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Robust control design for nonlinear time delay systems

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Abstract-This work is dedicated to the conception of a control law which stabilizes a family of nonlinear systems with distributed input delay. The main idea consists to use a new technique for construction of Lyapunov Krasovskii functional inspired by a new reduction model approach. We stabilize globally uniformly exponentially the origin of the considered system. Moreover, the Lyapunov-Krassovskki functional is used to get robustness properties in presence of uncertain additive terms. Then, we illustrate our construction using a numerical example

Keywords: **nonlinear, delay, distributed, Asymptotic stability**.

I. INTRODUCTION

The study of nonlinear delayed systems is motivated in particular by the fact that the delay can strongly influence on the performance of the model [17]. Two types of time delay have been used by the earlier researchers, namely, punctual and distributed delays. It has been found that distributed delay systems are more realistic. Studies show that time delays including distributed delays may destroy the stability [18], even cause oscillation.

In this context, a first method based on the reformulation of distributed delay system on a form of inequalities matrix LMI and which implies the use of Lyapunov-Krasovskii functional [6] [11]. A nother method based on a reduction model approach and Riccati equation approach [7] allows to show the stability of linear systems with distributed delay. In the case of input delays, the problem received considerable attention in the last few years [4] [16]. One of the most popular approaches used to cope with delays in the input is the reduction model approach, sometimes called finite spectrum assignment technique [14].

In the literature, we often find stabilizing controls for linear systems [2] [9]. Nevertheless, we show in this work that the same family of Lyapunov functions used in the context of linear systems can also be used in nonlinear systems [20] [10]. Having chosen a known and strict Lyapunov-Krasovskii functional allows to analyze the robustness of our method and justifies the advantage of our approach. The model reduction method [1] is discussed to demonstrate the stability of the reduced system obtained.

Although in order to help the reader to better understand the conceptual ideas of our methodology, we present the case of punctual delayed nonlinear systems [13], the control

law stabilizes globally exponentially [19] the origin of considered system, based on a reduction model approach which leads to obtain a new closed-loop system.
Consider the following system:

$$\dot{x}(t) = f(t, x(t)) + f_\tau(t-\tau, x(t-\tau))u(t-\tau) \tag{1}$$

with $x \in R^n, u \in R^P$ and $\tau > 0$ a punctual delay, f and f_τ two continuous and Lipshitz functions. The aim of the authors was to find a new control law which stabilizes globally exponentially the origin of the system (1).
When the following Assumption is satisfied: there exists a matrix L and Hurwitz Lipschitz continuous function $g(t,x)$ such that for all $x \in R^n$ and $t \geq -\tau$,

$$\overline{f}(t,x) = e^{-L\tau} f_\tau(t,x) g(t,x)$$

Then, the following representation is used:

$$\dot{x}(t) = Lx(t) + \overline{\tau}(t-\tau, x(t-\tau))u(t-\tau) + \overline{f}(t, x(t)).$$

The following operator

$$\alpha(t) = \int_{t-\tau}^{t} e^{L(t-l-\tau)} f_\tau(t, x(t))u(t),$$

is introduced. This leads to calculate the derivative along the trajectories of the operator. After some calculus and the choice of the control $u(t-\tau) = -g(t-\tau, x(t-\tau))$, the system obtained is

$$\dot{s}(t) = Ls(t), \tag{2}$$

with:

$$s(t) = x(t) - \int_{t-\tau}^{t} \overline{f}(l, x(l))dl. \tag{3}$$

The authors have shown the exponential stability of the global system (2) and (3), using a Lyapunov-Krasovskii functional when τ is enough small. The result which we are going to present in this work focuses on a different family of systems. Indeed, we are going to consider the distributed delay systems. We consider the following system:

$$\dot{x}(t) = f(t, x(t)) + \int_{t-\tau}^{t} f_\tau(\omega, x(\omega))u(\omega)d\omega. \tag{4}$$

with $x \in R^n, u \in R^P$ where τ, ω are positive constants. Motivated by the above discussions, our objective is to find a new control law which stabilizes globally exponentially the system (1).

This paper is organized as follows. Section 2 is devoted to include the assumptions and the theorem in order to demonstrate the stability of the system (1). In Section 3, we demonstrate the robustness of our approach of stabilization by studying the case of an uncertain additive term. An illustration of the results obtained is made in Section 4. In the end, a conclusion will be quoted in Section 5.

II. PRINCIPAL RESULT

A. Assumptions and results

Assumption 1: There exists a matrix $L \in R^{N \times N}$ Hurwitz and a Lipshitz continuous function $g(t,x)$ such that for all $x \in R^n$.

$$\bar{f}(t,x) = Mf_{\tau}(t,x)g(t,x), \tag{5}$$

With

$$M = \int_{-\tau}^{0} e^{-L(\tau+l)}dl, \tag{6}$$

And

$$\bar{f}(t,x) = f(t,x) - Lx. \tag{7}$$

In addition, the following inequality is satisfied:

$$\left|\bar{f}(l,x(l))\right| \le f_m \left|x(l)\right|, \ \forall x \in R^n, \tag{8}$$

where f_m is a positive constant.

Assumption 2: note that

$$k = \int_{-\tau}^{0} e^{|L|(l+\tau)}dl. \tag{9}$$

The following inequality

$$0 \le k^2 \le \frac{1}{4\tau f_m^2} \tag{10}$$

is satisfied.

Theorem 1: Consider the system (4) assume that it satisfies Assumptions H1 et H2. Then the closed-loop system with the control low.

$$u(l) = g(l,x(l)), \tag{11}$$

Admits the origin as a globally uniformly exponentially stable equilibrium point.

B. Proof

AccordingtoAssumption1, the system admits the following representation:

$$\dot{x}(t) = Lx(t) + \bar{f}(t,x(t)) + \int_{t-\tau}^{t} f_{\tau}(\omega,x(\omega))u(\omega)d\omega. \tag{12}$$

Consider the following operator:

$$\alpha(t) = \int_{t-\tau}^{t} e^{L(t-l-\tau)} \int_{l}^{t} f_{\tau}(\omega,x(\omega))u(\omega)d\omega dl. \tag{13}$$

Its derivative along the trajectories gives:

$$\dot{\alpha}(t) = L\alpha(t) - \int_{t-\tau}^{t} f_{\tau}(\omega,x(\omega))u(\omega)d\omega +$$

$$\int_{t-\tau}^{t} e^{L(t-l-\tau)} f_{\tau}(t,x(t))u(t)dl \tag{14}$$

Consider

$$s(t) = x(t) + \alpha(t). \tag{15}$$

and its time-derivative, satisfies:

$$\dot{s}(t) = Ls(t) + \int_{t-\tau}^{t} e^{L(t-l-\tau)} f_{\tau}(t,x(t))u(t)dl + \bar{f}(t,x(t)).$$

Therefore

$$\dot{s}(t) = Ls(t) + Mf_\tau(t, x(t))u(t) + \overline{f}(t, x(t)),$$

where M is defined in (6). Combining the equality $\overline{f}(t, x(t))$ with $Mf_\tau g(t, x)$, we obtain

$$\dot{s}(t) = Ls(t) + Mf_\tau(t, x(t))[u(t) + g(t, x(t))]. \quad (16)$$

The choice of the control law (11) gives

$$\dot{s}(t) = Ls(t), \tag{17}$$

And

$$s(t) = x(t) - \int_{t-\tau}^{t} e^{L(t-\tau-l)} \int_{l}^{t} \overline{f}(\omega, x(\omega)) d\omega dl. \tag{18}$$

We will show that the closed-loop system is globally exponentially stable by the construction of a Lyapunov function for systems (17) and (18). Consider the Lyapunov function

$$V_1(s) = s^T Ps \geq 0, \tag{19}$$

when $P \in R^{n \times n}$ is a symmetric positive definite matrix such that:

$$PL + L^T P \leq -I. \tag{20}$$

This inequality and (17) imply that:

$$\dot{V}_1(t) \leq -|s(t)|^2. \tag{21}$$

On the other hand, we deduce from (18) for all $t \geq 0$, knowing that for all functions α and for all $l \in [t - \tau, t]$

$$\int_{l}^{t} |\alpha(m)| dm \leq \int_{t-\tau}^{t} |\alpha(m)| dm$$

$$\left| \int_{t-\tau}^{t} e^{L(t-\tau-l)} \int_{l}^{t} \overline{f}(\omega, x(\omega)) d\omega dl \right| \leq \tag{22}$$

$$\left(\int_{t-\tau}^{t} e^{|L|(l-t+\tau)} dl\right)\left(\int_{t-\tau}^{t} \left|\overline{f}(\omega, x(\omega))\right| d\omega\right)$$

is satisfied. We deduce that

$$\left| \int_{t-\tau}^{t} e^{L(t-\tau-l)} \int_{l}^{t} \overline{f}(\omega, x(\omega) d\omega dl \right| \leq k\left(\int_{t-\tau}^{t} \left|\overline{f}(l, x(l))\right| dl\right), \tag{23}$$

Where

$$k = \int_{-\tau}^{0} e^{|L|(l+\tau)} dl. \tag{24}$$

We can rewrite (18) as

$$|x(t)| = |s(t)| + \left| \int_{t-\tau}^{t} e^{L(t-\tau-l)} \int_{l}^{t} \overline{f}(\omega, x(\omega)) d\omega dl \right|, \tag{25}$$

the inequality (23) implies that

$$|x(t)| \leq \left| s(t) + k \int_{t-\tau}^{t} \left|\overline{f}(l, x(l))\right| dl. \right. \tag{26}$$

According to the Assumption H2, we deduce:

$$|x(t)| \le |s(t)| + kf_m \int_{t-\tau}^{t} |x(l)| dl. \tag{27}$$

We introduce the following function $Q(x) = |x|^2$, then

$$Q(x(t)) \le 2|s(t)|^2 + 2f_m^2 k^2 \int_{t-\tau}^{t} |x(l)|^2 dl. \tag{28}$$

Let $\gamma(x_t) = \int_{t-\tau}^{t} Q(x(l))dl.$ We have

$$Q(x(t)) \le 2|s(t)|^2 + 2f_m^2 k^2 \gamma(x_t). \tag{29}$$

We introduce the following functional

$$V_2(x_t) = \frac{1}{\tau} \int_{t-\tau}^{t} \int_{m}^{t} Q(\phi(l))dldm. \tag{30}$$

Its derivative ensure:

$$\dot{V}_2(t) = Q(x(t)) - \frac{1}{\tau}\gamma(x_t) \tag{31}$$

We deduce from (28) that

$$\dot{V}_2(t) \le 2|s(t)|^2 + \frac{2\tau f_m^2 k^2 - 1}{\tau}\gamma(x_t). \tag{32}$$

According to Assumption H2 and (10) implies that

$$\dot{V}_2(t) \le 2|s(t)|^2 - \frac{1}{2\tau}\gamma(x_t) \tag{33}$$

(38) implies that

$$-\dot{V}_2(t) = -Q(x(t)) + \frac{1}{\tau}\gamma(x_t). \tag{34}$$

On the other hand:

$$2\dot{V}_2(t) \le 4|s(t)|^2 - \frac{1}{\tau}\gamma(x_t). \tag{35}$$

By addition (33) and (34), we obtain:

$$2\dot{V}_2(t) \le 4|s(t)|^2 - Q(x(t)). \tag{36}$$

Let's use $V_3 = 4V_1 + V_2$,

$$\dot{V}_3 \le -4|s(t)|^2 + 4|s(t)|^2 - Q(x(t)). \tag{37}$$

consequently,

$$\dot{V}_3 \le -Q(x(t)) \tag{38}$$

Since $Q(x(t))$ is positive definite, the Lyapunov-Krasovskii theorem ensures that the origin of (12) and (1) is globally uniformly asymptotically stable. Finally let:

$$V_4(t, x_t) = V_3(\varsigma(t, x_t), x_t) + c_1 \int_{t-\tau}^{t} \int_{l}^{t} Q(x(m)dmdl + c_2 \int_{t-\tau}^{t} Q(x(m)dm. \tag{39}$$

With

$$V_4(t,x_t)=4V_1(x(t)-\int_{t-\tau}^{t}e^{L(t-\tau-l)}\int_{l}^{t}\overline{f}(\omega,x(\omega))d\omega dl+$$

$$\frac{1}{\tau}\int_{t-\tau}^{t}\int_{m}^{t}Q(x(l))dldm+c_1\int_{t-\tau}^{t}\int_{l}^{t}Q(x(m)dmdl+c_2\int_{t-\tau}^{t}Q(x(m)dm. \quad (40)$$

Since P is symmetric and positive matrix, there exists a constant $c_3\in(0,1]$ when the following inequality is verified :

$$(v_1+v_2)^T P(v_1+v_2)\geq(1-c_3)v_1^T Pv_1+(1-\frac{1}{c_3})v_2^T Pv_2. \quad (41)$$

Since V_2 is positive definite and we assume $c_1\approx 1$:

$$V_4(t,x_t)\geq 4(1-c_1)x(t)^T Px(t)+4(1-\frac{1}{c_1})\phi(t,x)+$$

$$\frac{1}{\tau}\int_{t-\tau}^{t}\int_{m}^{t}Q(x(l))dldm\frac{1}{\tau}\int_{t-\tau}^{t}\int_{m}^{t}Q(x(l))dldm+$$

$$c_1\int_{t-\tau}^{t}\int_{l}^{t}Q(x(m))dmdl+c_2\int_{t-\tau}^{t}Q(x(m)dm. \quad (42)$$

Where

$$\phi(t,x_t)=(\int_{t-\tau}^{t}e^{L(t-\tau-l)}\int_{l}^{t}\overline{f}(\omega,x(\omega))d\omega dl)^T P$$

$$(\int_{t-\tau}^{t}e^{L(t-\tau-l)}\int_{l}^{t}\overline{f}(\omega,x(\omega))d\omega dl). \quad (43)$$

From Assumption H1 and with (23), there exists c_2 such that

$$\phi(t,x_t)\leq c_2\gamma(x(t)). \quad (44)$$

we rewrite (42) as

$$V_4(t,x_t)\geq 4((1-c_1)x(t)^T Px(t)+(1-\frac{1}{c_1})c_2\gamma(x(t)))+$$

$$\frac{1}{\tau}\int_{t-\tau}^{t}\int_{m}^{t}Q(x(l))dldm\frac{1}{\tau}\int_{t-\tau}^{t}\int_{m}^{t}Q(x(l))dldm+$$

$$c_1\int_{t-\tau}^{t}\int_{l}^{t}Q(x(m))dmdl+c_2\int_{t-\tau}^{t}Q(x(m))dm. \quad (45)$$

It follows that

$$V_4(t,x_t)\geq c_4 x(t)^T Px(t). \quad (46)$$

Similarly, we can show that there exist a constant c_5 such that:

$$V_4(t,x_t)\geq c_5\left|x(t)\right|^2_{[-\tau,0]} \quad (47)$$

We have from (38) and (47)

$$\dot{V}_4(t) \le -Q(x(t)) + c_2 \tau Q(x(t)) - c_2 \int_{t-\tau}^{t} Q(x(l))dl +$$

$$c_1 Q(x(t)) - c_1 Q(x(t-\tau)). \tag{48}$$

Let use:

$$c_1 + \tau c_2 = \frac{1}{2}.$$

$$\dot{V}_4(t) \le -\frac{1}{2} Q(x(t)) - c_2 \gamma(x(t)) - c_1 Q(t-\tau). \tag{49}$$

Since Q is positive definite, there exist a constant c_5 such that

$$\dot{V}_4(t) \le -c_6 V_4(t, x_t). \tag{50}$$

Finally, the inequalities (46), (47), (50) ensure that the closed loop system is globally uniformly exponentially stable

III. ROBUSTNESS RESULT

In this section, we use the Lyapunov-Krasovskii functional introduced in the previous section to establish, under extra assumption, the control (11) gives an ISS property to the system. Then, we consider the following system:

$$\dot{x}(t) = f(t, x(t)) + \int_{t-\tau}^{t} f_\tau(\omega, x(\omega)) u(\omega) d\omega + h(t, x(t)) \tag{51}$$

with $x \in R^n$, and $u \in R^P$, f et f_τ are two Lipshitz functions and h is an unknown nonnegative function. We introduce a new Assumption

Assumption 3: Consider the system (51), there exists h_m positive function such that

$$0 \le h_m \le \frac{1}{4\sqrt{2}\,|P|^2} \tag{52}$$

holds, where P verify this inequality

$$PL + L^T P \le -I \tag{53}$$

and for all $t \ge -\tau$ and $x \in R^n$

$$|h(t,x)| \le h_m |x| \tag{54}$$

Proof

We have the solution of (18) with the additive term in the system (51)

$$\dot{s}(t) = Ls(t) + h(t, x(t)) \tag{55}$$

We have:

$$\dot{V}_1(t) \le -|s(t)|^2 + 2|P||Is(t)||h(t, x(t))| \tag{56}$$

According to Assumption H3:

$$\dot{V}_1(t) \le -|s(t)|^2 + 2|P| h_m |Is(t)||x(t)| \tag{57}$$

We use the inequality $|x|^2 \le Q(x)$, and the inequality of Cauchy-Shwartz we obtain:

$$\dot{V}_1(t) \le -\frac{1}{2}|s(t)|^2 + 2|P|^2 h_m^2 Q(x(t)) \tag{58}$$

The inequality (36) doesn't change with the presence of $h(t, x(t))$ so we have

$$V_3 = V_2 + 8V_1 \tag{59}$$

$$\dot{V}_3 \le -Q(x(t)) + 16|P|^2 h_m^2 Q(x(t)) \tag{60}$$

according to Assumption (3):

$$\dot{V}_3 \le -\frac{1}{2} Q(x(t)) \tag{61}$$

We proceed similarly to the demonstration in the previous section, we can establish again that the closed loop system (51) with the additive term is globally uniformly exponentially stable.

IV. ILLUSTRATIVE EXAMPLE

Consider the following system:

$$\dot{x}(t) = -\sin(t)\frac{x(t)^3}{1+x(t)^2} + \int_{t-\tau}^{t} u(\omega)d\omega \tag{61}$$

For the purpose of simplifying the calculations and focusing on the development of the control law, we have made the following choice:

$L = -4, f_m = 5, f_\tau(\omega, x(\omega)) = 1.$

The Assumption H1 is satisfied with

$$g(t,x) = \frac{1}{M}(f(t,x) - Lx). \tag{62}$$

We calculate M and k

$$M = \int_{-\tau}^{0} e^{-L(\tau+l)}dl = \frac{1}{L}(e^{L_\tau} - 1)$$

$$k = e^{|L_\tau|}\left(\frac{1}{|L|} - \frac{1}{|L|e^{-\tau|L|}}\right) \tag{63}$$

$$k = \frac{1}{4}e^{4\tau}(1 - e^{-4\tau}) \tag{64}$$

According to Assumption H2 :

$$0 \le \left(\int_{-\tau}^{0} e^{4(l+\tau)}dl\right)^2 \le \frac{1}{100\tau} \tag{65}$$

We choose τ = 0.08 seconds in order to verify the stability conditions imposed in the Assumptions

$$g(t,x(t)) = \frac{1}{M}\left(-\sin(t)\frac{x(t)^3}{1+x(t)^2} + 4x(t)\right) \tag{66}$$

From inequality (11) in Theorem 1, we deduce that:

$$\dot{x}(t) = -\sin(t)\frac{x(t)^3}{1+x(t)^2} +$$

$$\int_{t-\tau}^{t} \frac{1}{M}\left(-\sin(\omega)\frac{x(\omega)^3}{1+x(\omega)^2}+4x(\omega)\right)d\omega \tag{67}$$

To trace the curve of the solution we need to use an approximation method to simplify the integral. The approximation is as follows:

$$\int_{a_i}^{a_i+\frac{\tau}{N}} \phi = \frac{\tau}{N}\left(\phi(a_i+\frac{\tau}{N})\right) \tag{68}$$

Where

$$a_i = t-\tau+i\frac{\tau}{N} \tag{69}$$

The choice of ϕ is given by $\phi(t)=\frac{1}{20}(e^{\cos(5\pi t/\tau)}-e^{-1})$, $t\in[t-\tau,t]$ (70)

In our case we take $N=4$, and the evolution of the solution is shown in **Fig. 1**.

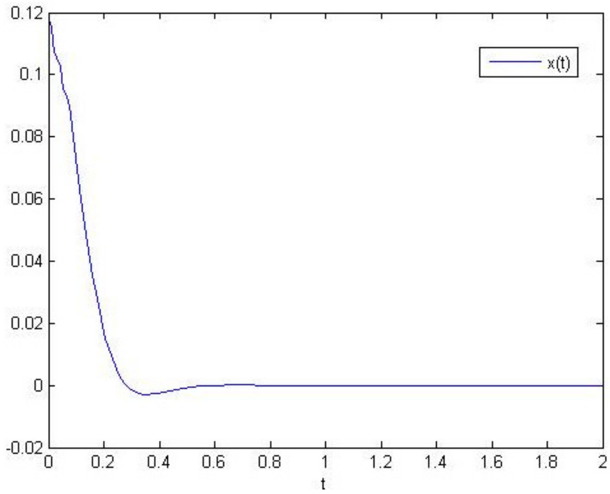

Fig 1. Simulation for $\tau = 0.08$ seconds

V. CONCLUSION

We have developed in this article a new approach of reduction model for a family of nonlinear systems with distributed delay based on the construction of a Lyapunov-Krasovskii functional that allows to stabilize globally uniformly exponentially the origin of the system. We also have demonstrated the robustness of the stabilization of the system in the presence of an additive uncertain term. We can extend our result to the systems with a combination of delays (punctual and distributed). Another study can be performed to determine a large family of positive solution component by component at any time and that using similar ideas to those used in [13]. Note also that we could complete our simulations, through other means of various approximation methods (Trapeze, Newton Cotes ... etc) or by increasing of the sampling

Appendix : Usual inequalities: For $A,B\in \mathrm{R}$ et $\rho\in(0,\infty)$ the inequalities

$$(A+B)^2 \le (1+\rho)A^2+(1+\frac{1}{4\rho})B^2$$

$$AB \le \frac{1}{2}A^2+\frac{1}{2}B^2 \quad \& \quad \left|e^A\right| \le e^{|A|}$$

are satisfied.

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