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Detection of Average Jump by Bayes Test with Adaptive Threshold

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The subject of this paper is to show how to implement a binary hypothesis testing problem. The test would allow the detection of low amplitude faults that affect a signal, as fast as possible and with a minimum risk of errors. It is a Bayes test of adaptive threshold based on the estimation of a priori probabilities through the Gaussian Kernels method. A comparative study is provided to show that such a test improves performance (the increase of detection probability and the reduction of false alarm probability) compared with Bayes's fixed threshold test.

Keywords: Bayes test, non parametric estimation, fixed threshold, adaptive threshold, false alarm probability, non detection probability, fault detection.

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1. INTRODUCTION

The complexity and sophistication of the current generation of industrial processes, and the growing need for autonomous agents that control physical systems, motivate the need for robust online monitoring and diagnosis of complex engineering systems. Failure diagnosis consists of providing information at the time and location of faults that occur in the supervised process. The former task is called fault detection problem, while the latter is referred to as fault isolation. The model based approach to fault detection and isolation (FDI) has received considerable attention in intelligent control systems. Many different approaches to fault detection in stochastic dynamical systems have been studied and summarized by [13], [11], [3], [19], which a statistical approach (statistical decision tools) is presented.

This approach was the subject of many works such as [15], [9], [1], and [17]. Based on the use of models, this approach may be divided into two main stages which are the residue generation beyond faults and the decision making. Concerning the first stage, we should generate signals highlighting the existence of fault, which is usually bias. If the residual signal is statistically null then, the system functions normally. Where as the existence of a fault when the residual signal is different from zero. This deviates significantly from the zero value. The second stage deals with the decision making (existence of faults, and the time of their appearance) and requires a statistical approach which refers to [16], [11], [17], [12]. Later on, the decision making issue will be dealt with, focusing on the statistical hypotheses tests. The analysis of fault indicators we can stems from the perception phase which this latter reflects the system functioning find a fault modes [10], [7], [16], [20], [8] allows the use of many techniques coming from classical methods (threshold logic), material redundancy and model-based methods [16], [5], [19], [14]. The aim of this work is to use a binary hypotheses statistical test that allows the detection of low amplitudes as fast as possible and with the least of errors (caused by false alarms and non detections). Later, we would like to focus on the use of hypotheses statistical test of fixed threshold. The paper

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is organized as follows: in section II, we will describe and we'll discuss the effect of threshold test on the model. In section III, we'll evoke the Bayes test technique highlighting its limits of use. In section IV, we will present an algorithm to estimate the a priori probabilities [18], [4], [6], [2] in order to use a binary hypotheses test of adaptive threshold, via an example. A comparative study of the fixed threshold test and the adaptive threshold test will be presented. The simulation results will show that the improvement of the decision making performance by the adaptive threshold test is better than by the fixed threshold test are presented in section V. Finally, we come to the conclusions of this paper.

2. PROBLEM STATEMENT

The statistical decision tools consist of deciding between the functioning modes of the industrial system. The problem of detecting a change in the scalar parameter of an independent random sequence is the survey paper [13].

Let use now the Bayesian approach in which a priori information about changes in the parameters of a probability distribution and will be estimated. We assume that this information is in the form of an a priori probability distribution.

The problem of detecting changes in properties of signals and dynamical systems is based in the case of a change in the mean of a Gaussian sequence. We consider a situation of a change in the mean level of a sequence of observation is locally characterized by an absolute value of the derivative of the sample observation. The sequence of random observations with conditional density $p(y_k / y_{k-1}, ..., y_1)$ made on the industrial system. In the case of a binary hypothesis test, a decision rule should be computed to test the two alternative hypotheses H_0 and H_1. These hypotheses often correspond respectively to normal and abnormal functioning modes of the system. In this case, the observation space is divided into two disconnected sub-spaces disjoined E_0 and E_1 which are respectively defined by :

$E_0 = \{y : y < \gamma\}$ and $E_1 = \{y : y > \gamma\}$,

with the following notations

E_0 : sub-space characterizing the hypothesis H_0,

E_1 : sub-space characterizing the hypothesis H_1,

γ : threshold of detection for a given observation.

Assuming that, the conditional probabilities densities $p(y/H_0)$ and $p(y/H_1)$ are known. Thus, we can define the conditional probabilities of false alarm and the probabilities of non detection that are obtained from [16] as follows:

$$P_F = \int_{E_1} p(y / H_0) dy \tag{1}$$

$$P_{ND} = \int_{E_0} p(y / H_1) dy \tag{2}$$

The Gaussian distributions of the two hypotheses are depicted in Fig. 1. As can be deduced, the two probabilities of risks of error (P_F, P_{ND}) are dependent, the importance of which will be discussed deeply.

Indeed,

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• if we choose the hypothesis H_0, we decrease P_F and we increase P_{ND},

• if we choose the hypothesis H_1, we decrease P_{ND} and we increase P_F .

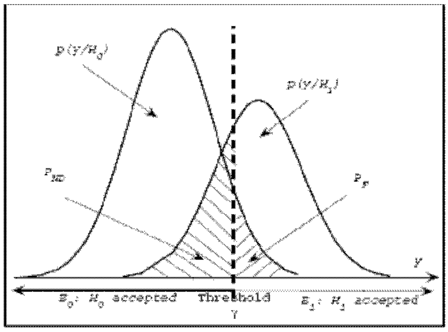

Figure 1: The regions of correct decision and incorrect decision with error risks

This highlights and reminds of the dilemma of the binary hypotheses test with fixed threshold. For this reason, we propose to make a binary hypotheses test with adaptive threshold. The later should be able to detect, as fast as possible and with the least errors, the faults of low amplitudes which are often drowned in the measurement noise.

3. BAYES TEST

The Bayes test detection problem, which is an off-line hypotheses test, can be formally stated as follows. Given a finite N size sample of independent observations $Y = [y_1, y_2, ..., y_N]^T$. A decision rule is computed to test between the two hypotheses H_0 and H_1 according to the likelihood ratio $\Lambda(Y)$.

$$\begin{aligned}\Lambda(Y) &= \frac{p(Y|H_1)}{p(Y|H_0)} \\ &= \prod_{i=1}^{N} \frac{p(y_i|H_1)}{p(yi|H_0)} \quad \underset{H_0}{\overset{H_1}{\gtrless}} \eta\end{aligned} \tag{3}$$

η : decision threshold.

The decision is a way of determining whether this likelihood ratio is above or below a threshold value.

• if $\Lambda(Y) \leq \eta$, then H_0 is accepted,

• if $\Lambda(Y) > \eta$, then H_1 is accepted,

The threshold η minimizes the risk of Bayes [16], [15], [4] given by :

$$\eta = \frac{P_0(C_{10} - C_{00})}{P_1(C_{01} - C_{11})} \tag{4}$$

where $P_0 = p(H_0)$ is the a prior probability of occurrence of the hypothesis H_0, $P_1 = p(H_1)$ is the priori probability of occurrence of the hypothesis H_1, C_{11}, C_{00} and C_{10}, C_{01} are respectively the costs of making a correct decision and the costs of making an incorrect decision with reference to H_0 and H_1.

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In Eq. (2), the threshold η in many practical cases can be simplified $C_{11} = C_{00} = 0$ and $C_{01} = C_{10}$, can then be expressed as :

$$\eta = \frac{P_0}{P_1} \tag{5}$$

This choice can be justified for the logical reasons, a correct decision being less penalizing than an incorrect decision; we have $C_{10} > C_{00}$ and $C_{01} > C_{11}$. In practical we can reason on two economic scales, while considering that these costs are relative and constant:

• in the first case, we consider that the cost C_{00} is null because the system of measurement functions normally and the corresponding decision is correct. On the other hand, the cost C_{01} corresponds to a penalty considered fixed which is due to false alarm;

• in the second case, we consider that the cost C_{11} associated with the good decision is null, because the system of measurement is not functional and making a correct decision. On the other hand, the cost C_{01} corresponds to a penalty, the cost of making an incorrect decision which represents a non detection.

The Bayes hypotheses test assumes that the number of hypotheses and their a priori probabilities are known. The costs of good and bad decision are also known. It is difficult to assign suitable values to a priori probabilities and to choose the weight of criterion. This assignment is often achieved heuristically, ponderations may form the adjustment parameters of the method. A good test is such where the false alarm probability should be the lowest possible and the detection probability the highest possible (such is the strongest test). The study of Bayes tests brings out many obstacles, particularly their application in the industrial systems:

• if we want to apply the Bayesian tests, the major difficulty consists in the knowledge of the laws of a priori probabilities (P_0 and P_1), the good functioning and the malfunctioning of binary hypotheses H_0 and H_1 respectively. The same difficulty emerges concerning conditional laws of observation probability density $p(Y/H_0)$ and $p(Y/H_1)$,

• the assessment of direct and indirect costs of a correct or an incorrect decision,

• the statistic independence of observations is also a difficulty that we encounter in order to use in particular the calculation the probability ratios.

One of the provided solutions to control the a priori probabilities laws (P_0 and P_1) is to estimate them directly basing an experimental data.

4. ESTIMATION OF THE A PRIORI PROBABILITIES

The estimation methods of a priori probabilities P_0 and P_1 compared with binary hypotheses H_0 and H_1 respectively,

are presented in many works by, among others, Naim and Kam [6]. One of the used techniques is the method of Gaussian kernels which allows the processing of data directly and provides an estimation of a priori probabilities.

More precisely, the measuring data are used directly to calculate weighting assigned to probability densities functions respective to hypotheses H_0 and H_1. Each kernel is characterized by two statistic parameters, the average μ and the standard deviation σ to which is added a parameter of weight correction ω_1 which has a function equivalent to that

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of a probability. Weights are corrected in a recursive way according to the available observations.

Each weight is equivalent to an a priori probability. After that, two kernels are taken into consideration whose average and variance are known and the weight will be adjusted recursively according to the provided algorithm [4].

4.1 Gaussian kernels algorithm for the a priori probability estimation

Let consider the date μ_0 , σ and μ_1 , σ of the hypotheses H_0, H_1, respectively. We consider a stop criterion ε which is the corresponding error to the difference between the estimated value and the true value. A parameter β is to consider, which allows adjusting the calculation of the adaptive gain α_k . The algorithm is given by the following stages.

❶ Set initial conditions, by randomly selecting a value for $p(H_1)$ within the range $0 < p(H_1) < 1$ which corresponds to the weight $\omega_{1,0}$, probability of having the hypothesis H_1.

Set the initial value α_0 for $k = 0$..

❷ To generate by randomly the observation y_k according of two Gaussian mixture given by

$$\left(1-\omega_{1,k}\right)p\left(y_k / H_0\right)+\omega_{1,k}.p\left(y_k / H_1\right)$$

❸ Calculate the weighted output for each of the two kernels:

$$S_{i,k} = \frac{\omega_{i,k}p_i\left(y_k / \mu_i, \sigma_i\right)}{\sum_{i=0}^{1}\omega_{i,k}p_i\left(y_k / \mu_i, \sigma_i\right)}, \quad for \quad \text{i= 0, 1}$$

❹ Update the weights :

$$\omega_{i,k+1} = \omega_{i,k} + \alpha_k\left(S_{i,k} - \omega_{i,k}\right), \quad \text{for} \quad \text{i= 0, 1}$$

❺ Adjust adaptive gain α :

$$\alpha_{k+1} = \frac{1}{k+\beta}$$

❻ $k = k+1$ go to step 2 while $\left|\left(\omega_{i,k+1} - \omega_{i,k}\right)/\omega_{i,k}\right| > \varepsilon$

4.2 Example of estimate of the a priori probability $P_1 = p(H_1)$

The results of the a priori probability estimate P1 associated with the hypothesis H_1 are depicted in Fig.2. The objective of this example is to show the influence of the parameter β on the estimation rules for these probabilities.

The estimate will then be used in the nominal decision rules making. The estimate data are:

- hypothesis H_0: $\mu_0 = 0$ and $\sigma = 0.2$
- hypothesis H_1: $\mu_1 = 3$ and $\sigma = 0.2$
- initial value of the weight : $\omega_0 = 0.5$
- the true value of the a priori probability of the hypothesis H_1 is : $p(H_1) = P_1 = 0.7$
- the stop criterion is : $\varepsilon = 10^{-3}$

We propose three configurations of estimate according to the following values of β, $\beta_1 = 1$, $\beta_2 = 50$ and $\beta_3 = 100$, this makes it possible to show the influence of the gain α in the algorithm given to paragraph 4.1, on the quality of the a priori probability estimate.

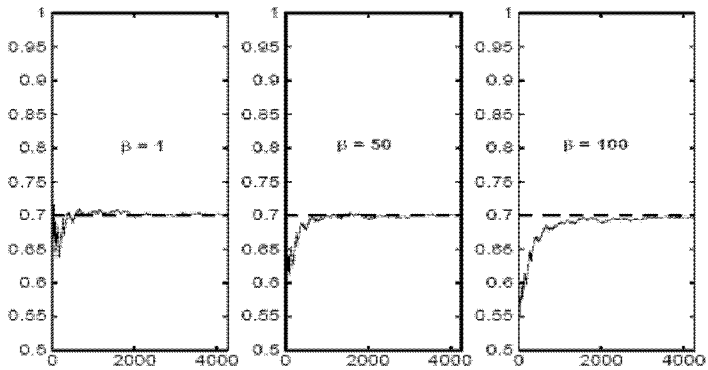

Figure 2: A priori probability estimate $P1$ for $\beta = 1,\ 50,\ 100$

The first curve in Fig.2 shows the optimal a priori probability estimate for β_1 obtained by using the algorithm of paragraph 4.1. These probabilities estimate in Fig.2 are compared to the exact value. In table 1 the comparison numerical values for the a priori probability is made and validated by the mean values and the standard deviations, which are calculated on a window of size ($N-L$). For reasons of calculation validity, we take 1000 last iterations in the permanent mode to calculate P_1 and its standard deviation (*std*). The mean values and the standard deviations of the a priori probability indicated in table 1 are evaluated with L = 3000 and N = 4000 by the following equations:

$$\overline{P}_1 = \frac{1}{(N-L+1)} \sum_{i=L}^{N} \omega_{1,i} \qquad (6)$$

$$std = \sqrt{\frac{1}{(N-L+1)} \sum_{i=L}^{N} \left(\omega_{1,i} - \overline{P}_1\right)^2} \qquad (7)$$

Table 1: The influence of β in Gaussian kernel algorithm

	$\beta_1 = 1$	$\beta_2 = 50$	$\beta_3 = 100$
$\overline{P}_1$	0.6980	0.6930	0.6845
std	0.0160	0.0174	0.0257

According to the results of simulation, a judicious choice is $\beta = 1$. Obviously the law of adaptation of gain α can be modified by the user to modify β the speed of the weights update ω.

5. COMPARATIVE STUDY OF BAYES TEST WITH FIXED THRESHOLD AND ADAPTIVE THRESHOLD

The implementation of Bayes test with a fixed threshold compared to an adaptive threshold is studied in this section. At the beginning, the implementation requires the generation of a random signal whose statistical characteristics are known. Thereafter, low amplitudes faults

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will be injected into the signal in order to evaluate the performances of Bayes test with adaptive threshold.

5.1 The Bayes test with fixed threshold

Let $(y_k)_{1\le k\le N}$ be a sequence random variable observations. The observations (y_k) are conditionally independent and identically distributed given two hypotheses, with a known conditional distribution $p(y/H_i)$, for i = 1, 2. Let consider the samples with a fixed size value of N and at the end of each sample, a decision rule is computed to choose between the two following hypotheses:

$H_0: \ y_k = \mu_0 + b_k, \quad \mu_0$ being known

$H_1: \ y_k = \mu_1 + b_k, \quad \mu_1$ being known

where b_k is the noises measurement. It is an independent Gaussian sequence with zero mean $E[b_k]=0$ and standard deviation $Var[b_k]=\sigma^2$.

For only one observation y_k, the probability density conditional respectively to the hypotheses H_0 and H_1 are supposed to be independent and Gaussian are given by:

$p(y/H_0) \sim \mathrm{N}(\mu_0, \ \sigma)$ and $p(y/H_1) \sim \mathrm{N}(\mu_1, \ \sigma)$

which we shall write as:

$$p(y/H_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu_i)^2}{2\sigma^2}\right), \quad \text{pour} \quad i=0, 1 \tag{8}$$

The construction of a binary hypotheses test is defined by a function of decision which is the likelihood ratio test of conditional probability densities noted $\lambda(y)$. It will be compared with a threshold η, which is determined by the minimization of a criterion of optimality, the likelihood ratio test can be expressed by

$$\lambda(y) = \frac{p(y/H_1)}{p(y/H_0)} = \exp\left(\frac{\mu_1-\mu_0}{\sigma^2}\left(y - \frac{\mu_1+\mu_0}{2}\right)\right) \tag{9}$$

In order to obtain an easily exploitable rule of decision from a numerical point of view. The likelihood ratio test between the hypotheses H_0 and H_1 can be expressed as

$$\ln[\lambda(y)] \underset{H_0}{\overset{H_1}{\gtrless}} \ln(\eta) \tag{10}$$

This can be rewritten as

$$y \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{11}$$

Where γ, threshold of detection for the observation y_k at a given time instant, is expressed by

$$\gamma = \frac{2\sigma^2 \ln(\eta) + (\mu_1^2 - \mu_0^2)}{2(\mu_1-\mu_0)} \tag{12}$$

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The Gaussian distributions of the two hypotheses H_0 and H_1 are depicted in Fig.1 the probability of false alarm, P_F and missed detection (or non detection) P_{ND}, can be defined by

$$P_F = \int_{\gamma}^{+\infty} p(y/H_0)\ dy \tag{13}$$

$$P_{ND} = \int_{-\infty}^{\gamma} p(y/H_1)\ dy \tag{14}$$

The risks of error P_F and P_{ND}, can be calculated by using the error function $erf(.)$:

$$P_F = \frac{1}{2}\left(1 - erf\left(\frac{\gamma - \mu_0}{\sigma_0\sqrt{2}}\right)\right) \tag{15}$$

$$P_{ND} = \frac{1}{2}\left(1 + erf\left(\frac{\gamma - \mu_1}{\sigma_1\sqrt{2}}\right)\right) \tag{16}$$

where the error function ca be expressed by

$$erf(y) = \frac{2}{\sqrt{\pi}}\int_0^y \exp\left(-t^2\right)\ dt \tag{17}$$

5.2 The Bayes test with adaptive threshold

For a false alarm probability given $P_F = 0.002$ and with the parameters, $\mu_0 = 0,\ \mu_1 = 0.5$ and $\sigma_1 = \sigma_0 = \sigma = 0.1$. The fixed threshold of detection relative to an observation y_k is given by the equations (12) et (15) such as :

$$\ln(\eta) = \frac{2\mu_1 y - \mu_1^2}{2\sigma^2} \tag{18}$$

With the preceding date, we deduced the fixed threshold value $\ln(\eta) = 3.5126$.

The choice of the average jump and the standard deviation is arbitrary to show the progression of adaptive threshold which is given by the algorithm in paragraph 4.1 by the following equation:

$$\ln(\eta_k) = \ln\left(\frac{1 - \omega_{1,k+1}}{\omega_{1,k+1}}\right) \tag{19}$$

where $\omega_{1,k+1}$ corresponds to the estimate probability $\overline{P}_1$ to have hypothesis H_1 at time $(k+1)$.

The decision rule is given by :

$$decision = \begin{cases} 0 & \text{if } \lambda(\mathrm{y}) < \eta, \quad H_0 \text{ is accepted} \\ 1 & \text{if } \lambda(\mathrm{y}) \geq \eta, \quad H_1 \text{ is accepted} \end{cases}$$

Fig.3, build a fault appearing at the moment t = 200s, that shows false alarm created during the normal functioning by using Bayes test with fixed threshold. That is unsuitable to detect the minimum value of jump magnitudes. Or the adaptive threshold test favored the hypothesis H_0 during the period of normal functioning, which the false alarm probability

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can be minimized. Thus, the increase of the probability of detection during the period of fault is shown. For the simulation data, $P_F = 0.002$, $\mu_0 = 0$, $\sigma = 0.1$ and we consider a lower jump magnitude fault $\mu_0 = 0.1$.

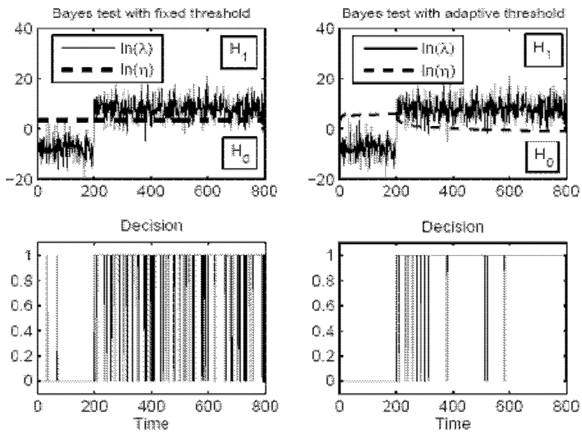

Figure 3: Bayes with fixed threshold and adaptive threshold for $\mu_1 = 0.4$

The results of simulation in Fig.4 show that the threshold of decision adapts with the signal fluctuations. Indeed, in the absence of a fault, the adaptive threshold test favors the decision making of the hypothesis H_0 in order to decrease the probability of false alarm. In the presence of a fault, the threshold adapts in order to favor the decision making of the hypothesis H_1. The results show the probability of non-detection decrease and the results reveal the most stringent test.

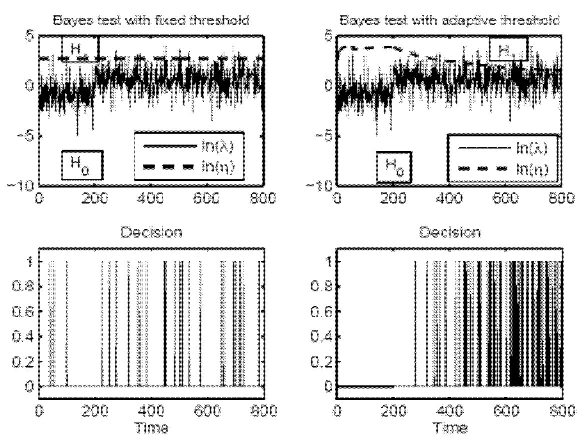

Figure 4: Bayes with fixed threshold and adaptive threshold for $\mu_1 = 0.1$

5.3 Performance evaluation of Bayes test with adaptive threshold

When the a priori probabilities estimated, using equations (5), (12), (15), (16) as design rules, the error risks can be estimated. We represent the performances evolution, where the system is in malfunctioning (hypothesis H_1).

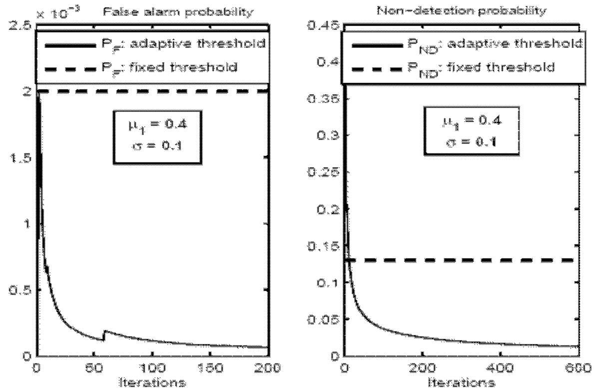

Figure 5: False alarm and non-detection probabilities progression

Let consider a different fault magnitude $\mu_1 = 0.4$ and 0.1, the progression of false alarm and non-detection probabilities are depicted in Fig.5 and Fig.6 that show for minimum values jump magnitude μ_1, the false alarm probability decrease.

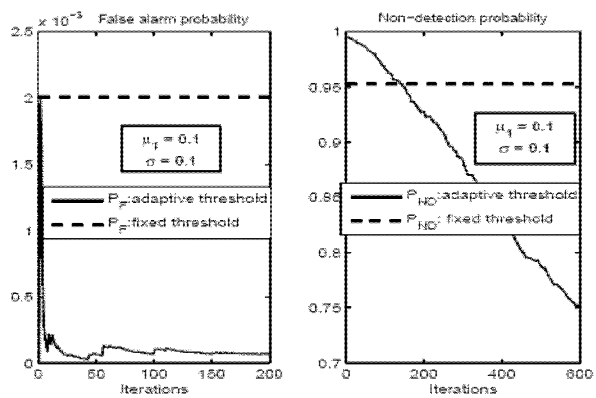

Figure 5: False alarm and non-detection probabilities progression

However, we notice also a light increase in the non-detection probability which tends to decrease according to the adaptation of the threshold for numbers of points considered.

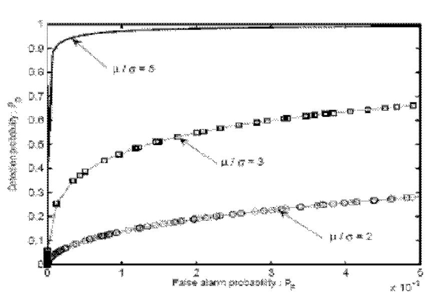

Figure 7: Detection probabilities progression $PD = f(PF)$

Fig.7 shows the resulting receiver operating characteristic (ROC) is concave downward for three ratios given of μ / σ, where $\mu = \mu_1 - \mu_0$. We notice that, if the ratio increases, the test will be the most stringent.

6 CONCLUSIONS

In this paper, we implemented the Bayesian technique with adaptive threshold for change detection in a signal can illustrate the faults appearance. That gives more accurate results by the introduction of an estimate of the a priori probabilities. This study shown that, the improvement of performances can be presented; we decrease false alarm probability and non detection probability. The technique used here is based on the estimate of the a priori probabilities by a non parametric method using Gaussian kernels.

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