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Geometric Modeling of the Par4 Parallel Robot Using the Modified Denavit Hartenberg Method

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The determination of the Forward kinematic model (FKM) of parallel robot is very difficult in comparison to the serial manipulators. It is the famous problem of research. This model is almost impossibility to solve it analytically. Most researchers have resort to numerical method. But the common drawback of these methods is the convergence problem. In this paper, Modified Denavit Hartenberg Method (MDHM) is used to determine the FKM of the parallel robot Par4. The Inverse Kinematic Model (IKM) of Par4, inspired from the Nabat model, is used for the verification of our work. The results show the validity of the calculated model.

Keywords: *Modified Denavit Hartenberg Method (MDHM); geometric modeling; closed loop; parallel robot; Par4*

1. INTRODUCTION

Parallel manipulators have received a lot of attention from researchers over the past couple decades [1] [2] [3], due to the advantages such as more accuracy and more rigidity which they present over their serial counterpart. They have an important ratio of load capacity and masse. A parallel manipulator, generally, consists in mobile platform that is connected to a fixed base by several limbs in parallel. Despite their advantages, FKM is a famous problem of research. Unlike the serial robot, the FKM of parallel robot is a composed task; the direct kinematic problem pertains to the determination of the actual pose: position and orientation of the end-effecter relative to the base from a set of joint position. The problem of FKM of parallel robot is the almost impossibility to be solved analytically. Numerical methods are one of the common solutions of this problem.

Traditional methods to solve FKM of parallel robot have focused on using algebraic formulations to generate a high degree polynomial or a set of nonlinear equations. Then methods such as internal analysis [4], algebraic elimination [5] and continuation [6] are used to find the root of the polynomials or solve nonlinear equations.

Also, the Newton Raphson method is used by Nabat [7] for modeling parallel robot Par4. It is also used by Arshad [8] to model the Stewart platform. But, the convergence problem of these methods is the drawback of using them and numerical iteration is usually sensitive to the choice of initial values and nature of resulting constraint equations.

In this paper, we used MDHM to model the Par4 robot. This method is an evolution of Denavit Hartenberg Method (DHM)[9]. It is the most popular method in the world of robotics for modeling serials robots. It is proposed by Denavit and Hartenbergin1955. It has an ambiguity for closed and branched chains. In 1986, Khalil and Kleinfinger [10] made some changes to this method to be adapted to all types of robots. It is named Modified Denavit Hartenberg Method or Khalil_Kleinfinger method. It is the most widely used method to describe the geometry of all types of robots. MDHM process uses a minimum number of parameters. It is fast and it hasn't got a convergence problem.

In the next section, we present the MDHM for different robots structures: simple open structure, tree structure and closed loop structure. In Section III, we determine the FKM of Par4 using MDHM and the IKM using an inspiration of Nabat model [7]. Then, we give some simulation results in Section VI. Finally, a conclusion is given in section V.

2. MODIFIED DENAVIT HARTENBERG METHOD

In this section, we give a geometric description of different robots structures by using the MDHM [10]: simple open structure, tree structure and closed loop structure.

2.1 Simple open loop structure

Considering a simple loop structure composed of $n+1$ rigid body: $C_0,\ldots,C_n$. C_0 This is the fixed body. C_n This is the terminal body. The joint i connects the body $i-1$ to the body i.

The determination of the geometric model for the simple loop robot structure is based on the following steps:

Step 1: Attaching an axe to each robot body.

Each body C_i of the robot is connected to a frame R_i. The z_i axeswhich are taken along the axes of the link connecting the body $C_i$to C_{i-1}.x_i axes which are taken by the common normal to z_i andz_{i+1}.y_iCompletes the direct frame with the axes xi and z_i (is generally not presented).

Step 2: Giving parameters of MDH.

The transformation matrix for frame R_{i-1} to $R_i$gives the following parameters called parameters of MDH. These parameters are shown in Figure1.

- d_i: distance between z_{i-1} and z_i along x_{i-1}, this is the minimum distance between two consecutive axes of joints.
- α_i: angle around x_{i-1} between z_{i-1} and z_i.
- r_i: distance between $x_{i-1}$and$x_i$along z_i. It is the joint variable of the prismatic link.
- θ_i: angle around z_i betweenx_{i-1}andx_i. It is the joint variable of the pivot link.

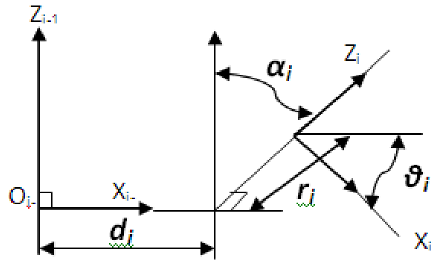


Figure 1 Parameters of MDH convention.

Step 3: Calculating Homogeneous Transformation Matrices between each body.

The Homogeneous Transformation Matrices is given by (1). This matrix allows the passage of a body $i-1$ (frame R_{i-1}) to the body i (frame R_i).

$$T_i^{i-1} = Rot(x,\alpha_i).Trans(x,d_i).Rot(z,\theta_i).Trans(z,r_i) \quad (1)$$

$$T_i^{i-1} = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & di \\ C\alpha_i.S\theta_i & C\alpha_i.C\theta_i & -S\alpha_i & -ri.S\alpha_i \\ S\alpha_i.S\theta_i & S\alpha_i.C\theta_i & C\alpha_i & ri.C\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$T_i^{i-1} = \begin{bmatrix} A_i^{i-1} & P_i^{i-1} \\ 0_{1\times 3} & 1 \end{bmatrix} \quad (3)$$

A_i^{i-1} is the rotation matrix giving the orientation of the frame R_i in the frame R_{i-1}.

- P_i^{i-1} is the vector of O_i point coordinates in the R_{i-1} frame.
- C. is the $\cos(.)$ and S. is the $\sin(.)$.

Step 4: Determining the FKM of the robot.

The homogeneous matrix between the base of the robot and the terminal member is the multiplication of different homogeneous matrices between each body.

$$T_i^{i-1} = \begin{bmatrix} C\gamma_i.C\theta_i - S\gamma_i.C\alpha_i.S\theta_i & -C\gamma_i.S\theta_i - S\gamma_i.C\alpha_i.C\theta_i & S\gamma_i.S\alpha_i & d_iC\gamma_i + r_i.S\gamma_i.S\alpha_i \\ S\gamma_i.C\theta_i + C\gamma_i.C\alpha_i.S\theta_i & -S\gamma_i.S\theta_i + C\gamma_i.C\alpha_i.C\theta_i & -C\gamma_i.S\alpha_i & d_i.S\gamma_i - r_i.C\gamma_i.S\alpha_i \\ S\alpha_i.S\theta_i & S\alpha_i.C\theta_i & C\alpha_i & r_i.C\alpha_i + b_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4)

$$T_p^0 = T_1^0.T_2^1.\ldots.T_n^{n-1}.T_p^n \quad (5)$$

2.2 Tree structure robot

A tree structure robot is an extension of the simple open loop structure. It has two minimum end organs. It is composed of $n+1$ rigid body: C_0, C_n. We chose a main chain. The others are considered secondary. C_0 is the first body of the main chain. C_n is the end body of the robot. The determination of the geometric model for the tree structure robot is based on the following steps:

Step 1: Attaching the axes to each robot body, than the simple open loop structure. If the body C_i supports two or more bodies, x_i is chosen perpendicularly to the common z_i and another axis z_j or z_k. Generally the main chain is the longest chain.

Step 2: Giving the parameters of MDHM (from $R_i$to R_j).

If we study the main chain, we assume that the work would be the same as in the case of the serial robot. The parameters of MDHM are the same as for the case of the simple open loop structure. Moreover, we add another vector U at the tree. $U$is the common perpendicular to z_i and z_j. Two other parameters will be added γ_i and b_i (see Figure2). So, we obtain six parameters of MDHM:

- γ_i: angle between $x_i$and U around z_i.
- b_i: distance between $x_i$and U alongz_i.
- α_i: angle between z_i and z_j around U.
- d_i: distance between z_i and z_j along U.
- θ_i: angle between U and x_j around z_j.
- r_i: distance between U and x_j along z_i.

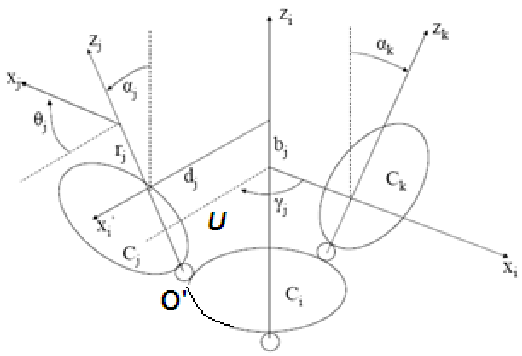


Figure 2 The MDH parameters for tree structure robot.

Step 3: The homogeneous matrices between each body.

If x_i is perpendicular to z_i and z_j, the homogeneous matrix is as in the case of the simple open loop structure. If xi is not the perpendicular to z_i and z_j, the homogeneous matrices between each body (passage of the frame R_i to R_j)are given by:

$$T_j^{j-1} = Rot(z_{i-1}, \Upsilon_j).Trans(z_{i-1}, \varepsilon_j).Rot(x_{i-1}, \alpha_i).$$
$$Trans(x_{i-1}, d_i).Rot(x_{i-1}, \alpha_i).Trans(x_{i-1}, d_i) \quad (5)$$

After development, we obtain:

Step 4: Determining the forward kinematics model of the robot.

We determine the forward kinematics model like the case of the open loop structure for each chain.

2.3 Closed loop structure

A closed loop robot is composed by B loops calculated by the following equation: $B = L - n$. Where: $n + 1$ is the number of the robot bodies and L is the number of motorized and non-motorized joints. Each closed loop system complies with the following conditions :

- This structure is compatible with the loop closure constraints.
- The number of motorized joints is equal to the number of degree of freedom of the robot.
- The motorized joints variables can determine the configuration of the robot.

To determine the KM of this robot type, we determined the KM of each separate chain and the loop closure equations:

Forward Kinematics Model (FKM) is obtained by determining the FKM of each separate loop of the robot. The geometric parameters of closed system structure are determined as the case of tree structure robot [10][11][12].

Loop Closure Equation is used to determine the passive angles depending on actives angles. Determining the equation of the closed loop structure is based on the following steps:

Step 1: One virtually cuts all closed loops on one of its joints (usually passive joint). The closed system becomes equivalent to a tree system.

Step 2: One numbers the body of the new system. Then, we place the frames. We determine the geometric parameters by applying the rules of the tree system.

Step3: One numbers joints cut from $n + 1$to L. In each cut, we add two confused frames (z_k and z_{k+B}), as shown in Figure 3. The frame R_k is attached to the body C_i. The other frame $R_{k+B}$is attached to the body C_j. With $C_i = C_{a(j)}$, ($a(j)$ is antecedent to i), x_k is carried by the common perpendicular to z_k and z_j, x_{k+B} is carried by the common perpendicular to z_{k+B} ($z_{k+B} = z_k$) and z_j.

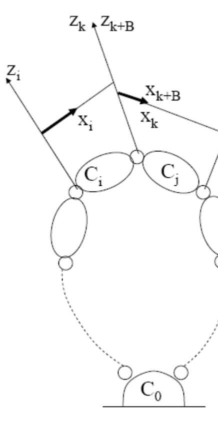


Figure 3 Closed loop structure

Step4: The determination of passive angles depending to active angles allows solving the equation of the closed loop:

$$T_{K+B}^{K} = I_4. \qquad (6)$$

We obtain twelve nonlinear equations for each loop. For a spatial loop, there will be six independent equations. For a flat loop, there will be three independent equations.

3. FORWARD KINEMATIC MODEL (FKM) OF THE PAR4

The Par4 robot, shown in Figure4, is a robot with four degrees of freedom dedicated to pick-and-place manipulation like Delta Robot (three translations and one rotation). It is composed of a movable platform and four identical kinematics chains. It is a very lightweight robot. It can rotate 360°. The Par4 is commercialized under the name Quattro [7] [11].

Figure 4 Quattro robot

Each chain of the Par4 is composed of one bare and two barrettes connected by one pivot joint and two ball joints.

In the literature, there are various methods for determining the forward kinematics model of the Par4 robot. In this part, we will use MDHM for closed structure robot.

We assume that one chain is composed by five pivots joint where the two pivots between bare and barrette and the two pivots between barrette and mobile platform have the same center. The kinematic schema of the main chain is given by Figure 5. The schema of the secondary chain is given by Figure 6 (chain3).

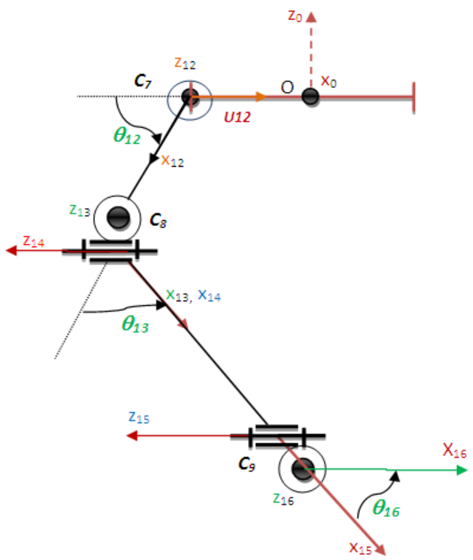


Figure 5 Parameter of the Par4

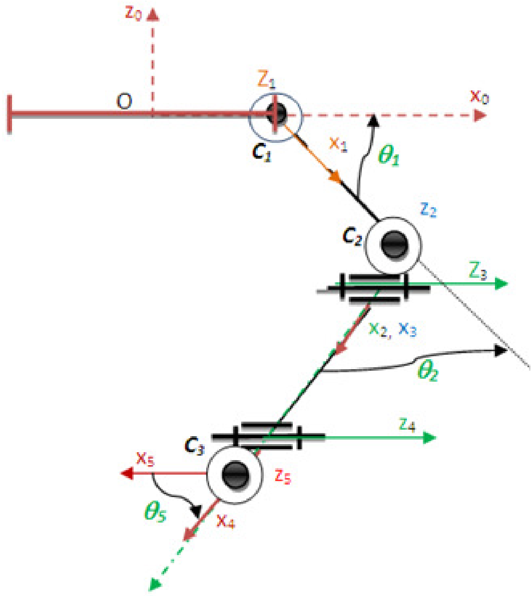


Figure 6 Parameters of arm3 of the Par 4

The parameters of MDHM of the main chain and the third chain are presented in TABLE I and TABLE II respectively.

TABLE I. PARAMETER OF THE FIRST ARM

I	**a(i)**	**σ_i**	**a_i**	**d_i**	**θ_i**	**r_i**
1	**0**	**0**	$\frac{\pi}{2}$	**R**	**$-\theta 1$**	**0**
2	**1**	**0**	**0**	**L**	**$-\theta 2$**	**0**
3	**2**	**0**	$-\frac{\pi}{2}$	**0**	**$\theta 3$**	**0**
4	**3**	**0**	**0**	**l**	**$\theta 4$**	**0**
5	**4**	**0**	$\frac{\pi}{2}$	**0**	**$-\theta 5$**	**0**
Pc	**5**	**0**	$\frac{\pi}{2}$	**r**	**$-\pi$**	**0**

TABLE II. PARAMETER OF THE THIRD ARM

i	a(i)	u_i	σ_i	∂_i	b_i	a_i	d_i	θ_i	r_i
12	0	1	0	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$	-R	$-\pi+\theta_{12}$	0
13	12	0	0	0	0	0	L	θ_{13}	0
14	13	0	0	0	0	$\frac{\pi}{2}$	0	θ_{14}	0
15	14	0	0	0	0	0	l	θ_{15}	0
16	15	0	0	0	0	$-\frac{\pi}{2}$	0	θ_{16}	0
Pc	16	$-\frac{\pi}{2}$	0	0	0	$-\frac{\pi}{2}$	r	$-\frac{\pi}{2}$	0

The Kinematic Model of arm1 and arm3 are given respectively by:

$$T_5^0 = T_1^0 . T_2^1 . T_3^2 . T_4^3 . T_5^4 \qquad (7)$$

$$T_{16}^{0} = T_{12}^{0}.T_{13}^{12}.T_{14}^{13}.T_{15}^{14}.T_{16}^{15} \quad (8)$$

We do the same work for chain 2 and chain 4.

The passage of the point P_c, center of mobile platform, to P is obtained by homogeneous transformation matrices (see Figure7).

<u>Loop closure equation</u>

Par4 has a closed structure with three identical loops. The tree equivalent structure is obtained by choosing a principal chain and cutting the three other chains to one of their passive joints (V_2, V_3, and V_4 as shown in Figure7).

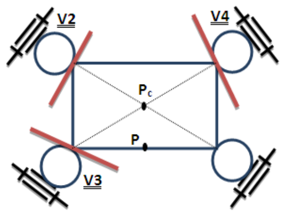


Figure 7 Virtual cuts in the base

At each cut, we add two identical frames R_{Ai} and R_{Bi}. The kinematic schema of one loop is given by the Figure8.To determine the passive angles depending to active angles; we solve the closed loop equation:

$$T_{Bi}^{Ai} = I4 \quad (8)$$

There are twelve equations to solve for each loop.

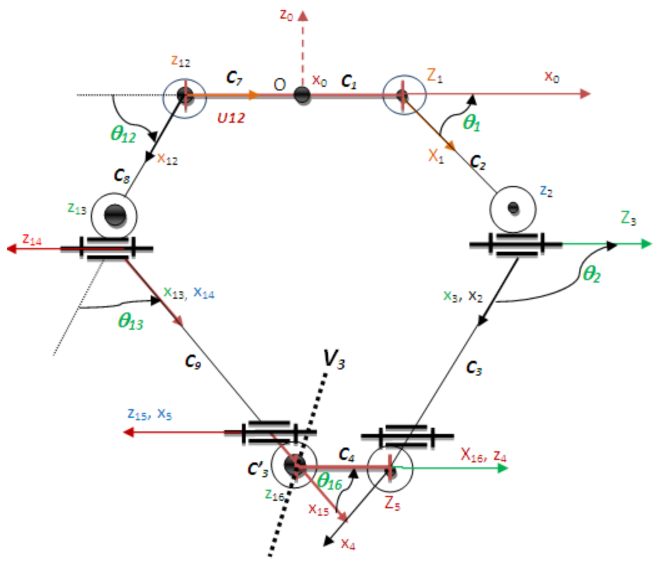


Figure 8 Robot arm with a cut V3 at point C'3

We determine also the loop closure equation for loop 1 and 2 and 1 and 4.

4. INVERSE KINEMATICS MODEL (IKM) OF THE PAR4

In this part, the IKM used for the verification of the FKM of Par4, will be explained. The IKM is used to describe the articulated variables according operational coordinates (positions and orientations).To determine the IKM, we base on the idea of Nabat [7] with some modifications. We changed the direction of the frames. We assumed, too, that d_i and h_i are negligible. We get the following figure. The IKM is obtained from the closing loop equations:

$\|A_i C'_i\|^2 = L_i^2$ i=1, 2, 3 or 4 (9)

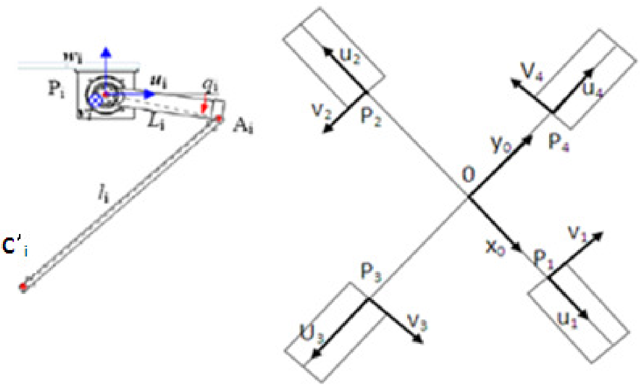


Figure 9 Arm of the Par4 and his fixed body.

$\alpha_1 = 0,\ \alpha_2 = \pi, \alpha_3 = \frac{3\pi}{2}, \alpha_4 = \frac{\pi}{2}$ (α_i is the angle between x_0 and U_i).

In the frame R_0, the coordinates of the points A ($A = [A_1 A_2 A_3 A_4]$) and P ($P = [P_1 P_2 P_3 P_4]$) are defined in full by:

$$A = P + \begin{bmatrix} L_1 Cq_1 & -L_2 Cq_2 & 0 & 0 \\ 0 & 0 & -L_3 Cq_3 & L_4 Cq_4 \\ -L_1 Sq_1 & -L_2 Sq_2 & -L_3 Sq_3 & -L_4 Sq_4 \end{bmatrix} \quad (10)$$

With: $$P = \begin{bmatrix} x_{p1} & x_{p2} & x_{p3} & x_{p4} \\ y_{p1} & y_{p2} & y_{p3} & y_{p4} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

The coordinates of points C' is given by the following expressions: $C' = [C'_1 C'_2 C'_3 C'_4]$ with:

$$C'_1 = \begin{bmatrix} x + \frac{d}{2}.C\frac{\pi}{4} \\ y + \frac{d}{2}.S\frac{\pi}{4} \\ z \end{bmatrix},\ C'_2 = \begin{bmatrix} x - \frac{d}{2}.C\frac{\pi}{4} \\ y - \frac{d}{2}.S\frac{\pi}{4} \\ z \end{bmatrix}$$

$$C'_2 = \begin{bmatrix} x + \left(-\frac{d}{2} - h.S\theta\right).C\frac{\pi}{4} - h.C\theta.S\frac{\pi}{4} \\ y + \left(-\frac{d}{2} - h.S\theta\right).S\frac{\pi}{4} + h.C\theta.C\frac{\pi}{4} \\ z \end{bmatrix} \quad (12)$$

$$C'_4 = \begin{bmatrix} x + \left(\frac{d}{2} - h.S\theta\right).C\frac{\pi}{4} - h.C\theta.S\frac{\pi}{4}) \\ y + \left(-\frac{d}{2} - h.S\theta\right).S\frac{\pi}{4} + h.C\theta.C\frac{\pi}{4}) \\ z \end{bmatrix}$$

h is the length of the mobile platform. The parameters of the Par4 are: $L = 0.8m;\ l = 0.350m;\ d = h = 0.1m$. From equation (9), we obtain:

$$I_i.Sq_i + J_i.Cq_i + K_i = 0 \quad (13)$$

With

$$I_i = 2zL_i \quad (14)$$

$$J_i = 2L_i.P_iC_i.y_0 \quad (15)$$

$$K_i = \|P_iC_i\|^2 - L_i^2 - l_i^2 \quad (16)$$

We set $t_i = \tan(\frac{q_i}{2})$.The trigonometric system becomes:

$$[K_i - J_i]t_i^2 + [2I_i]t_i + [J_i + K_i] = 0 \quad (17)$$

The resolution of this system gives,

$$q_i = 2\text{atan}\left(\frac{-I_i \pm \sqrt{\Delta_i}}{K_i - J_i}\right) \quad (18)$$

With: $\Delta_i = I_i^2 - K_i^2 + J_i^2$

This resolution is true if: $\Delta_i > 0$ and $K_i - J_i \neq 0$

5. Simulation Results

In order to verify our work, we set a desired trajectory of the point P (P_x^*, P_y^*, P_z^*) of the mobile platform. The desired trajectory is an ellipse with big diameter $0.6m$ along y and small diameter $0.2m$ along x. These verifications are based on the steps shown in Figure10.

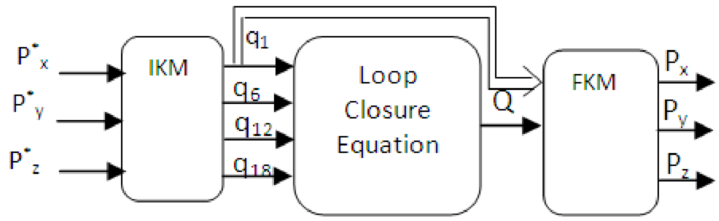
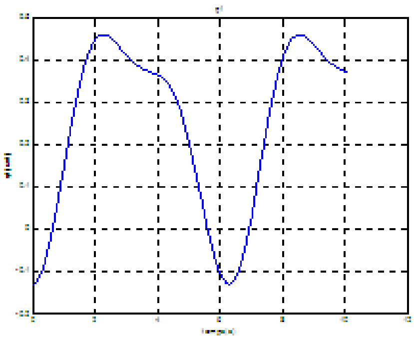


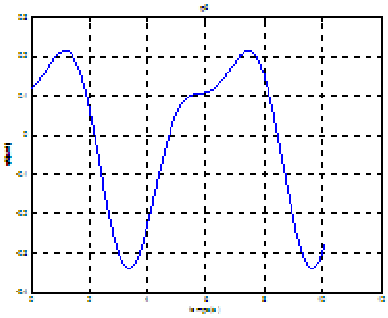
Figure 10 Verification of the forward path

With $Q = [q_2, q_3, q_4, q_5, q_7, q_8, q_9, q_{10}, q_{13}, q_{14}, q_{15}, q_{16}, q_{19}, q_{20}, q_{21}, q_{22}]$ are the non-motorized angles.

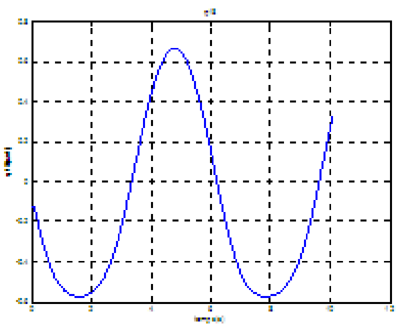
The motorized angles q_1, q_6, q_{12} and q_{18}, presented in Figure11, are given from the IKM corresponding to the desired trajectory of P.



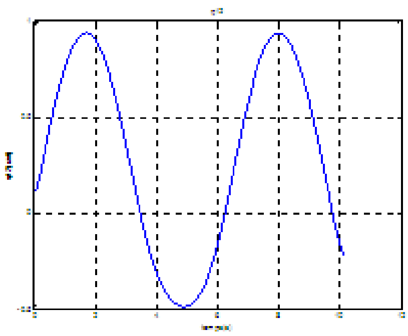
A. Evolution of angle q_1



b. Evolution of angle q_6



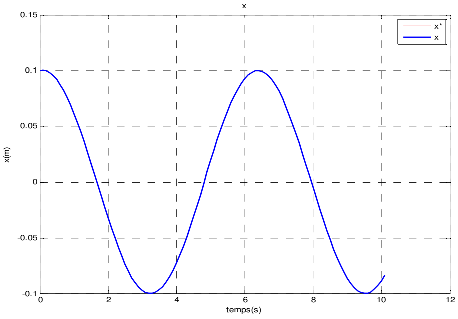
c. Evolution of angle q_{12}



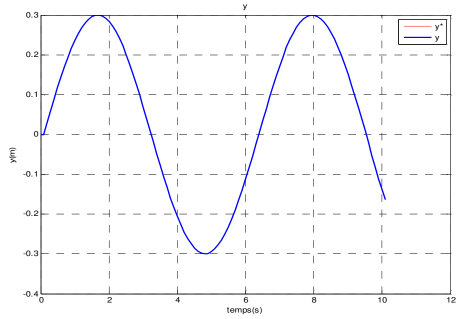
d.Evolution of angle q_{18}

Figure 11 Simulation of IKM

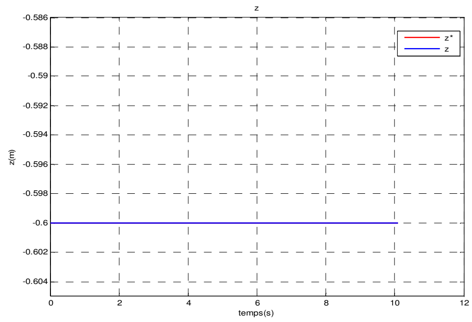
From Figure11, we notice that the angle q_1 and q_2 are respectively complementary asq_3 and q_4 angles. Then, we determined the non-motorized Par4 angles from the loop closure equation depending motorized angles. We obtain the Q angles. To verify our work, we solve the FKM of Par4 by using motorized and non-motorized joints. We obtain the final position of the mobile platform (P_x, P_y, P_z) as is shown in Figure12.



a. Evolution of position along x



b. Evolution of position along y



c. Evolution of position along z

Figure 12 Simulation of the FKM

Figure12 shows that the desired and calculated position coordinates of the mobile platform are almost identical.

The red trajectory is the desired trajectory. The blue is the calculated trajectory. Figure13 shows that these two trajectories, desired and obtained, are nearly coincident.

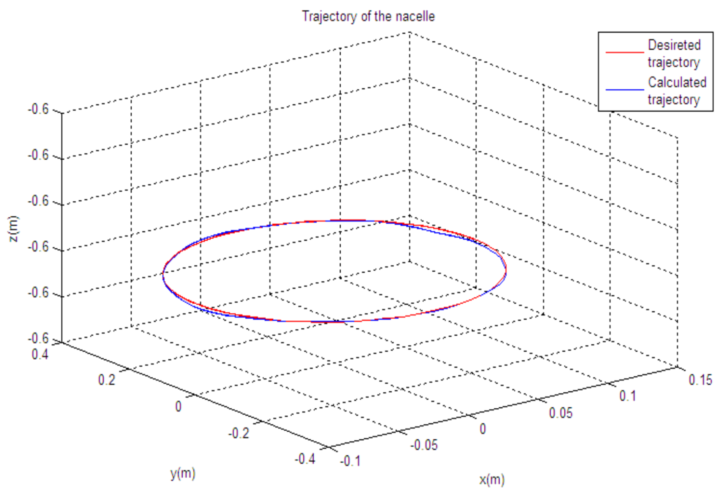


Figure 13 Trajectoire of the mobile platform

These simulations show that the determination of the FKM of Par4 is verified. This small difference between the two trajectories is caused by the static and dynamic deformation of the robot. This influence, caused by geometrical parameters, is very negligible.

6. Conclusion

The FKM of the parallel robot is a complex task. The most of the methods used to solve this problem are numerical methods that present some disadvantages. In this paper, we used the MDHM to determine the FKM of parallel robot Par4. It is an analytical method which is used in the literature to model all types of robots: serial or parallel robot. The principle of this method is to cut virtually some passive joints to obtain a tree structure robot and apply the MDHM for this structure. The closed loop equation is used to determine the passive joints useful for the calculation of the FKM of Par4. The IKM is obtained using a method based on the idea of Nabat. A simulation results are done to validate our work.

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