hat{b}_{i,n_i}(k)\; \hat{b}_{ij,1}(k) \cdots \hat{b}_{ij,n_i}(k)\; \hat{a}_{ij,1}(k) \cdots \hat{a}_{ij,n_i}(k)] \tag{12}$$

We define the *a priori* prediction error $\varepsilon_i(k)$, which represents the difference between the output $y_i(k)$ of the interconnected system S_i and that predicted by the corresponding adjustable model, by the following expression:

$$\varepsilon_i(k) = y_i(k) - \hat{\theta}_i^T(k-1)\psi_i(k) \tag{13}$$

Thus, in order to obtain an optimal estimated parameter vector $\hat{\theta}_i(k)$, within the meaning of an estimate with minimum variance, we can consider a quadratic criterion that is related to the prediction error $\varepsilon_i(k)$.

4. ANALYSIS OF PARAMETRIC ESTIMATION METHODS

With the beginning of this section, we treat the case of a large-scale time-invariant system that is composed of several interconnected systems, i.e., the parameters intervening in these interconnected systems are supposed unknown, but constant.

The output $y_i(k)$ of the interconnected time-invariant system $\mathcal{S}_i$, $1 \le i \le N$, can be rewritten in the following matrix form:

$$y_i(k) = \theta_i^T \psi_i(k) + e_i(k) \tag{14}$$

where the observation vector $\psi_i(k)$ is given by (9) and the true parameter vector θ_i is described as follows:

$$\theta_i^T(k) = [a_{i,1} \cdots a_{i,n_i} \;\; b_{i,1} \cdots b_{i,n_i} \;\; b_{ij,1} \cdots b_{ij,n_i} \;\; a_{ij,1} \cdots a_{ij,n_i}] \tag{15}$$

The estimate of the parameter vector θ_i, which is given by (15), can be ensured on the basis of the ordinary Recursive Least Squares (RLS) method and by using the prediction error method. We can show easily that the RLS parametric estimation algorithm, which permits to estimate this parameter vector θ_i, is described by:

$$\begin{aligned} \hat{\theta}_i(k) &= \hat{\theta}_i(k-1) + P_i(k)\psi_i(k)\varepsilon_i(k) \\ P_i(k) &= P_i(k-1) - \frac{P_i(k-1)\psi_i(k)\psi_i^T(k)P_i(k-1)}{1 + \psi_i^T(k)P_i(k-1)\psi_i(k)} \\ \varepsilon_i(k) &= y_i(k) - \hat{\theta}_i^T(k-1)\psi_i(k) \end{aligned} \tag{16}$$

where $P_i(k)$ denote the adaptation gain matrix, which corresponds to the variance-covariance matrix of the noise $e_i(k)$.

Practical Implementation of the RLS parametric estimation algorithm (16) will be carried out starting from the knowledge of several measured values of the inputs and the outputs resulting from all interconnected systems $\mathcal{S}_i$, $i = 1, \cdots, N$, and the initial conditions $\theta_i(0)$ and $P_i(0)$.

It is important to indicate that the RLS parametric estimation algorithm (16) cannot apply to an interconnected system with time-varying parameters, as given by (6), and its use can lead to a failure (bad quality of estimate). This comes owing to the fact that the structure of this algorithm is not likely to follow the parametric variations of such interconnected system. Let us note however that the principal element in the structure of the RLS parametric estimation algorithm corresponds to its adaptation gain matrix. Thus, we can affirm that this algorithm is not likely to follow the evolution of the time-varying parameters of the interconnected system $\mathcal{S}_i$, $1 \le i \le N$,. This affirmation was confirmed in several examples of numerical simulations, which are not deferred here. Let us add that the parameters of the adaptation gain matrix $P_i(k)$ of the RLS parametric estimation algorithm (16) decrease quickly towards low values. Thus, the trace of this adaptation gain matrix $P_i(k)$ passes from an important value at the beginning of the iterations (e.g., $\mathrm{tr}[P(0)] = 8000$, where $\mathrm{tr}[.]$ denotes trace) to a low value after some iterations (e.g., $\mathrm{tr}[P(10)] = 1$).

It is thus obvious that, if a parameter (possibly several parameters) of the interconnected system $\mathcal{S}_i$, $1 \le i \le N$, varies after some iterations, then the adaptation gain matrix $P_i(k)$ does not permit at the RLS parametric estimation algorithm to follow the evolution of the variation of this parameter. In other words, this algorithm is not able to make corrections at the appropriate time, for being likely to ensure a better quality of estimate. It is thus judicious to formulate adequate recursive parametric estimation methods by primarily modifying the computation procedure of the adaptation gain matrix, in order to apply them to interconnected time-varying systems. Thus, we can ensure a good follow-up of the parametric variations of these interconnected systems and an acceptable quality of the estimate. This constitutes the objective of this section.

In recent years, there has been an increased interest in the area of parametric estimation time-varying dynamical systems, notably linear SISO systems, and a variety of recursive parametric estimation methods have been developed by using different techniques for the computation procedure of the adaptation gain matrix (see, e.g., [4], [15-18]).

This section aims at the analysis of recursive parametric estimation methods which can be applied to large-scale time-varying systems composed of several interconnected SISO systems, as described by the IDARMA mathematical model (7). In this case, three recursive parametric estimation methods that incorporate weighted functions into the adaptation gain matrices of their recursive parametric estimation algorithms are analysed. It is about the Recursive Least Squares method with Forgetting Factor (RLSFF), the Recursive Least Squares method with Weighted Parameters (RLSWP) and the Recursive Least Squares method using Kalman Filter (RLSKF). Recursive parametric estimation algorithms are consequently developed making it possible to estimate the parameters of the considered interconnected systems. Let us note that these algorithms permit to follow the parametric variations of each interconnected system in order to ensure a good quality of the estimate.

The techniques of the practical implementation of the developed recursive parametric estimation algorithms are given. The performance and the effectiveness of these algorithms are tested in numerical simulation. Let us add that a comparative study in numerical simulation between these algorithms is given.

4.1 Recursive least squares method with forgetting factor

We will treat, in this subsection, the parametric estimation problem of the interconnected time-varying system $\mathcal{S}_i$, $1 \le i \le N$, that is given by (7), by using the RLSFF parametric estimation method. This method is basis on the introduction of a forgetting factor in the computation procedure of the adaptation gain matrix of the corresponding parametric estimation algorithm.

The problem arising here consists of the estimate of parameter vector $\theta_i(k)$, which is defined by (8), while being based on the knowledge of several measurements (inputs and outputs) resulting from the considered interconnected systems. The formulation of the RLSFF parametric estimation algorithm can be carried out starting from the minimization of the following quadratic criterion:

$$J_i(M_i) = \sum_{k=1}^{M_i} \lambda_i^{M_i - k}(k)\varepsilon_i^2(k) \tag{17}$$

where λ_i is a forgetting factor, which can be chosen as follows: $0 < \lambda_i < 1$.

In certain case, we can opt a time-varying forgetting factor $\lambda_i(k)$, with $0 < \lambda_i(k) < 1$.

The choice of a certain value of the forgetting factor λ_i, within the meaning of ensuring a better quality of the estimate, is not obvious. It proves that the expertise of the control engineer and its know-how can condition this choice.

The forgetting factor λ_i permits to introduce a forgetting of the influence of the measured old values with the profit of the new measured values; this makes to weight less and less the old observations. In this direction, the measured values of the inputs and the outputs at the discrete-time $k-\tau$ are balanced by λ^{τ}.

We can show easily that the RLSFF parametric estimation algorithm, which can be applied to the interconnected time-varying system $\mathcal{S}_i$, $1 \leq i \leq N$, is described by:

$$\hat{\theta}_i(k) = \hat{\theta}_i(k-1) + P_i(k)\psi_i(k)\varepsilon_i(k)$$

$$P_i(k) = \frac{1}{\lambda_i}\left[P_i(k-1) - \frac{P_i(k-1)\psi_i(k)\psi_i^T(k)P_i(k-1)}{\lambda_i + \psi_i^T(k)P_i(k-1)\psi_i(k)}\right] \quad (18)$$

$$\varepsilon_i(k) = y_i(k) - \hat{\theta}_i^T(k-1)\psi_i(k)$$

The procedure, which consists in introducing a forgetting factor λ_i into the adaptation gain matrix $P_i(k)$ of a recursive parametric estimation algorithm, makes it possible to improve the capacity of this adaptation gain matrix, while ensuring best followed variable parameters in the time of the system considered. In such a procedure, the forgetting factor λ_i prevents that the values of the parameters of the adaptation gain matrix $P_i(k)$ do not become too small so that all new data (measured values of the inputs and the outputs) in the observation vector continuous to have an effect on the quality of the estimate.

It is advisable to indicate that the RLSFF parametric estimation algorithm (18) can be applied easily to the interconnected systems with slowly time-varying parameters. Indeed, it does not allow a best follow-up of the parameters being able to vary in an abrupt way during time and within a short time. Let us add that this algorithm is also appropriate for the interconnected systems where the quality of the experimental measurements resulting from each interconnected system is not same at any discrete-time. These results were confirmed by numerical simulations.

4.2 Recursive least squares method with weighted parameters

This subsection propose a RLSWP parametric estimation method in order to estimate the unknown time-varying parameters of the interconnected system $\mathcal{S}_i$, $1 \leq i \leq N$, as described by the IDARMA mathematical model (7). Thus, a RLSWP parametric estimation algorithm is developed, in which two weighted parameters are included in the computation procedure of its adaptation gain matrix. The formulation of this algorithm is based on the knowledge of the measured values of the inputs and the outputs of the considered interconnected systems at each step of computation, and by considering a certain previous measured values.

The techniques, which include of weighted parameters in the recursive parametric estimation algorithms of systems with time-varying parameters became increasingly widespread in the industrial applications. Indeed, these weighted parameters offer more flexibility to a recursive parametric estimation algorithm for following the parametric variations of the system (see, e.g., [4, 16, 17]).

We propose to express the output of the interconnected system S_i, $1 \le i \le N$, which is described by the IDARMA mathematical model (6) at the discrete-time k and the previous discrete-time $k-1, k-2, \cdots, k-m_i$ as follows:

$$\begin{aligned}
y_i(k-m_i) &= -a_{i,1}(k)y_i(k-m_i-1) - \cdots - a_{i,n_i}(k)y_i(k-m_i-n_i) \\
&\quad + b_{i,1}(k)u_i(k-m_i-1) + \cdots + b_{i,n_i}(k)u_i(k-m_i-n_i) \\
&\quad + b_{ij,1}(k)u_j(k-m_i-1) + \cdots + b_{ij,n_i}(k)u_j(k-m_i-n_i) \\
&\quad + a_{ij,1}(k)y_j(k-m_i-1) + \cdots + a_{ij,n_i}(k)y_j(k-m_i-n_i) + e_i(k-m_i) \\
y_i(k-m_i+1) &= -a_{i,1}(k)y_i(k-m_i) - \cdots - a_{i,n_i}(k)y_i(k-m_i+1-n_i) \\
&\quad + b_{i,1}(k)u_i(k-m_i) + \cdots + b_{i,n_i}(k)u_i(k-m_i+1-n_i) \\
&\quad + b_{ij,1}(k)u_j(k-m_i) + \cdots + b_{ij,n_i}(k)u_j(k-m_i+1-n_i) \\
&\quad + a_{ij,1}(k)y_j(k-m_i) + \cdots + a_{ij,n_i}(k)y_j(k-m_i+1-n_i) + e_i(k-m_i+1) \\
&\vdots \\
y_i(k) &= -a_{i,1}(k)y_i(k-1) - \cdots - a_{i,n_i}(k)y_i(k-n_i) \\
&\quad + b_{i,1}(k)u_i(k-1) + \cdots + b_{i,n_i}(k)u_i(k-n_i) \\
&\quad + b_{ij,1}(k)u_j(k-1) + \cdots + b_{ij,n_i}(k)u_j(k-n_i) \\
&\quad + a_{ij,1}(k)y_j(k-1) + \cdots + a_{ij,n_i}(k)y_j(k-n_i) + e_i(k)
\end{aligned} \tag{19}$$

where the parameter m_i corresponds to a certain horizon of work.

The output $y_i(k-m_i)$ of the interconnected system S_i, $1 \le i \le N$, can be defined in the following matrix form:

$$y_i(k-m_i) = \theta_i^T(k)\psi_i(k-m_i) + e_i(k-m_i) \tag{20}$$

where the parameter vector $\theta_i(k)$ is given by (8) and the observation vector $\psi_i(k-m_i)$ is defined as follows:

$$\begin{aligned}
\psi_i^T(k-m_i) = [&-y_i(k-m_i-1) - \cdots - y_i(k-m_i-n_i)\ u_i(k-m_i-1)\cdots u_i(k-m_i-n_i) \\
&u_j(k-m_i-1)\cdots u_j(k-m_i-n_i)\ y_j(k-m_i-1)\cdots y_i(k-m_i-n_i)]
\end{aligned} \tag{21}$$

with $j = 1,\cdots,N$, $j \ne i$.

Let us notice that this observation vector $\psi_i(k-m_i)$ corresponds, in fact, to a vector containing old observations (inputs, outputs) compared to the discrete-time k, and this, according to the choice of the value of the parameter m_i. Results of several numerical simulations, no reported here, have been shown that the choice of the value of the parameter m_i is connected to the type of the parametric variations of the considered interconnected systems. In this direction, we can indicate that in the case of interconnected systems with slowly time-varying parameters, then we can opt a weak value for the parameter m_i.

The RLSWP parametric estimation algorithm, which permits to estimate the parameters intervening in the parameter vector $\theta_i(k)$, as defined by (8), can be described by the following four steps (see [4]):

Step 1: computation of the auxiliary estimated parameter vector $\hat{\theta}_i^*(k)$ from:

$$\hat{\theta}_i^*(k) = \hat{\theta}_i(k-1) - K_i^*(k)[y_i(k-m_i) - \hat{\theta}_i^T(k-1)\psi_i(k-m_i)]$$
$$K_i^*(k) = \frac{\varsigma_i P_i(k-1)\psi_i(k-m_i)}{1-\varsigma_i\psi_i^T(k-m_i)P_i(k-1)\psi_i(k-m_i)} \tag{22}$$

Let us note that the parameters intervening in the estimated vector $\hat{\theta}_i(k-1)$ and the adaptation gain matrix $P_i(k-1)$, which are expressed at the discrete-time $k-1$, must be known at the discrete-time k; they can correspond to initial conditions for this first step;

Step 2: computation of the auxiliary adaptation gain matrix $P_i^*(k)$ from:

$$P_i^*(k) = P_i(k-1) + \frac{\varsigma_i P_i(k-1)\psi_i(k-m_i 1)\psi_i^T(k-m_i)P_i(k-1)}{1-\varsigma_i\psi_i^T(k-m_i)P_i(k-1)\psi_i(k-m_i)} \tag{23}$$

Step 3: estimate of the parameter vector $\theta_i(k)$ from:

$$\hat{\theta}_i(k) = \hat{\theta}_i^*(k) + K_i(k)[y_i(k) - \hat{\theta}_i^{*T}(k)\psi_i(k)]$$
$$K_i(k) = \frac{P_i^*(k)\psi_i(k)}{\rho_i + \psi_i^T(k)P_i^*(k)\psi_i(k)} \tag{24}$$

Step 4: computation of the adaptation gain matrix $P_i(k)$ from:

$$P_i(k) = \frac{1}{\rho_i}\left[P_i^*(k) - \frac{P_i^*(k)\psi_i(k)\psi_i^T(k)P_i^*(k)}{\rho_i + \psi_i^T(k)P_i^*(k)\psi_i(k)}\right] \tag{25}$$

These four steps proceed sequentially, and this, at each discrete-time k ($k = 1, \cdots, M_i$).

The practical implementation of the RLSWP parametric estimation algorithm can be made while supposing known their initial conditions, the values of the two weighted parameters ς_i and ρ_i, and the values of the inputs and the outputs intervening in the information sequence $I_i(k)$ resulting from the considered interconnected systems.

Let us note however that the quality of the estimate that we can obtain using the developed RWLS parametric estimation algorithm, which is defined by the equations (22)-(25), depends on the choice of the parameter m_i and the values of the two weighted parameters ς_i and ρ_i. Thus, this choice must be made in an adapted way, in order to ensure a best quality of estimate. The choice of the two weighted parameters ς_i and ρ_i must satisfy the following conditions, respectively (see [4]):

$$\varsigma_i \geq 0 \tag{26}$$

and

$$0 < \rho_i \leq 1 \tag{27}$$

To more improve the capacity of adaptation gain matrix of the RWLS parametric estimation algorithm (22)-(25), we can choose a weighted variable parameters $\varsigma_i(k)$ and $\rho_i(k)$. However, the choice of the time-variation functions of these parameters is not obvious. It is thus judicious to bind these choices to the type of the parametric variations of each interconnected system. We propose to generate the sequences of the weighted variable parameters $\varsigma_i(k)$ and $\rho_i(k)$ on the basis of the knowledge of the value of the variance of the prediction error $\varepsilon_i(k)$ and by using advanced analysis techniques (fuzzy logic, supervision, expert system, etc). This constitutes our future works.

4.3 Recursive least squares method using Kalman Filter

This subsection is related to the development of a recursive parametric estimation algorithm for the interconnected time-invariant system $\mathcal{S}_i$, $1 \leq i \leq N$, which is described by the IDARMA mathematical model (7), using the Kalman filter approach. The formulation of this algorithm is made while retaining the assumption made in sub-section 2.1 concerning the noise $e_i(k)$, which affects the output $y_i(k)$ of this interconnected system. Thus, we can write the following expressions:

$$\mathcal{E}[e_i(k)] = 0 \tag{28}$$

and

$$\mathcal{E}[e_i^{\,2}(k)] = \sigma_i^{\,2} \tag{29}$$

where $\mathcal{E}$ represents the symbol of the expectation.

The use of the techniques of filtering, within the meaning of kalman, in parametric estimation of the dynamical systems, in particular with time-varying parameters, is an approach that drew the attention of several researchers (see, e.g., [4, 19]).

The time-varying parameters vector $\theta_i(k)$, which is defined by (8), can be described by a process of Gauss-Markov, that is to say:

$$\theta_i(k) = \Phi_i \theta_i(k-1) + v_i(k) \tag{30}$$

where Φ_i is a matrix, which is related mainly to the nature of the parametric variations of the considered interconnected system $\mathcal{S}_i$, $1 \leq i \leq N$, and $v_i(k)$ is a noise vector, such as: $v_i^{\,T}(k) = [v_{i,1}(k), \cdots, v_{i,4ni}(k)]$, which consists of sequences $\{v_{i,r}(k); r = 1, \cdots, 4n_i\}$ of an independent random variables with zero mean and variance-covariance matrix $Q_i(k)$, such that:

$$\mathcal{E}[v_i(k)] = 0 \tag{31}$$

and

$$\mathcal{E}[v_i(k) v_i^{\,T}(j)] = Q_i(k)\delta_{kj} \tag{32}$$

where δ_{kj} denote the Kronecker symbol, such as: $\delta_{kj} = 0$, if $k \neq j$, and $\delta_{kj} = 1$, if $k = j$; with k and j positive whole numbers.

Let us notice that the equation (30) corresponds to a state-space equation, in which Φ_i represents the dynamic matrix of the state. Indeed, the parameters of this matrix make to describe the dynamics of the parametric variations of the interconnected system S_i, $1 \leq i \leq N$. In this context, and if these parametric variations would be abrupt and/or fast, then we can choose a matrix of the state with time-varying parameters (i.e., $\Phi_i(k)$). However, to simplify the practical implementation of the parametric estimation algorithm of the time-varying parameter vector $\theta_i(k)$, defined by (8), we suppose that the parameters of the matrix Φ_i are constant and well known.

We supposed here that the parameters of the variance-covariance matrix $Q_i(k)$ of the noise vector $v_i(k)$ are not known. In such a situation, we must proceed to a stage of estimate of these parameters. Let us note however that, in certain industrial applications, the control engineer can fix the parameters of this matrix, in particular its diagonal parameters, which correspond to the variances of the noises $v_{i,r}(k)$, $r = 1, \cdots, 4n_i$. The initial state $\theta_i(0)$ of the state equation (30) being supposed a normal distribution with an average θ_{i0} and a variance-covariance matrix $Q_i(0)$. We suppose that the noise $e_i(k)$ is independent of the components of two vectors $v_i(k)$ and θ_{i0}.

From the equations (7) and (30), we can obtain the following state-space mathematical model, which allows to describe, in fact, the interconnected time-varying system S_i, $1 \leq i \leq N$:

$$\begin{aligned} \theta_i(k) &= \Phi_i \theta_i(k-1) + v_i(k) \\ y_i(k) &= \psi_i^T(k)\theta_i(k) + e_i(k) \end{aligned} \tag{33}$$

We propose to estimate the time-varying parameter vector $\theta_i(k)$, while basing ourselves on state estimation approach of the dynamical systems, within the meaning of the Kalman filter. This state estimation will be carried out starting from the knowledge of the several signals (inputs, outputs) resulting from the set of the interconnected systems S_i, $i = 1, \cdots, N$. In this case, we must consider the information sequence $I_i(k)$, which is defined in subsection 2.1.

Taking into account the two equations (8) and (30), we can suppose that the conditional distribution of the time-varying parameter vector $\theta_i(k)$, using the sequence of information $I_i(k)$, is a Gaussian distribution, with an average $\hat{\theta}_i(k)$ and a variance-covariance matrix $R_i(k)$. We propose to compute recursively this average value $\hat{\theta}_i(k)$, which corresponds to the best estimated value (within the meaning of the minimum variance) of the parameter vector $\theta_i(k)$, starting from the following RLSKF parametric estimation algorithm:

$$\begin{aligned} \hat{\theta}_i(k) &= \Phi_i \hat{\theta}_i(k-1) + K_i(k)\varepsilon_i(k) \\ K_i(k) &= \frac{\Phi_i R_i(k-1)\psi_i(k)}{\sigma_i^2 + \psi_i^T(k) R_i(k-1)\psi_i(k)} \\ R_i(k) &= [\Phi_i - K_i(k)\psi_i^T(k)] R_i(k-1)\Phi_i^T + Q_i(k) \\ \varepsilon_i(k) &= y_i(k) - \hat{\theta}_i^T(k-1)\psi_i(k) \end{aligned} \tag{34}$$

The implementation of the RLSKF parametric estimation algorithm (34) requires the knowledge of the initial conditions $\hat{\theta}_i(0)$ and $R_i(0)$, which are relating to the estimated parameter vector $\hat{\theta}_i(k)$ and the variance-covariance matrix $R_i(k)$, respectively. Let us add that the values of the variance σ_i^2 of the noise $e_i(k)$ and the parameters intervening in the variance-covariance matrix $Q_i(k)$ must be known. In several types of industrial applications, we can have an *a priori* information on the values of the variances of the noises $v_{i,r}(k)$, $r = 1, \cdots, 4n_i$, which correspond to the diagonal parameters of the variance-covariance matrix $Q_i(k)$. In such a situation, we can use a time-invariant variance-covariance matrix Q_i, i.e.: $Q_i(k) = Q_i$, $\forall k$. Moreover, the choice of all the parameters intervening in this variance-covariance matrix Q_i, which must be made in an adapted way, makes to simplify the practical implementation of the RLSKF parametric estimation algorithm (34). However, we propose to use procedures allowing to estimate in real-time the parameters intervening in the variance-covariance matrix $Q_i(k)$. This constitutes our future works.

5. SIMULATION EXAMPLE

In this section, the performance and the effectiveness of the developed RLSWP parametric estimation algorithm (22)-(25) are tested in numerical simulation. Also, a comparative study in numerical simulations between the developed RLSFF, RLSWP and RLSKF parametric estimation algorithms that are defined by (18), (22)-(25) and (34), respectively, is given. We will show the role of the weighted parameters, which are introduced in the computation procedure of the adaptation gain matrices of these algorithms, in order to estimate the interconnected time-varying systems.

Let us consider a large-scale time-varying system $\mathcal{S}$ that is constituted by two interconnected SISO systems $\mathcal{S}_1$ and $\mathcal{S}_2$, which are described by the following IDARMA mathematical models, respectively:

$$y_1(k) = -a_{1,1}(k)y_1(k-1) + b_{1,1}u_1(k-1) + b_{12,1}u_2(k-1) + a_{12,1}y_2(k-1) + e_1(k) \tag{35}$$

and

$$y_2(k) = -a_{2,1}(k)y_2(k-1) + b_{2,1}u_2(k-1) + b_{21,1}u_1(k-1) + a_{21,1}y_1(k-1) + e_2(k) \tag{36}$$

where $u_1(k)$, $y_1(k)$ and $e_1(k)$ denote the input, the output and the noise of the interconnected system $\mathcal{S}_1$, respectively, $u_2(k)$, $y_2(k)$ and $e_2(k)$ represent the input, the output and the noise of the interconnected system $\mathcal{S}_2$, respectively.

The parameters intervening in the IDARMA mathematical models (35) and (36) are supposed unknown; moreover, $a_{1,1}(k)$ and $a_{2,1}(k)$ are time-varying parameters.

The output $y_1(k)$ of the interconnected system $\mathcal{S}_1$ can rewrite as follows:

$$y_1(k) = \theta_1^T(k)\psi_1(k) + e_1(k) \tag{37}$$

where the parameter vector $\theta_1(k)$ and the observation vector $\psi_1(k)$ are defined by:

$$\theta_1^T(k) = [a_{1,1}(k)\ b_{1,1}\ \ b_{12,1}\ \ a_{12,1}] \tag{38}$$

and

$$\psi_1^T(k) = [-y_1(k-1)\ u_1(k-1)\ u_2(k-1)\ y_2(k-1)] \tag{39}$$

We can rewrite the output $y_2(k)$ of the interconnected system S_2 in the following form:

$$y_2(k) = \theta_2^T(k)\psi_2(k) + e_2(k) \tag{40}$$

where the parameter vector $\theta_2(k)$ and the observation vector $\psi_2(k)$ are defined by:

$$\theta_2^T(k) = [a_{2,1}(k)\ b_{2,1}\ \ b_{21,1}\ \ a_{21,1}] \tag{41}$$

and

$$\psi_2^T(k) = [-y_2(k-1)\ u_2(k-1)\ u_1(k-1)\ y_1(k-1)] \tag{42}$$

In the first numerical simulation series, the objective is to estimate the parameter vectors $\theta_1(k)$ and $\theta_2(k)$ by using the RLSWP parametric estimation algorithm (22)-(25) in order to test its performance and its effectiveness.

For the purpose of the simulation example, we assume the following numerical values for the parameters intervening in the IDARMA mathematical models (35) and (36): $a_{1,1}(k) = 0.0044k\sin(0.0400k)$, for $k = 1,\cdots,50$, $a_{1,1}(k) = -0.7200$, for $k = 51,\cdots,100$, $a_{1,1}(k) = -0.9700$, for $k = 101,\cdots,200$, $b_{1,1} = 0.2400$, $b_{12,1} = 0.1800$, $a_{12,1} = 0.1200$, $a_{2,1}(k) = -0.8800 + 0.0020k$, $k = 1,\cdots,100$, $a_{2,1}(k) = -0.6800$, $k = 101,\cdots,200$, $b_{2,1} = 0.3000$, $b_{21,1} = 0.2200$, $a_{21,1} = -0.1000$. Let us add that the noise sequences $\{e_1(k)\}$ and $\{e_2(k)\}$ are supposed to be independent and correspond to the Gaussian distribution, with zero means and variances $\sigma_1^2 = 0.0005$ and $\sigma_2^2 = 0.0080$, respectively.

The relative data to this example of numerical simulation, for the practical implementation of the RLSWP parametric estimation algorithm (22)-(25), are given hereafter:

1. The input $u_1(k)$ applied to the interconnected system S_1 being a pseudo-random binary sequence, with length 1023 and level [+1, – 1];
2. The input $u_2(k)$ applied to the interconnected system S_2 being a pseudo-random binary sequence, with length 1023 and level [+2, – 2];
3. The initial values of the two parameter vectors $\hat{\theta}_i(0)$ and the two adaptation gain matrices $P_i(0)$ are chosen, such as: $\hat{\theta}_i(0) = 0$ and $P_i(0) = 1000I$, where I is an identity matrix, $i = 1,2$;
4. The weighted parameters ς_1, ρ_1, ς_2 and ρ_2 are chosen, such as: $\varsigma_1 = 0.0200$, $\rho_1 = 0.6400$, $\varsigma_2 = 0.0400$ and $\rho_2 = 0.5200$;
5. The number of measurements M_i being chosen, such as: $M_i = 1,\cdots,200$.

We can define the estimation error $\delta_{a_{i,1}}(k)$ of the parameter $a_{i,1}(k)$ as follows:

$$\delta_{a_{i,1}}(k) = a_{i,1}(k) - \hat{a}_{i,1}(k) \tag{43}$$

The global quality of the obtained estimate in the numerical simulations can be made starting from the calculation of the following parametric distance $d_i(k)$:

$$d_i(k)=\left[\left(\frac{a_{i,1}(k)-\hat{a}_{i,1}(k)}{a_{i,1}(k)}\right)^2+\left(\frac{b_{i,1}(k)-\hat{b}_{i,1}(k)}{b_{i,1}(k)}\right)^2+\left(\frac{a_{ij,1}(k)-\hat{a}_{ij,1}(k)}{a_{ij,1}(k)}\right)^2+\left(\frac{b_{ij,1}(k)-\hat{b}_{ij,1}(k)}{b_{ij,1}(k)}\right)^2\right]^{0.5} \tag{44}$$

with $i,j=1,2$, $j\neq i$.

Some obtained numerical simulation results are given in Figures 1 and 2. Thus, the evolution curves of the estimated parameters $\hat{a}_{1,1}(k)$, $\hat{b}_{1,1}(k)$, $\hat{b}_{12,1}(k)$ and $\hat{a}_{12,1}(k)$, and the true parameters of the interconnected system S_1 are given in Figure 1. Figure 2 represents the evolution curves of the estimated parameters $\hat{a}_{2,1}(k)$, $\hat{b}_{2,1}(k)$, $\hat{b}_{21,1}(k)$ and $\hat{a}_{21,1}(k)$, and the true parameters of the interconnected system S_2. We can note that the evolution curves of the different estimated parameters shows well the good quality of the estimate.

These numerical simulation results confirm well the interest of the developed RLSWP parametric estimation algorithm (22)-(25). The interpretation of the obtained evolution curves proves the effectiveness and the performances (in particular a good follow-up of the parameters $a_{1,1}(k)$ and $a_{2,1}(k)$), which can be assured by using this algorithm.

In the second numerical simulation series, we propose to make a comparative study between the developed RLSFF, RLSWP and RLSKF parametric estimation algorithms, which are given by (18), (22)-(25) and (34), respectively. Let us consider the same example of numerical simulation, which is treated above for the RLSWP parametric estimation algorithm (22)-(25). We retain the same data for the implementation of these algorithms. Moreover, the forgetting factors λ_1 and λ_2 intervening in the RLSFF parametric estimation algorithm (18) are chosen as follows: $\lambda_1=0.9700$, $\lambda_1=0.9800$. Also, for the implementation of the RLSKF parametric estimation algorithms (34), we suppose that:

1. The components $v_{i,1}(k)$ and $v_{i,2}(k)$, $v_{i,3}(k)$ and $v_{i,4}(k)$ of the noise vector $v_i(k)$ are consisted of the sequences of an independent random variables with zero mean;
2. The noise vectors $v_1(k)$ and $v_2(k)$, and the noise $e_i(k)$ are supposed independent;
3. The variance-covariance matrix Q_1 of the noise vector $v_1(k)$, the variance-covariance matrix Q_2 of the noise vector $v_2(k)$, and the initial condition of the variance-covariance matrix are chosen as follows:

$$Q_1=\begin{bmatrix}0.5000 & 0.0010 & 0.0010 & 0.0001\\ 0.0010 & 0.3000 & 0.0010 & 0.0001\\ 0.0010 & 0.0010 & 0.4000 & 0.0002\\ 0.0001 & 0.0001 & 0.0002 & 0.4000\end{bmatrix},\ Q_2=\begin{bmatrix}0.4000 & 0.0010 & 0.0001 & 0.0001\\ 0.0001 & 0.3000 & 0.0010 & 0.0002\\ 0.0010 & 0.0010 & 0.2000 & 0.0001\\ 0.0001 & 0.0002 & 0.0001 & 0.3000\end{bmatrix},$$

$$R_i(0)=\begin{bmatrix}0.5000 & 0.0010 & 0.0010 & 0.0001\\ 0.0010 & 0.3000 & 0.0010 & 0.0001\\ 0.0010 & 0.0010 & 0.4000 & 0.0002\\ 0.0001 & 0.0001 & 0.0002 & 0.4000\end{bmatrix},\ i=1,2\,.$$

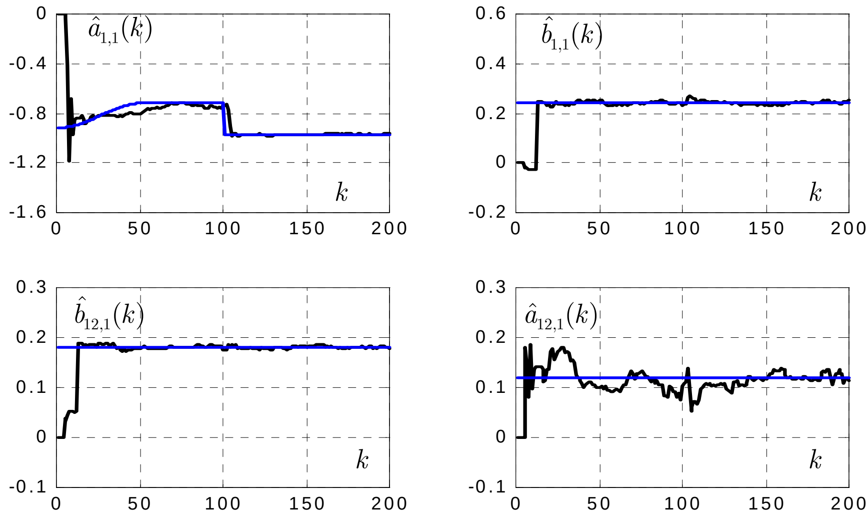

Figure 1. Evolution curves of the estimated parameters $\hat{a}_{1,1}(k)$, $\hat{b}_{1,1}(k)$, $\hat{b}_{12,1}(k)$ and $\hat{a}_{12,1}(k)$, and the true parameters (--- blue) of the interconnected system S_1.

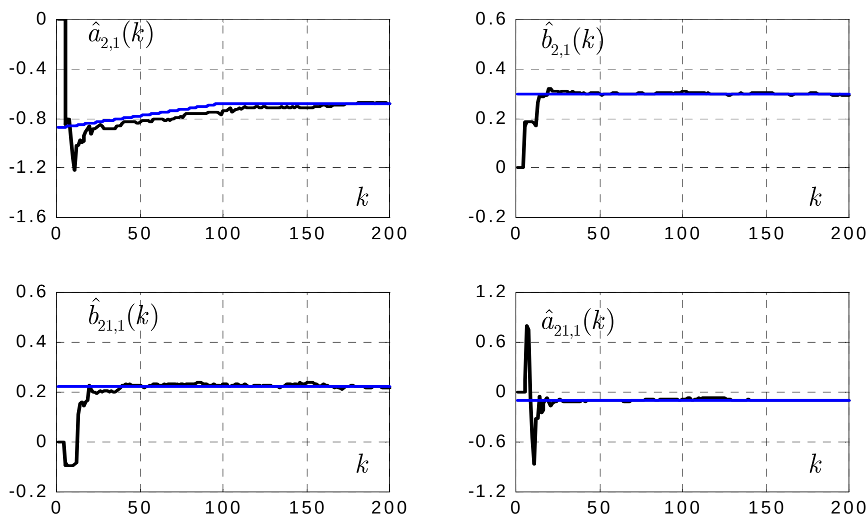

Figure 2. Evolution curves of the estimated parameters $\hat{a}_{2,1}(k)$, $\hat{b}_{2,1}(k)$, $\hat{b}_{21,1}(k)$ and $\hat{a}_{21,1}(k)$, and the true parameters (--- blue) of the interconnected system S_2.

The principal obtained numerical simulation results are given in Tables 1 and 2. Thus, Table 1 presents the values of the statistical averages $\overline{m}_{\hat{a}_{1,1}}$, $\overline{m}_{\hat{b}_{1,1}}$, $\overline{m}_{\hat{b}_{12,1}}$ and $\overline{m}_{\hat{a}_{12,1}}$ of the estimated parameters of the interconnected system $\mathcal{S}_1$, and the values of the statistical averages $\overline{m}_{\hat{a}_{1,1}}$, $\overline{m}_{\hat{b}_{1,1}}$, $\overline{m}_{\hat{b}_{12,1}}$ and $\overline{m}_{\hat{a}_{21,1}}$ of the estimated parameters of the interconnected system $\mathcal{S}_2$, and this for $k = 151, \cdots, 200$.

Table 1. Statistical averages $\overline{m}_{\hat{a}_{1,1}}$, $\overline{m}_{\hat{b}_{1,1}}$, $\overline{m}_{\hat{b}_{12,1}}$ and $\overline{m}_{\hat{a}_{12,1}}$ of the estimated parameters of the interconnected system $\mathcal{S}_1$, and statistical averages $\overline{m}_{\hat{a}_{1,1}}$, $\overline{m}_{\hat{b}_{1,1}}$, $\overline{m}_{\hat{b}_{12,1}}$ and $\overline{m}_{\hat{a}_{21,1}}$ of the estimated parameters of the interconnected system $\mathcal{S}_2$.

Algorithm	$\overline{m}_{\hat{a}_{1,1}}$	$\overline{m}_{\hat{b}_{1,1}}$	$\overline{m}_{\hat{b}_{12,1}}$	$\overline{m}_{\hat{a}_{12,1}}$	$\overline{m}_{\hat{a}_{2,1}}$	$\overline{m}_{\hat{b}_{2,1}}$	$\overline{m}_{\hat{b}_{21,1}}$	$\overline{m}_{\hat{a}_{21,1}}$
RLSWP	–0.9676	0.2417	0.1813	0.1232	–0.6879	0.2976	0.2230	–0.1019
RLSKF	–0.9640	0.2403	0.1787	0.0938	–0.7035	0.2991	0.2255	–0.0999
RLSFF	–0.9422	0.2509	0.1698	0.1342	–0.7466	0.2803	0.2075	–0.0911

Table 2 gives the values of the statistical averages $\overline{m}_{\delta_{a1,1}}$ of the estimation error $\delta_{a_{1,1}}(k)$ of the parameter $a_{1,1}(k)$, $\overline{m}_{d_1}$ of the parametric distance $d_1(k)$, $\overline{m}_{\varepsilon_1}$ of the prediction error $\varepsilon_1(k)$ and $\overline{m}_{\sigma_{\varepsilon 1}^2}$ of the variance of the prediction error $\varepsilon_1(k)$ of the interconnected system $\mathcal{S}_1$, and the values of the statistical averages $\overline{m}_{\delta_{a2,1}}$ of the estimation error $\delta_{a_{2,1}}(k)$ of the parameter $a_{2,1}(k)$, $\overline{m}_{d_2}$ of the parametric distance $d_2(k)$, $\overline{m}_{\varepsilon_2}$ of the prediction error $\varepsilon_2(k)$ and $\overline{m}_{\sigma_{\varepsilon 2}^2}$ of the variance of the prediction error $\varepsilon_2(k)$ of the interconnected system $\mathcal{S}_2$. Let us note that the computation of the values $\overline{m}_{\delta_{a1,1}}$ and $\overline{m}_{\delta_{a2,1}}$ is made for $k = 1, \cdots, 200$. However, the computation of the other values is made for $k = 151, \cdots, 200$.

The interpretation of the obtained numerical results, which are given in Tables 1 and 2, shows well the good quality of the estimate, which is obtained by the developed RLSFF RLSWP and RLSKF parametric estimation algorithms. However, let us indicate that the best quality of the estimate is obtained by using the RLSWP parametric estimation algorithm (22)-(25). Also, The RLSKF parametric estimation algorithm (34) permits to ensure a quality of estimate better than that which the RLSKF parametric estimation algorithm (18) can ensure.

Table 2. Value of the statistical averages $\overline{m}_{\delta_{a1,1}}$, $\overline{m}_{d_1}$, $\overline{m}_{\varepsilon_1}$ and $\overline{m}_{\sigma_{\varepsilon 1}^2}$ of the interconnected system $\mathcal{S}_1$, and value of the statistical averages $\overline{m}_{\delta_{a2,1}}$, $\overline{m}_{d_2}$, $\overline{m}_{\varepsilon_2}$ and $\overline{m}_{\sigma_{\varepsilon 2}^2}$ of the interconnected system $\mathcal{S}_2$.

Algorithm	$\overline{m}_{\delta_{a1,1}}$	$\overline{m}_{d_1}$	$\overline{m}_{\varepsilon_1}$	$\overline{m}_{\sigma_{\varepsilon 1}^2}$	$\overline{m}_{\delta_{a2,1}}$	$\overline{m}_{d_2}$	$\overline{m}_{\varepsilon_2}$	$\overline{m}_{\sigma_{\varepsilon 2}^2}$
RLSWP	–0.0024	0.0652	–0.0037	0.0007	0.0079	0.0453	0.0067	0.0084
RLSKF	–0.0060	0.2189	–0.0046	0.0013	0.0235	0.0518	0.0058	0.0087
RLSFF	–0.0081	0.2033	–0.0059	0.0014	0.0411	0.0606	0.0063	0.0092

It is important to note than globally, the convergence time of a recursive parametric estimation algorithm depend on the choice of the values on the initial conditions $\hat{\theta}_i(0)$ and $P_i(0)$. Indeed, if the initial condition of the estimated parameter vector $\hat{\theta}_i(0)$ is chosen near to the true parameter vector $\theta_i(k)$ and the value of the initial condition of the adaptation gain matrix is small (e.g., $P_i(0) = I$), then the estimated parameter vector $\hat{\theta}_i(k)$ converge rapidly to the true parameter vector $\theta_i(k)$.

Let us note the that the *a priori* prediction error $\varepsilon_i(k)$ corresponds to the best estimate of the noise $e_i(k)$, $i = 1, 2$, which is not measurable. For example, the computed value $\bar{\mathrm{m}}_{\sigma_{\varepsilon 1}{}^2}$ of the statistical average of the variance of the prediction error $\varepsilon_1(k)$ obtained by the RLSWP parametric estimation algorithm (22)-(25) is near of the variance $\sigma_1{}^2 = 0.0005$ of the noise $e_1(k)$. The variation enters the values of these two variances being equal to 0.0002.

Finally, we can affirm that the obtained numerical simulation results, in this example of numerical simulation, are satisfactory and show well the performances, which can ensure the developed parametric estimation algorithms (notably the RLSWP parametric estimation algorithm) during the estimate of the parameters of the two interconnected time-varying systems, which are described by the IDARMA mathematical models (35) and (36).

6. CONCLUSION

The purpose of this paper was to analyse three recursive parametric estimation methods, which permit to estimate the parameters of the large-scale time-varying systems that are composed into several interconnected SISO systems. These methods are based upon the prediction error method and by using the least squares techniques. It is about the Recursive Least Squares method with Forgetting Factor (RLSFF), the Recursive Least Squares method with Weighted Parameters (RLSWP) and the Recursive Least Squares method using Kalman Filter (RLSKF). Recursive parametric estimation algorithms corresponding to the three methods were developed. Let us note that the key idea of the considered recursive parametric estimation methods is to incorporate weighted factors in the computation procedure of the adaptation gain matrices of these algorithms.

We have considered the class of large-scale time-varying systems that can be decomposed of several interconnected SISO systems. The considered interconnected systems are described by discrete-time input-output mathematical models, SISO, linear, with known structure parameters and with unknown time-varying parameters.

Numerical simulation results were presented in order to validate the effectiveness and the performance of the developed parametric estimation algorithms.

The obtained numerical simulation results have shown the good follow-up of the parametric variations of the interconnected time-varying systems and the best quality of the estimate, which are ensured by using the developed parametric estimation algorithms.

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