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Trajectory tracking control of robot manipulators using disturbance observer based iterative learning law

F. Bouakrif, D. Boukhetala and F. Boudjema

This paper presents a disturbance observer based iterative learning control for robotic manipulators. In this control scheme, the whole control law consists of two parts, the feedback control law, and the disturbance estimated given by a disturbance observer, which has an iterative form. Using Lyapunov method, the asymptotic stability of the whole system is guaranteed, and the external disturbances are compensated. Simulation results on two-link manipulator show the asymptotic convergence of tracking error.

Keywords: Disturbance observer, iterative learning control, robot manipulator.

1. INTRODUCTION

The design of a controller for dynamical system is typically divided into two different design problems, the first design problem is a regulation problem which consists of finding a control law that manipulates the input variable so that the system automatically holds the output at a constant value, even when unknown disturbances try to move output away from this constant set point. The second one is the trajectory tracking problem which consists to force the output response to follow a desired trajectory as close as possible. On the other hand , many industrial machines executes the same task repeatedly over a finite time-interval, such as robotic manipulator, and most existing control methods are not able to fully capture and utilize the information available through the underlying nature of the system repeatability.

Iterative Learning Control "ILC" was proposed to best meet this kind of control tasks. The idea of ILC is straightforward, use the control information of the preceding trial to improve the control performance of the present trial, in order to enable the controlled system to perform progressively better from operation to operation. This is realized through memory based learning. Note, the first academic contribution to what today is called ILC appears to be a paper by Uchiyama [1]. Since it was published in Japanese only, the ideas did not become widely spread. Indeed, from an academic perspective it was not until 1984 that ILC started to become an active research area. In 1984 Arimoto et al. [2] published paper about a method that iteratively could compensate for model errors and disturbances. Afterwards, this technique has been the center of interest of many researchers over the last two decades (see for instance [3]-[6]). On the other hand, another type of ILC algorithms has been developed using Lyapunov-like methods. In fact, in [7], Xu and Qu utilized a Lyapunov-based approach to illustrate how an ILC can be combined with a variable structure controller to handle a broad class of non linear systems, in [8], Ham *et al.* utilized Lyapunov-based techniques to develop an ILC that is combined with a robust control design to achieve global uniformly ultimately bounded link position tracking for robot manipulators, the applicability of this design was extended to a broader class of nonlinear

Laboratoire de Commande des Processus, Ecole Nationale Polytechnique, 10, Avenue Pasteur BP 182, El Harrach, Alger, Algeria.

systems in [9]. Using Lyapunov-like function, Bouakrif *et al.* presented in [10] two ILC schemes for the trajectory tracking problem of rigid robot manipulators, and Tayebi derived in [11] an adaptive ILC schemes for the same problem.

The control problem for a nonlinear system under disturbances has been developed and applied in engineering over two decades. Nakao *et al.* [12] proposed firstly the concept of disturbance observer 'DO' as compensating unknown disturbance. Furthermore, friction is a common phenomenon in mechanical systems. One of the most promising methods is observer-based control, where a variable structure DO has been proposed [13], and a nonlinear observer for a special kind of friction, i.e., Coulomb friction, has been proposed by Friedland and Park [14]. It has been further modified and implemented on robotic manipulators by Tafazoli *et al.* [15]. In [16] a DO based control approach for nonlinear systems under disturbances has been proposed, but only semi global stability condition of the composite controller-observer has been established.

To cancel out the disturbance, we present in this paper a disturbance observer based learning control. In this control scheme, the whole control law consists of two parts, the feedback control law and the disturbance estimated given by a disturbance observer based learning law. The asymptotic stability condition of the proposed controller is established, this result is based on Lyapunov theory. Simulation results on two-link manipulator show the asymptotic convergence of tracking error.

2. DYNAMIC EQUATIONS OF ROBOT MANIPULATORS

We consider a robot manipulator that is composed of serially connected rigid links. The motion of the manipulator with n-links is described by the following dynamic equation

$$\tau_k = M(q_k)\,\ddot{q}_k + C(q_k,\dot{q}_k)\,\dot{q}_k + G(q_k) + d(t) \tag{1}$$

where t denotes the time and the nonnegative integer k denotes the operation or iteration number. The signals $q_k(t), \dot{q}_k(t), \ddot{q}_k(t) \in R^n$ denote the link position, velocity, and acceleration vectors, respectively. $M(q_k(t)) \in R^{n\times n}$ represents the link inertia matrix, $C(q_k(t), \dot{q}_k(t)) \in R^{n\times n}$ represents centripetal-Coriolis matrix, $G(q_k(t)) \in R^{n\times 1}$ represents the gravity effects, $\tau_k(t) \in R^{n\times 1}$ represents the torque input vector, and $d(t) \in R^{n\times 1}$ is a disturbance torque or force vector. It should be noted that $d(t)$ has different meanings in different observer applications. For example, it can be friction in friction compensation, reaction torque or force in force control, and unmodeled dynamics in independent joint control.

In the sequel, $q_d(t), \dot{q}_d(t), \ddot{q}_d(t) \in R^n$ denote the desired link position, velocity, and acceleration vectors, respectively.

The norm of a vector X is defined as

$$\|X\| = \sqrt{X^T X} \tag{2}$$

and the norm of a matrix A as

$$\|A\| = \sqrt{\lambda_{\max}(A^T A)} \tag{3}$$

with $\lambda_{\max}(.)$ denotes the maximum eigenvalue of A.

The dynamic equation of (1) has the following properties [17]-[19] that will be used in the controller development and analysis.

P 1: The inertia matrix $M(q_k(t))$ is symmetric, positive definite and bounded as

$$0_n < \beta_1 < \|M(q_k)\| < \beta_2 \tag{4}$$

where $q_k \in R^n$, and $\beta_2 > \beta_1 > 0$.

P 2: $M(q(t))$ is globally Lipschitz continuous in their arguments as follows

$$\|M(q_{k+1}) - M(q_k)\| \le l_m \|q_{k+1} - q_k\| . \tag{5}$$

where l_m a positive constant.

P 3: The inertia and centripetal-Coriolis matrices satisfy the following skew-symmetric matrix

$$X^T(\ \dot{M}(q_k) - 2\,C(q_k,\dot{q}_k)\ )\ \ X = 0 \qquad \forall X \in \Re^n . \tag{6}$$

P 4: The norm of the centripetal-Coriolis is bounded as follows

$$\|C(q_k, \dot{q}_k)\| \ \le \ C_m \|\dot{q}_k\| . \tag{7}$$

where C_m denotes known positive bounding constant.

P 5: $G(q_k)$ is globally Lipschitz continuous in their arguments and bounded as

$$\|G(q_{k+1}) - G(q_k)\| \le g_m \|q_{k+1} - q_k\| \tag{8}$$

$$\|G(q_k)\| \le l_g . \tag{9}$$

where g_m and l_g denote known positives bounding constants.

In this paper, the following lemma is used

Lemma 1 [18] The inertia matrix $M(q_k)$ has the following property

$$\|M^{\ 1}(q_{k+1}) - M^{\ 1}(q_k)\| \quad \le \qquad l_m \beta_1^{\ 2} \|q_{k+1} - q_k\| . \tag{10}$$

Proof

$$M^{\ 1}(q_{k+1}) - M^{\ 1}(q_k) = -M^{\ 1}(q_{k+1})(M(q_{k+1}) - M(q_k))M^{\ 1}(q_k) \tag{11}$$

from properties 1 and 2, we have

$$\|M^{-1}(q_{k+1}) - M^{-1}(q_k)\| \quad \le \qquad l_m \beta_1^{-2} \|q_{k+1} - q_k\| . \tag{12}$$

The following assumptions are imposed.

A1: The robot velocity is bounded by a known constant V_m such that

$$\|\dot{q}(t)\| \le V_m , \forall t \in [0, T] . \tag{13}$$

A2: The disturbance $d(t)$ is repetitive and bounded as follows

$$\|d(t)\| \le l_d \,, \forall t \in [0,T]. \tag{14}$$

A3: The reference trajectory and its first and second time-derivative are bounded, and the resetting condition is satisfied, i.e.

$$\dot{q}_d(0) - \dot{q}_k(0) = q_d(0) - q_k(0) = 0 \,, \forall k \in N \tag{15}$$

A4: The admissible range for the control input u_k is given by $\|u_k\| \le l_u$, where l_u is known value obtained from the system's physical limitations.

Assumption (A4), which is not very restrictive from a practical point of view, is introduced for a technical reason guaranteeing the system stability.

Our objective is to design a control law $\tau\,(t)$ guaranteeing the convergence of $q_k(t)$ to the desired reference trajectory $q_d(t)$, and $\dot{q}_k(t)$ to the desired reference velocity $\dot{q}_d(t)$, for all $\forall t \in [0,T]$ when k tends to infinity.

In the following, we will introduce a tracking control algorithm which is referred to as learning law based disturbance observer, and the asymptotic stability is guaranteed.

3. DISTURBANCE OBSERVER BASED ITERATIVE LEARNING CONTROL

We propose the following control

$$\tau\,(t) = M(q)[\ddot{q}_d(t) + K_v\dot{e} + K_p e] + C(q,\dot{q})\dot{q} + \hat{d}(t) \tag{16}$$

where $e(t) = q_d(t) - q(t)$ is the tracking error vector, $\dot{e}(t) = \dot{q}_d(t) - \dot{q}(t)$ is the velocity error vector, $K_v = k_v I_{n\times n}$, $K_p = k_p I_{n\times n}$, $k_p = \sigma\, k_v$ with k_p, k_v and σ are positive scalar constants, and $I_{n\times n} \in R^{n\times n}$ is an identity matrix.

For symmetric matrix $H \in R^{n\times n}$, $\lambda_{\max}(H)$ and $\lambda_{\min}(H)$ are maximum and minimum eigenvalues of H respectively.

$\hat{d}(t)$ Represents the estimated disturbance, it is given by the learning law as follows

$$\hat{d}_{k+1}(t) = \hat{d}_k(t) + \eta\ \ z_k(t) \tag{17}$$

where $z_k(t) = \dot{e}_k(t) + \sigma\, e_k(t)$, $\eta = \mu\, K_v$.

Then the following theorem is given.

Theorem

Given the robot dynamics (1) with the tracking controller (16), where $\hat{d}(t)$ is given by the iterative learning law (17), and let assumptions (A1-A3) be satisfied. If

1. $\sigma^2 \ge (2\psi + 4C_m V_m K_{vM})\beta_1{}^{1}$
2. $\sigma \ge 6C_m V_m \beta_1{}^{1}$
3. $\sigma\left(\lambda_{\min}(M(K_v - \sigma I)) - 2C_m V_m\right) \ge \sqrt{2}\ \psi$
4. $\lambda_{\min}(M(K_v - \sigma I)) \ge 2\sqrt{2}\ C_m V_m + \lambda_{\max}(\eta)$.

Then

$$\lim_{k\to\infty} e_k(t) = \lim_{k\to\infty} \dot{e}_k(t) = 0 . \tag{18}$$

Where $\psi = g_m + \beta_2 \beta_1^{\,2} l_m (l_u + l_d + l_g), K_{vM} = \|K_v\|$.

Proof

Substituting (16) to (1), we obtain at k^{th} iteration

$$\ddot{e}_k + K_v \dot{e}_k + K_p e_k = M_k^{\,1}\left[d(t) + G_k(q) - \hat{d}_k(t)\right]. \tag{19}$$

From (17) and (19), we obtain at $(k+1)^{th}$ iteration

$$\ddot{e}_{k+1} + K_v \dot{e}_{k+1} + K_p e_{k+1} = M_{k+1}^{\,1}\left[d(t) + G_{k+1}(q) - \hat{d}_k(t) - \eta z_k(t)\right]. \tag{20}$$

Subtracting (19) from (20), we have

$$\begin{aligned} \dot{\tilde{z}}_k + (K_v - \sigma I)\tilde{z}_k + \sigma^2 \tilde{e}_k &= \left(M_{k+1}^{\,1} - M_k^{\,1}\right)(d - \hat{d}_k) + \left(M_{k+1}^{\,1} - M_k^{\,1}\right)G_k \\ &\quad + M_{k+1}^{\,1}\left(G_{k+1} - G_k\right) - M_{k+1}^{\,1}\eta z \end{aligned} \tag{21}$$

where $\tilde{e}_k = e_{k+1} - e_k$ and $\tilde{z}_k = z_{k+1} - z_k$.

Consider the Lyapunov function candidate

$$V_k(t) = \int_0^t \left(z_k(t)^T \eta z_k(t)\right) d\tau \quad \text{for } t \in [0, T] \tag{22}$$

hence

$$V_{k+1} = V_k + \int_0^t \left[\tilde{z}_k^T \eta \tilde{z}_k + 2\, \tilde{z}_k^T \eta z_k\right] d\tau . \tag{23}$$

Let's define

$$\Delta V_k = V_{k+1} - V_k . \tag{24}$$

Taking summation of $\Delta V_k(t)$, we have

$$V_k(t) = V_1(t) + \Delta V_k(t) \tag{25}$$

Stability analysis: The stability analysis consists of two parts, 1/ the first part, the proof that V_1 is bounded is given, and 2/ in the second part, we demonstrate that $\Delta V_k(t) \le 0$.

Part 1**:** The time derivative of (22) for the first iteration, is given by

$$\dot{V}_1 = \dot{e}_1^T \alpha \dot{e}_1 + e_1^T \sigma \alpha \sigma\, e_1 + 2 e_1^T \sigma \alpha \dot{e}_1 . \tag{26}$$

According to assumption (A3), we have

$$\dot{V}_1 = e_1^T \sigma \alpha \sigma\, e_1 + 2\int_0^t (e_1^T \sigma \alpha\, \ddot{e}_1 + \dot{e}_1^T \sigma \alpha\, \dot{e}_1) d\tau + 2\int_0^t \dot{e}_1^T \alpha\, \ddot{e}_1\, d\tau . \tag{27}$$

From (19) and (27), we obtain

$$\dot{V}_1 = e_1^T \sigma\alpha\sigma\, e_1 - 2\int_0^t (e_1^T \sigma\alpha K_v \dot{e}_1 + e_1^T \sigma\alpha K_p e_1 - e_1^T \sigma\alpha M_1^{-1}(d + G_1 - \hat{d}_1))d\tau$$
$$+ \int_0^t (\dot{e}_1^T \sigma\alpha\, \dot{e}_1)\, d\tau - 2\int_0^t (\dot{e}_1^T \alpha K_v \dot{e}_1 + \dot{e}_1^T \alpha K_p e_1 - \dot{e}_1^T \alpha M_1^{-1}(d + G_1 - \hat{d}_1))d\tau \tag{28}$$

According to assumption (A3), we have

$$\dot{V}_1 = -e_1^T(\sigma\alpha K_v - \sigma\alpha\sigma)e_1 - 2\int \dot{e}_1^T(\alpha K_v - \sigma\alpha)\dot{e}_1 d\tau - 2\int_0^t e_1^T \sigma\alpha K_p e_1 d\tau$$
$$-2e_1^T \alpha K_p e_1 + 2\int_0^t (e_1^T \sigma\alpha M_1^{-1}(d + G_1 - \hat{d}_1))d\tau + 2\int_0^t (\dot{e}_1^T \alpha M_1^{-1}(d + G_1 - \hat{d}_1))d\tau. \tag{29}$$

Choosing $(K_v - \sigma)$ and αK_p positive definite matrices, we can write

$$\dot{V}_1 \le -2\int_0^t e_1^T \sigma\alpha K_p e_1 d\tau + 2\int_0^t (e_1^T \sigma\alpha M_1^{-1}(d + G_1 - \hat{d}_1))d\tau$$
$$+2\int_0^t (\dot{e}_1^T \alpha M_1^{-1}(d + G_1 - \hat{d}_1))d\tau. \tag{30}$$

Taking the norms, and according to (P1, P5, A2 and A4), we obtain

$$\dot{V}_1 \le -2\int_0^t (\lambda_{\min}(\sigma\alpha K_p)\|e_1\|^2)d\tau + 2\int_0^t \gamma\|e_1\|d\tau + 2\int_0^t \delta\|\dot{e}_1\|d\tau \tag{31}$$

Where $\delta = \|\alpha\|\beta_1^{-1}(l_g + l_d + l_u), \gamma = \|\sigma\|\delta$.

Using Young's inequality, for any $\phi > 0$, we have

$$\gamma\|e_1\| \le \phi\|e_1\|^2 + \frac{1}{4\phi}\gamma^2. \tag{32}$$

Hence

$$\dot{V}_1 \le -2\int_0^t (\lambda_{\min}(\sigma\alpha K_p) - \phi)\|e_1\|^2 d\tau + \frac{1}{4\phi}\gamma^2 + 2\int_0^t \delta\|\dot{e}_1\|d\tau \tag{33}$$

Let $0 < \phi < \lambda_{\min}(\sigma\alpha K_p)$, thus we can write

$$\dot{V}_1 \le \frac{1}{4\phi}\gamma^2 + 2\int_0^t \delta\|\dot{e}_1\|d\tau. \tag{34}$$

According to assumption (A1), $\|\dot{e}_1\|$ is bounded, therefore, $\dot{V}_1$ is bounded, this implies that V_1 is uniformly continuous in time, thus it is bounded.

Part 2: From (21), (22), (23) and (24), we can write

$$\Delta V_k = \int_0^t \left[\tilde{z}_k^T \eta \tilde{z}_k - 2\tilde{z}_k^T M_{k+1}\dot{\tilde{z}}_k - 2\tilde{z}_k^T M_{k+1}(K_v - \sigma)\tilde{z} \right.$$
$$-2\tilde{z}_k^T M_{k+1}\sigma^2 \tilde{e}_k + 2\tilde{z}_k^T (G_{k+1} - G_k) + 2\tilde{z}_k^T M_{k+1}\left(M_{k+1}^{1} - M_k^{1}\right)(d - \hat{d}_k) \tag{35}$$
$$\left. +2\tilde{z}_k^T M_{k+1}\left(M_{k+1}^{1} - M_k^{1}\right)G_k \right] d\tau$$

Adding and subtracting $\tilde{z}_k^T \dot{M}_{k+1}\tilde{z}_k$, $2\tilde{z}_k^T C_{k+1}\tilde{z}_k$, $\tilde{e}_k^T \dot{M}_{k+1}\sigma^2 \tilde{e}_k$, $2\tilde{e}_k^T C_{k+1}\sigma^2 \tilde{e}_k$, $2\sigma\tilde{e}_k^T \dot{M}_{k+1}(K_v - \sigma I)\tilde{e}_k$, $4\sigma\tilde{e}_k^T C_{k+1}(K_v - \sigma I)\tilde{e}_k$ and according to property (P.3), and assumption (A3), we have

$$\Delta V_k = -\tilde{z}_k^T M_{k+1}\tilde{z}_k - \tilde{e}_k^T M_{k+1}\sigma^2 \tilde{e}_k + \sigma\tilde{e}_k^T \eta\, \tilde{e}_k - 2\tilde{e}_k^T M_{k+1}(K_v - \sigma I)\tilde{e}_k$$
$$+ \int_0^t \left[\dot{\tilde{e}}_k^T \eta\, \dot{\tilde{e}}_k + \sigma^2 \tilde{e}_k^T \eta\, \tilde{e}_k + 2\dot{\tilde{e}}_k^T C_{k+1}\dot{\tilde{e}}_k + 2\sigma^2 \tilde{e}_k^T C_{k+1}\tilde{e}_k + 4\sigma\dot{\tilde{e}}_k^T C_{k+1}\tilde{e}_k \right.$$
$$- 2\dot{\tilde{e}}_k^T M_{k+1}(K_v - \sigma I)\dot{\tilde{e}}_k - 2\sigma^2 \tilde{e}_k^T M_{k+1}(K_v - \sigma I)\tilde{e}_k + 4\sigma\dot{\tilde{e}}_k^T C_{k+1}(K_v - \sigma I)\tilde{e}_k \tag{36}$$
$$+ 2\tilde{e}_k^T C_{k+1}\sigma^2 \tilde{e}_k - 2\tilde{e}_k^T \sigma M_{k+1}\sigma^2 \tilde{e}_k + 2\tilde{z}_k^T (G_{k+1} - G_k)$$
$$\left. + 2\tilde{z}_k^T M_{k+1}\left(M_{k+1}^{1} - M_k^{1}\right)(d - \hat{d}_k) + 2\tilde{z}_k^T M_{k+1}\left(M_{k+1}^{1} - M_k^{1}\right)G_k \right] d\tau.$$

According to properties (1, 4, 5), and using assumptions (A1-A4) and lemma 1, we obtain

$$\Delta V_k \le -\lambda_{\min}(M)\, \|\tilde{z}\|^2 - \left(\sigma^2 \lambda_{\min}(M) - 2\psi\right) \|\tilde{e}\|^2$$
$$-2\lambda_{\min}(M(K_v - \sigma I))\|\tilde{e}\|^2 + \int_0^t W(\tau) d\tau. \tag{37}$$

where

$$W(t) = -\sigma\left(\sigma^2\beta_1 - 4C_m V_m K_{vM} - 2\psi\right)\|\dot{\tilde{e}}\|^2 - \sigma^2\left(\sigma\beta_1 - 6C_m V_m\right)\|\tilde{e}\|^2 - \sigma^2\alpha\,\|\tilde{e}\|^2$$
$$- \alpha\left\|\dot{\tilde{e}}\right\|^2 + 2\psi\left\|\dot{\tilde{e}}\right\|\|\tilde{e}\| - \delta\left\|\dot{\tilde{e}}\right\|^2 - \sigma^2\delta\,\|\tilde{e}\|^2 + 4\sigma C_m V_m \left\|\dot{\tilde{e}}\right\|\|\tilde{e}\|. \tag{38}$$

with $\alpha = \lambda_{\min}(M(K_v - \sigma I)) - 2C_m V_m, K_{vM} = \|K_v\|$, $\delta = \lambda_{\min}(M(K_v - \sigma I)) - \lambda_{\max}(\eta)$, $\psi = g_m + \beta_2\beta_1{}^2 l_m(l_u + l_d + l_g), \lambda_{\min}(M) = \beta_1$

We note that

$$2\psi\|\tilde{e}_k\|\left\|\dot{\tilde{e}}_k\right\| \le 2\|\tilde{e}_k\|\psi\left[\frac{2}{\alpha}\right]^{1/2}\left[\frac{\alpha}{2}\right]^{1/2}\left\|\dot{\tilde{e}}_k\right\|$$
$$\le \|\tilde{e}_k\|^2 \psi^2 \frac{2}{\alpha} + \left\|\dot{\tilde{e}}_k\right\|^2 \frac{\alpha}{2}. \tag{39}$$

And

$$2(2\sigma C_m V_m)\|\tilde{e}_k\|\left\|\dot{\tilde{e}}_k\right\| \le (2\sigma C_m V_m)^2 \frac{2}{\delta}\|\tilde{e}_k\|^2 + \frac{\delta}{2}\left\|\dot{\tilde{e}}_k\right\|^2. \tag{40}$$

Therefore, we obtain

$$W(t) \le -\sigma\left(\sigma^2\beta_1 - 4C_m V_m K_{vM} - 2\psi\right)\|\tilde{e}\|^2 - \sigma^2\left(\sigma\beta_1 - 6C_m V_m\right)\|\tilde{e}\|^2 - \frac{\alpha}{2}\left\|\dot{\tilde{e}}\right\|^2$$
$$-(\sigma^2\alpha - 2\frac{\psi^2}{\alpha})\|\tilde{e}\|^2 - \frac{\delta}{2}\left\|\dot{\tilde{e}}\right\|^2 - (\sigma^2\delta - 2\frac{(2\sigma C_m V_m)^2}{\delta}\|\tilde{e}\|^2 \quad (41)$$

From (37) and (41), if we choose

1. $\sigma^2 \ge (2\psi + 4C_m V_m K_{vM})\beta_1^{\ 1}$,
2. $\sigma \ge 6C_m V_m \beta_1^{\ 1}$,
3. $\sigma\left(\lambda_{\min}(M(K_v - \sigma I)) - 2C_m V_m\right) \ge \sqrt{2}\ \psi$,
4. $\lambda_{\min}(M(K_v - \sigma I)) \ge 2\sqrt{2}\ C_m V_m + \lambda_{\max}(\eta)$.

Then

$\Delta V_k \le 0 \quad t \in [0,T]$. (42)

Therefore $V_k(t)$ is bounded $\forall k \in N$ and $t \in [0,T]$, hence

$$\lim_{k\to\infty} V_k(t) = V_1(t) + \lim_{k\to\infty} \Delta V_k(t). \quad (43)$$

Since $V_1(t)$ is finite and $V_k(t)$ is positive, then, one can conclude that

$$\lim_{k\to\infty} e_k(t) = \lim_{k\to\infty} \dot{e}_k(t) = 0,\ t \in [0,T]. \quad (44)$$

This completes the proof.

4. NUMERICAL SIMULATION RESULTS

Consider a two-link manipulator with masses m_1, m_2, lengths l_1, l_2, and angles q_1, q_2; then the model equations can be written as (1) with

The elements of $M(q)$ are given by

$$m_{11} = m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) + (m_1 + m_2)l_1^2,$$
$$m_{12} = m_{21} = m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2), m_{22} = m_2 l_2^2.$$

The elements of $C(q,\dot{q})$ are given by

$$C_{11} = -m_2 l_1 l_2 \sin(q_2)\dot{q}_2, C_{12} = -m_2 l_1 l_2 \sin(q_2)\dot{q}_2,\ C_{21} = m_2 l_1 l_2 \sin(q_2)\dot{q}_1, C_{22} = 0.$$

The elements of $G(q)$ are given by

$$G_1 = m_2 l_2 g\cos(q_1 + q_2) + (m_1 + m_2)l_1 g\cos(q_1),\ G_2 = m_2 l_2 g\cos(q_1 + q_2).$$

The external disturbance considered is given by

$$d_1(t) = 100\sin(\pi\, t) + 50\sin(2\pi\, t),\ d_2(t) = 100\sin(\pi\, t) + 50\sin(2\pi\, t)$$

Simulation parameters:

$m_1 = 0.5[kg]$, $m_2 = 0.5[kg]$, $l_1 = 1[m]$, $l_2 = 1.4[m]$,
$K_D = diag\{150,150\}, K_p = diag\{18000,18000\}$, η=diag{3,3}, σ =120.

C_m is given by [20], $C_m \geq n^2 \left(\sup_{i,j,k,q} \left| c_{ijk}(q) \right| \right)$

where $c_{ijk}(q)$ is the ijk Christoffel symbol [21].

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial M_{kj}(q)}{\partial q_i} + \frac{\partial M_{ki}(q)}{\partial q_j} - \frac{\partial M_{ij}(q)}{\partial q_k} \right].$$

g_m is given by [20]

$$g_m \geq n \left(\sup_{i,j,q} \left| \frac{\partial g_i(q)}{\partial q_j} \right| \right).$$

Therefore, we find that $g_m = 89.5[kg.m^2/\sec^2]$ $C_m = 1.4[kgm^2]$, $l_g = 17[m/\sec^2]$, $V_m = 10[rad/s]$, $l_u = 100N.m$, $\beta_1 = 1[kgm^2]$, β_2 is given by Geršgorin theorem [21], then we have $\beta_1 = m_1 l_2^2 + m_2\left(l_1^2 + 2l_2^2 + 3l_1 l_2\right)$, hence $\beta_2 = 5.6[kgm^2]$.

The desired trajectories are

$$q_{d1}(t) = -\frac{\pi}{2} + \frac{1}{4}\left(2\pi\frac{t}{4} - \sin\left(2\pi\frac{t}{4}\right)\right) \ rad, \ 0 \leq t \leq 4$$

$$q_{d2}(t) = \frac{1}{4}\left(2\pi\frac{t}{4} - \sin\left(2\pi\frac{t}{4}\right)\right) \ rad, \ 0 \leq t \leq 4.$$

The simulation results for real and desired position trajectories, and position error for 1st and 20th iteration of each joint are shown in fig.(1-4) and fig. (5-8), respectively. We can see that the real trajectory follows the desired trajectory through learning iterations. Therefore, the robot executes 20 iterations so that the real trajectory follows the desired trajectory without error.

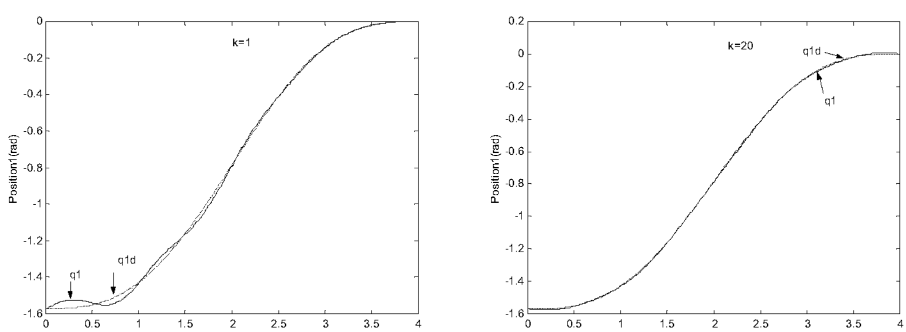

Fig.1 Real and desired position for first joint (k=1) **Fig**.2 Real and desired position for first joint (k=20)

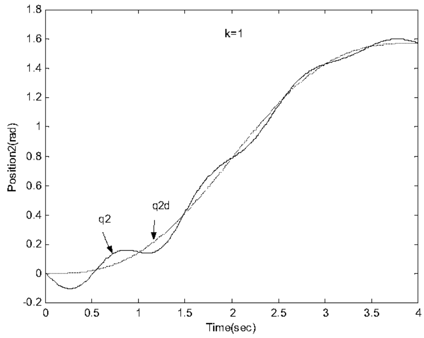

Fig.3 Real and desired position for second joint (k=1)

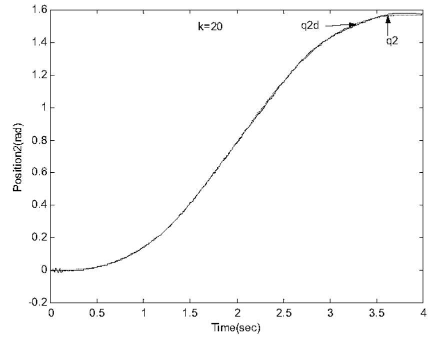

Fig.4 Real and desired position for second joint (k=20)

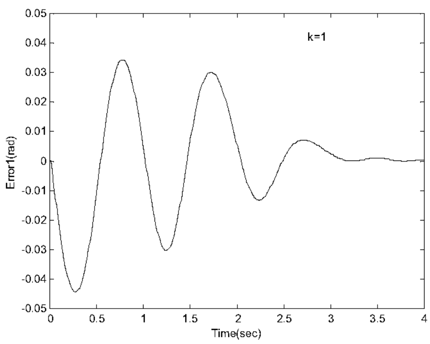

Fig.5 Position error for first joint (k=1)

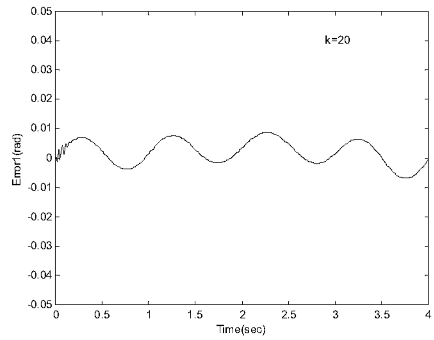

Fig.6 Position error for first joint (k=20)

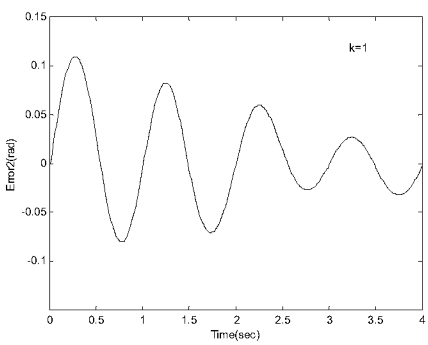

Fig.7 Position error for second joint (k=1)

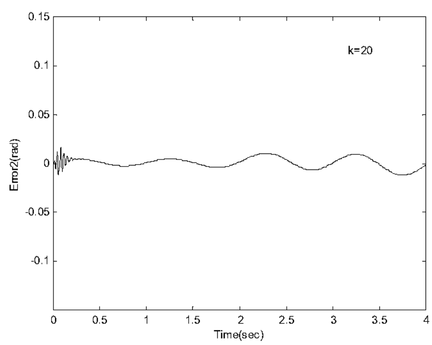

Fig.8 Position error for second joint (k=20)

5. CONCLUSION

This paper has presented a disturbance observer based iterative learning control scheme for the position tracking problem of rigid robot manipulators with subject to external disturbances. The proposed controller is based upon a feedback controller plus an iterative term represents the disturbance estimated for cope with the term of gravitational and disturbances. The proof of convergence is based upon the use of a Lyapunov-like positive definite sequence, which is shown to be monotonically decreasing under the proposed control scheme. Simulation results on two-link manipulator show the asymptotic convergence of tracking error.

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