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Observer-Based Robust Switched Dynamic Output Control

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This paper discusses the stabilization of discrete time, polytopic uncertain switched systems, focusing on the design of a robust dynamic output feedback control, based on a Luenberger observer's. The results are derived using the direct Lyapunov approach and the polyquadratic function concept. The stabilization conditions are written through linear matrix inequalities relations. The polyquadratic Lyapunov approach provides a constructive way to tackle uncertainty in the switched framework. The feasibility is illustrated on an example of discrete time uncertain switched systems, when one of used actuators broke down.

Keywords: Polyquadratic stability, switched discrete time system, polytopic uncertainty, Linear Matrix Inequality, switched observer, output feedback control.

1. INTRODUCTION

The recent publications shows a growing interest on switched systems since switched control systems exist widely in engineering technology and social systems [8]. Switched systems are an important class of hybrid systems defined by a set whose elements are dynamic continuous (or discrete) time models and a commutation law which governs, in time, the jumps between the elements, defining a non stationary dynamic system.

During the last couple of decades, the switched systems were the subject of several publications in theory of system control. In fact, several industrial applications impose a changing in the regime of functioning generally leading to a modification of system dynamics. Take for example the regime of changing speed in vehicle [29], the control of some robots and the flexible workshops [14, 25, 26, 27]. The functioning of human heart can be also modeled by hybrid system [24].

Many important progress and remarkable achievements have been made on issues such as controllability, reachability and stabilizability [4, 9, 10], control and switching law design [11, 12; 13, 14], optimal control [15, 16]. We can quote the work of Petterson and Lennarston [28] who assume a perfect knowledge of linear system and the command laws. They present a method for a synthesis, if it is possible, for a stabilized switching law. In 1994, wicks [23] assumed a switched law for the stabilization of a linear system by using two switched commands. In 1997, Skafidas [7] presents a necessary and sufficient condition for the robust stability of systems commanded by synchronous switched controllers. In 1998, Branicky [19] presents the use of several Lyapunov functions as tool for the study of switched command systems. In 1998, Savkin lays down sufficient condition for the robust stabilization by output feedback with switched synchronous controllers.

Among others, stability analysis and stabilization control are two hot topics. The basic

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problems considered include stability analysis for systems with specific switching laws [19], stability analysis for systems with arbitrary switching laws [13] and design of stabilization switching laws [14]. Many contributions proposed to analyze stability of arbitrary switched systems use some conservative arguments to conclude. The most pessimistic arguments assume the existence of a common Lyapunov function [17, 18]. Even if these conditions are easily tractable, they can be used in a very few applications. More recently, less conservative conditions, using multiple Lyapunov function, have been proposed [5, 19, 20]. In [2] where a sufficient (but relatively non restrictive compared to the quadratic approach) stability condition for discrete switched systems is provided using the poly-quadratic approach proposed by J. Daafouz and J. Bernussou 2001[1] for stability analysis and stabilization control of Linear Time Varying systems. An extension of this works in the case when the switches are made among uncertain LTI systems [3] had been proposed in [21] for the stability analysis and the state feedback control.

In this paper we propose, for this kind of systems, a method to design a dynamic output feedback control based on a Luenberger observer which, of course, is a more realistic framework than the state feedback control and less restrictive one than the static output feedback control.

The rest of paper is organized as follows. The second section introduces problem formulation for uncertain polytopic switched system. The third and fourth sections put emphasis on theorems allowing stabilization of such kind of systems. Using observers, the fifth section deals with a synthesis of dynamic output controller based on LMIs formulation. Finally an illustration example is given.

2. PROBLEM FORMULATION

We consider an uncertain discrete time switched system where the so-called subsystems are uncertain with a polytopic uncertainty, as roughly illustrated in (fig. 1). Where the uncertainty domains are polytopic shaped with different number of vertices to cope with maximal generality. The model can be stated as follows:

$$\begin{aligned} x(k+1) &= \sum_{l=1}^{M} \xi_l(k)\left(\sum_{i=1}^{Nl} \alpha_{i(l)} A_{i(l)} x(k) + B_l u(k)\right) \\ y(k) &= \sum_{l=1}^{M} \xi_l(k) C_l x(k) \end{aligned} \tag{1}$$

where *l* is the switching index, and *i* is the «uncertainty» one.

Let M be the number of uncertain system domains. D_l is the uncertainty domain for subsystem *l* defined by:

$$D_l = \left\{ A_\alpha : A_\alpha = \sum_{i(l)=1}^{Nl} \alpha_{i(l)} A_{i(l)}, \alpha_{i(l)} \geq 0, \sum_{i(l)=1}^{Nl} \alpha_{i(l)}(k) = 1 \right\} \tag{2}$$

Nl is the number of the vertices for uncertain domain D_l.

The $\alpha_{i(l)}$ are invariant in time

$$\xi_l(k) = \begin{cases} 1 \text{ when the state matrix is defined in the domain } D_l. \\ 0 \text{ otherwise} \end{cases} \tag{3}$$

then:

$$\xi_l(k) \geq 0, \sum_{l=1}^{M} \xi_l(k) = 1 \tag{4}$$

Let

$$\xi(k) = [\xi_1(k), \ldots\ldots, \xi_N(k)]^T \tag{5}$$

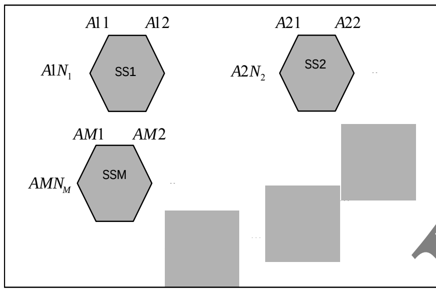

Fig. 1: Autonomous uncertain switched system

3. UNCERTAIN SWITCHED SYSTEMS STABILITY ANALYSIS

Starting from the results of J. Daafouz G. Millerioux and C. Iung, 2002 [2], the analysis of the polyquadratic stability of the uncertain switched systems is developed in [21] to propose the following condition.

First we consider an autonomous system with same description in (1).

Theorem 1: The system described by (1), in autonomous case is polyquadratically stable if and only if there exist H symmetric positive definite matrices $S_{11},\ldots,S_{MN}$ and M matrices $G_1,\ldots,G_M$ of appropriate dimension solutions of the LMIs:

$$\begin{bmatrix} G_l + G_l^T - S_{i(l),l} & G_l^T A_{i(l),l}^T \\ A_{i(l),l} G_l & S_{i(j),j} \end{bmatrix} > 0 \tag{6}$$
$$\forall (l, i(l), i(j), j) \in (e \times e_l \times e_j \times J_l)$$

With

$$H = \sum_{l=1}^{M} N_l \tag{7}$$

$e = \{1 \ldots M\}$, $e_l = \{1 \ldots N_l\}$, $e_j = \{1 \ldots N_j\}$, $J_l \subseteq e$ is an under set characterizing the admissible switch from l.

4. OBSERVER FORMULATION FOR UNCERTAIN SWITCHED SYSTEM

This section is directed towards the design of a dynamic output control based on a switched observer for system (1). Let first write the state space representation of the observer assuming that the subsystems matrices are not uncertain (i.e at every time the dynamic system matrix is known), then:

$$
\begin{aligned}
\hat{x}(k+1) &= \sum_{l=1}^{M} \xi_l(k)\Big(\sum_{i=1}^{Nl} \alpha_{i(l)}(A_{i(l)}\hat{x}(k)) + B_l u(k) + L_l(y(k)-\hat{y}(k))\Big) \\
\hat{y}(k) &= \sum_{l=1}^{M} \xi_l(k) C_l \hat{x}(k)
\end{aligned} \tag{8}
$$

The observer (8) is obviously not feasible [29, 30] since for uncertain subsystems the dynamic *A* matrix is not known. As an attempt to overcome this difficulty one may think in choosing for the observer a dynamic $\hat{A}$ matrix which is inside the uncertainty domains at each time. Then the observer state space representation becomes:

$$
\begin{aligned}
\hat{x}(k+1) &= \sum_{l=1}^{M} \xi_l(k)(\hat{A}_l \hat{x}(k) + B_l u(k) + L_l(y(k)-\hat{y}(k))) \\
\hat{y}(k) &= \sum_{l=1}^{M} \xi_l(k) C_l \hat{x}(k)
\end{aligned} \tag{9}
$$

The observation gains *L* must be computed to achieve stability of the estimation error; ε is the error between the switched system state (1) and the observer state (9).

$$
\varepsilon(k) = x(k) - \hat{x}(k) \tag{10}
$$

Using (9), the dynamic estimation error is:

$$
\varepsilon(k+1) = \sum_{l=1}^{M} \xi_l(k)\Big(\sum_{i=1}^{Nl} \alpha_{i(l)}(A_{i(l)} - \hat{A}_{i(l)})x(k) + (\hat{A}_{i(l)} - L_l C_l \varepsilon(k))\Big) \tag{11}
$$

5. DYNAMIC OUTPUT CONTROL DESIGN

To close the loop and realize a dynamic output control a natural way is then to compute with the polyquadratic approach a state feedback and apply this feedback gain with the observer state, mimicking what is done in the separation principle. Of course the closed loop stability is not implied by such an approach and has to be checked using, for instance, the polyquadratic analysis approach.

To support such an approach it is easy to prove that such a control determined for non uncertain switched systems (nominal systems) would provide a stabilizing gain for sufficiently small uncertainties around the nominal and it is also possible to give bounds for these uncertainties.

To apply this approach let us first compute a state feedback control gains

5.1 State feedback control

Consider the uncertain switched system described by (1) , where all states are measurable $(C = I)$. The stabilization problem of the switched system through state feedback consists in determining a control law of the form:

$$
u(k) = K_l x(k) \tag{12}
$$

$$
x(k+1) = \tilde{A}_{l\alpha} x(k) \quad with \quad \tilde{A}_{l\alpha} = A_{l\alpha} + B_l K_l \tag{13}
$$

Introducing the dynamic matrix $\tilde{A}_{l\alpha}$ in the condition (6) and after a linearizing change of variables ($R_l = K_l G_l$) the theorem 1 gives rise to the following result in terms of LMI:

Theorem 2: The system (1) can be stabilized by a state feedback if there exists H symmetric positive definite matrices $S_{11},\ldots,S_{MN}$, M matrices $G_1,\ldots,G_M$ and M matrices $R_1,\ldots,R_M$ of appropriate dimension solutions of the LMIs:

$$\begin{bmatrix} G_l + G_l^T - S_{i(l),l} & (A_{i(l),l}G_l + B_l R_l)^T \\ A_{i(l),l}G_l + B_l R_l & S_{i(j),j} \end{bmatrix} > 0 \tag{14}$$

$$\forall (l, i(l), i(j), j) \in (e \times e_l \times e_j \times J_l)$$

With $H = \sum_{l=1}^{M} N_l$

$e = \{1 \ldots M\}$, $e_l = \{1 \ldots N_l\}$, $e_j = \{1 \ldots N_j\}$, $J_l \subseteq e$ is an under set characterizing the admissible switch from l.

The state feedback gains are then defined by

$$K_l = R_l G_l^{-1} \tag{15}$$

The following parts present the main results of this paper.

5.2. Augmented system formulation

Now we consider the augmented system representing the dynamic of the state and the estimation error:

$$\begin{pmatrix} x(k+1) \\ \varepsilon(k+1) \end{pmatrix} = \sum_{1}^{M} \xi_l(k) \sum_{1}^{Nl} \alpha_{i(l)} \Phi_{i(l),l} \begin{pmatrix} x(k+1) \\ \varepsilon(k+1) \end{pmatrix} \tag{16}$$

with

$$\Phi_{i(l),l} = \begin{pmatrix} A_{i(l),l} + B_l K_l & -B_l K_l \\ \Delta_{i(l),l} & \hat{A}_l - L_l C_l \end{pmatrix} \tag{17}$$

and

$$\Delta_{i(l),l} = A_{i(l),l} - \hat{A}_l \tag{18}$$

5.3 Observer computing

Given the K_l, the observation gains L_l are to be computed to achieve stability of the augmented system (16). Mimicking the procedure from the separation principle, one can compute the observation gains L_l solving the dual problem of theorem 2 as done in [22]. In this case the closed loop stability has to be checked after hand.

The following prevents from this last a posteriori analysis step by proposing a sufficient condition from which the observation gains can be computed while checking in the same time the closed loop stability.

From theorem 1 with $G_l = \begin{pmatrix} G_{l,1} & G_{l,2} \\ G_{l,3} & G_{l,4} \end{pmatrix}$ one gets:

Theorem 3: The system described by (9), is polyquadratically stable if there exist H symmetric positive definite matrices $S_{11},\ldots,S_{MN}$ and M matrices $G_1,\ldots,G_M$ of appropriate dimension solutions of:

$$\begin{bmatrix} G_l + G_l^T - S_{i(l),l} & \begin{matrix} G_{l,1}^T(A_{i(l),l} + B_lK_l) + G_{l,3}^T A_{i(l),l} - R_l & -G_{l,1}^T B_l K_l + R_l - U_l C_l \\ G_{l,2}^T(A_{i(l),l} + B_lK_l) + G_{l,4}^T A_{i(l),l} - F_l & -G_{l,2}^T B_l K_l + F_l - V_l C_l \end{matrix} \\ (\bullet)^T & S_{i(j),j} \end{bmatrix} > 0 \quad (19)$$

With

$$\begin{bmatrix} R_l \\ F_l \end{bmatrix} = \begin{bmatrix} G_{l,3}^T \\ G_{l,4}^T \end{bmatrix} \hat{A}_l \quad \text{and} \quad \begin{bmatrix} U_l \\ V_l \end{bmatrix} = \begin{bmatrix} G_{l,3}^T \\ G_{l,4}^T \end{bmatrix} L_l \quad (20)$$

Proof:

Substituting $A_{i(l),l}$ by $\Phi_{i(l),l}^T$ in theorem 1 one gets:

$$\begin{bmatrix} G_l + G_l^T - S_{i(l),l} & \begin{matrix} G_{l,1}^T(A_{i(l),l} + B_lK_l) + G_{l,3}^T(A_{i(l),l} - \hat{A}_l) & -G_{l,1}^T B_l K_l + G_{l,3}^T \hat{A}_l - G_{l,3}^T L_l C_l \\ G_{l,2}^T(A_{i(l),l} + B_lK_l) + G_{l,4}^T(A_{i(l),l} - \hat{A}_l) & -G_{l,2}^T B_l K_l + G_{l,4}^T \hat{A}_l - G_{l,4}^T L_l C_l \end{matrix} \\ (\bullet)^T & S_{i(j),j} \end{bmatrix} > 0 \quad (21)$$

Equation (21) is a Bilinear Matrix Inequality due to the presence of the following products: $G_{l,3}^T L_l$, $G_{l,4}^T L_l$, $G_{l,3}^T \hat{A}_l$ and $G_{l,4}^T \hat{A}_l$. To come over this difficulty we propose the following change of variables:

$$\begin{aligned} G_{l,3}^T L_l &= U_l \\ G_{l,4}^T L_l &= V_l \\ G_{l,3}^T \hat{A}_l &= R_l \\ G_{l,4}^T \hat{A}_l &= F_l \end{aligned} \quad (22)$$

Then the formulation (19) and (20) can easily be obtained. □

Of course (20) are non linear equalities which could be written as range conditions without improvement [31].

So the problem (19), (20) is not LMI. Below two structural constraints on *G* are proposed which enables to get new sufficient LMI conditions:

1) $G_{l,3} = 0$

In this case the conditions (19) and (20) become:

$$\begin{bmatrix} G_l + G_l^T - S_{i(l),l} & \begin{matrix} G_{l,1}^T(A_{i(l),l} + B_l K_l) & -G_{l,1}^T B_l K_l \\ G_{l,2}^T(A_{i,l(l)} + B_l K_l) + G_{l,4}^T A_{i(l),l} - F_l & -G_{l,2}^T B_l K_l + F_l - V_l C_l \end{matrix} \\ (\bullet)^T & S_{i(j),j} \end{bmatrix} > 0 \quad (23)$$

which is linear depending on $S_{i(l),l}, S_{i(j),j}, G_l, F_l$ and V_l.

Then

$$\hat{A}_l = G_{l,4}^{T\,-1} F_l \quad \text{and} \quad L_l = G_{l,4}^T V_l \qquad (24)$$

2) $G_{l,3} = G_{l,4}$

In this case the conditions (19) and (20) become:

$$\begin{bmatrix} G_l + G_l^T - S_{i(l),l} & \begin{matrix} G_{l,1}^T(A_{i(l),l} + B_l K_l) + G_{l,3}^T A_{i(l),l} - F_l & -G_{l,1}^T B_l K_l + F_l - V_l C_l \\ G_{l,2}^T(A_{i(l),l} + B_l K_l) + G_{l,4}^T A_{i(l),l} - F_l & -G_{l,2}^T B_l K_l + F_l - V_l C_l \end{matrix} \\ (\bullet)^T & S_{i(j),j} \end{bmatrix} > 0 \quad (25)$$

which is linear depending on $S_{i(l),l}, S_{i(j),j}, G_l, F_l$ and V_l.

Then

$$\hat{A}_l = G_{l,4}^{T\,-1} F_l \quad \text{and} \quad L_l = G_{l,4}^T V_l \qquad (26)$$

It is to be notified that such condition, propose the determination of both the «observers» matrices and gains $\hat{A}_l, L_l$. Which can be «favorable» point for the satisfaction of the respective conditions.

6. STUDY CASE

To illustrate the polyquadratic stabilization through dynamic output feedback the discrete time switched system given in P. Perez and J. Geromel 1993 [6] is considered with some modifications:

$$X(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = AX(k) + Bu(k) \qquad (27)$$

$$y(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$A = \begin{bmatrix} 0.2113 & 0.0087 & 0.4524 \\ 0.0824 & 0.8096 & 0.8075 \\ 0.7599 & 0.8474 & 0.4832 \end{bmatrix} (1+\alpha) I$$

(28)

$$B = \begin{bmatrix} 0.6135 & 0.6538 \\ 0.2749 & 0.4899 \\ 0.8807 & 0.7741 \end{bmatrix}$$

It is assumed that one of the two actuators can break down so that the *B* matrix can take 3 different values. A global uncertainty is put on the *A* matrix with $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$. Based on that, the system is an uncertain discrete time switched system represented by 3 uncertain sub-systems, characterized by three control matrices.

$$B_1 = \begin{bmatrix} 0.6135 & 0.6538 \\ 0.2749 & 0.4899 \\ 0.8807 & 0.7741 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 & 0.6538 \\ 0 & 0.4899 \\ 0 & 0.7741 \end{bmatrix} \quad B_3 = \begin{bmatrix} 0.6135 & 0 \\ 0.2749 & 0 \\ 0.8807 & 0 \end{bmatrix} \tag{29}$$

For $-0.22 \leq \alpha \leq 0.22$, using theorem 2 we compute the switched state feedback gains:

$$K_1 = \begin{bmatrix} -1.5119 & 0.6172 & 1.7030 \\ 0.8147 & -1.5515 & -2.5006 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.6575 & -1.0011 & -0.9105 \\ 0 & 0 & 0 \end{bmatrix} \tag{30}$$

$$K_3 = \begin{bmatrix} 0 & 0 & 0 \\ -0.5625 & -0.9588 & -0.9337 \end{bmatrix}$$

Then we use condition (23) to design the switched observers.

$$\hat{A}_1 = \begin{bmatrix} 0.2438 & 0.0463 & 0.3899 \\ 0.1366 & 0.8884 & 0.6933 \\ 0.8144 & 0.9404 & 0.4165 \end{bmatrix}, L_1 = \begin{bmatrix} 0.3988 & 0.3612 \\ 0.4754 & 1.4862 \\ 1.1007 & 1.3300 \end{bmatrix}$$

$$\hat{A}_2 = \begin{bmatrix} 0.1946 & 0.0640 & 0.4621 \\ 0.0900 & 0.9053 & 0.7857 \\ 0.8435 & 0.9273 & 0.3908 \end{bmatrix}, L_2 = \begin{bmatrix} 0.3754 & 0.3635 \\ 0.4504 & 1.4713 \\ 1.0984 & 1.3303 \end{bmatrix} \qquad 31)$$

$$\hat{A}_3 = \begin{bmatrix} 0.1864 & 0.0588 & 0.4281 \\ 0.1106 & 0.9083 & 0.7222 \\ 0.8571 & 0.9426 & 0.3693 \end{bmatrix}, L_3 = \begin{bmatrix} 0.3790 & 0.3548 \\ 0.4816 & 1.4753 \\ 1.1017 & 1.3426 \end{bmatrix}$$

For simulation the actuators break down are chosen randomly.

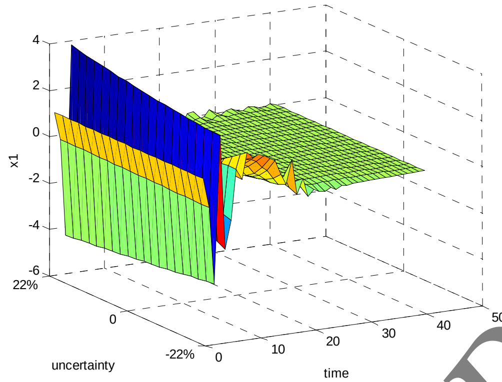

Fig. 2: x_1 trajectory.

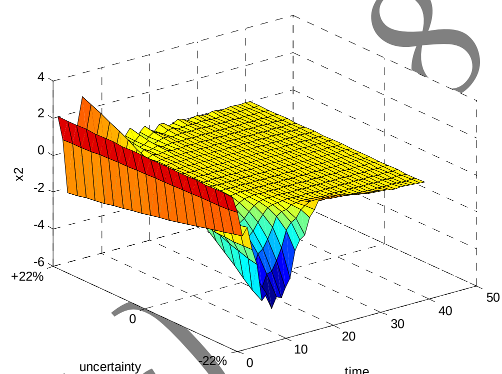

Fig. 3: x_2 trajectory.

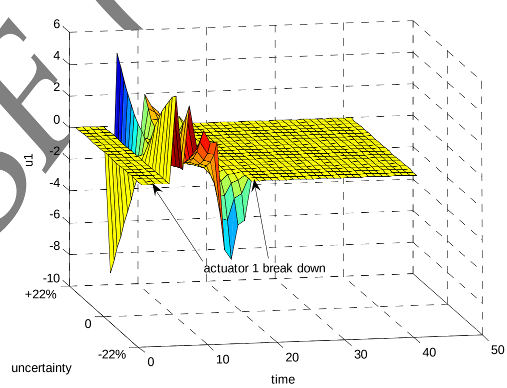

Fig. 4: First actuator control.

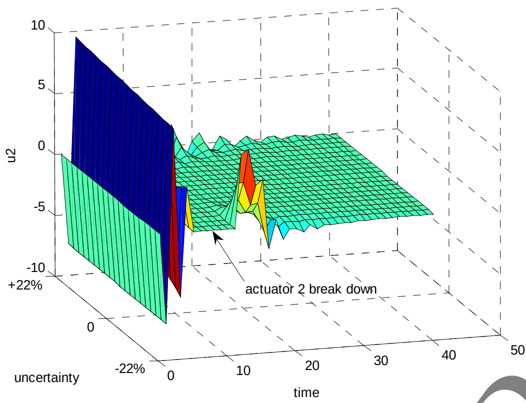

Fig. 5: Second actuator control.

The figures 2 and 3 shows that despite the actuators break down (figures 4 and 5) the control law ensure a robust stability of the global system.

7. CONCLUSION

The problem of designing a stabilizing dynamic output feedback is a complex one for switched systems with uncertain sub-systems. It is known that output control design is difficult when the uncertainties are structured ones, which is the case for polytopic uncertainty. In this paper we propose sufficient conditions using polyquadratic stability to find robust dynamic output feedback controls, based on a Luenberger observer, for this kind of systems. One can develop this work using some relaxation algorithms to resolve the Bilinear Matrix Inequality given in (21).

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