A Differential Evolution Algorithm for the Solution of Optimal Power Flow with Consideration of FACTS Devices "UPFC"

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Abstract_ **This paper present the solution of optimal power flow problems using a Differential Evolution algorithm (DE) with consideration of unified power flow controller (UPFC). Differential Evolution method is one of the best evolutionary algorithms for global optimization, it using three basic operations, namely: mutation, crossover, and selection operators. The objective of this paper is to minimize the total fuel cost of generation, total real power loss and also maintain an acceptable system performance in terms of limits on generator real power, bus voltages. In this work, the proposed method (DE) has been applied to solve the optimal power flow problems with and without installation of UPFC device on the standard IEEE 30-bus system.**

Index Terms_ **Optimal power flow, Differential evolution (DE), FACTS, Unified Power Flow Controller (UPFC).**

1. Introduction

The optimal power flow problem (OPF) has been one of the most usually studied subjects in the power system community [1]. He was first discussed by Carpentier in 1962 [2]. OPF problem has been generally considered as the minimization of an objective function representing the generation cost and the transmission loss. The constraints concerned are the physical laws governing the power generation-transmission systems and the operating limitations of the equipment.

The unified power flow controller (UPFC) is an advanced devices member of the group of Flexible Alternating Current Transmission Systems (FACTS) [3]. This device can control many parameters, so it is the combination of the properties of two FACTS: static synchronous compensator STATCOM and a static synchronous series compensator SSSC [4]. It is able to control all the parameters affecting power flow in the transmission line: voltage, impedance, and phase angle [5].

Differential evolution (DE) is a population-based algorithm with the generation-and-test feature for global optimization problems using real-valued parameters, It was invented by Price and Storn in 1995 [6]. Differential evolution improves a population of candidate solutions over several generations using three basic operations in order to reach an optimal solution, the operations are, mutation, crossover, and selection operators [7].

In this paper voltage source model of unified power flow controller is integrated in Newton Raphson algorithm in order to investigate the control of power flow using DE algorithm [8].the simulations has been implemented by the use of MATLAB software and have been tested on the IEEE 30-bus power system.

The rest of this paper is planned as follows. Section 2 describes the mathematical formulation of optimal power flow problem. Sections 3 present the mathematical equivalent model of UPFC. The DE algorithm is represented in section 4. Section 5 present results and simulation. The last section, Sect. 6, is devoted to conclusions and future work.

2. Problem Formulation

The mathematical formulation of optimal power flow problem can be written in the following from :

$$Minimize\ \left(F(x)\right) \tag{1}$$

$$Subject\ to:\ g(x) = 0 \tag{2}$$

$$h(x) \leq 0 \tag{3}$$

Where: $F(x)$ is the objective function, $g(x)$ is the equality constraints, $h(x)$ is the inequality constraints and x is the vector of control variables, It can be generated active and reactive power P_g, Q_g , generation bus magnitudesV_g, and transformers tap setting T, … etc.

$$x = \left[P_g, Q_g, V_g, T \ldots\right] \tag{4}$$

2.1 Objective function

In this paper, The OPF problem can be expressed as minimizing the cost of production of the real power which is given by a quadratic function of generator power output as [9,10].

$$F(x) = \sum_{i=1}^{ng} (A_i + B_i P_{Gi} + C_i P_{Gi}^2) \tag{5}$$

Where:
$F(x)$ is the fuel cost function.
A_i, B_i, C_i are the fuel cost coefficients.

i represent the corresponding generator (1,2,.....ng).
P_{G_i} is the generated active power at bus *i*.
ng is number of generators including the slack bus.

2.2 Equality and Inequality constraints

OPF constraint can be split into two parts: The equality and inequality constraint:

The equality constraints reflects the physics of the power system, equality constraints *g(x)* are the power flow equations describing bus injected active and reactive powers of the i^{th}bus, expressed as

$$P_{g_i} - P_{d_i} = V_i \sum_{j=1}^{N} V_j \left(g_{ij} \cos \delta_{ij} + z_{ij} \sin \delta_{ij}\right) \quad (6)$$

$$Q_{g_i} - Q_{d_i} = V_i \sum_{j=1}^{N} V_j \left(g_{ij} \sin \delta_{ij} + z_{ij} \cos \delta_{ij}\right) \quad (7)$$

Where P_{g_i}, Q_{g_i} are the active and the reactive power generation at bus i

P_{d_i}, Q_{d_i} are the real and the reactive power demand at bus *i*

V_i, V_j the voltage magnitude at bus i, j, respectively

g_{ij}, z_{ij} are the real and imaginary part of the admittance (*Yij*)

δij is the phase angle difference between buses *i* and *j* respectively

N is the total number of buses.

The inequality constraints reflect the limits on physical devices in the power system as well as the limits created to ensure system security that they are presented in the following inequalities:

- Limits on active power at generator buses:
  $P_{g_i}^{min} \leq P_{g_i} \leq P_{g_i}^{max}$ (8)
- Limits on reactive power at generator buses:
  $Q_{g_i}^{min} \leq Q_{g_i} \leq Q_{g_i}^{max}$ (9)
- Limits on voltage magnitude of at the all buses:
  $V_i^{min} \leq V_i \leq V_i^{max}$ (10)
- Limits on tap positions of a transformer :
  $T^{min} \leq T \leq T^{max}$ (11)
- Limits on the bus voltage phase angles:
  $\theta_i^{min} \leq \theta_i \leq \theta_i^{max}$ (12)
- Limits on transmission lines loading :
  $S_{li}^{min} \leq S_{li} \leq S_{li}^{max}$ (13)
- Limits on FACTS controllers :
  $X^{min} \leq X_{FACTS} \leq X^{max}$ (14)

3. Modeling of UPFC

The Unified Power Flow Controller (UPFC), which was proposed by L. Gyugyi in 1991 [11-13], is one of the most complex FACTS devices in a power system today. It is a multipurpose FACTS's device which allows simultaneous control of active and reactive power flow and the voltage magnitude at the UPFC terminals [14].

UPFC is modeled as a series reactance together with the dependent loads injected at each end of the series reactance. The model is simple and helpful in understanding the UPFC impact on the power system.
A simpler schematic representation of UPFC is shown in figure 1 with its equivalent circuit [15].

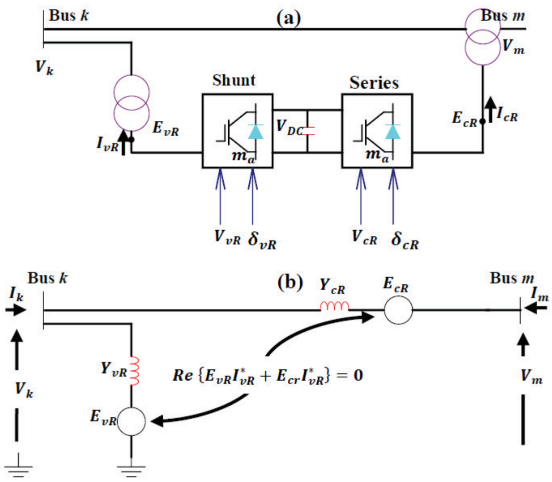

Fig.1. The UPFC equivalent circuit

The equivalent circuit of UPFC shown in Figure 1(b) consists of a shunt-connected voltage source, a series-connected voltage source, and an active power constraint equation, which links the two voltage sources. The two voltage sources are connected to the AC system through inductive reactance representing the VSC transformers.

The UPFC voltage sources E_{vR} and $E_{cR}$are [18]:

$$E_{vR} = V_{vr}(cos\, \delta_{vR} + j\, sin\, \delta_{vR}) \quad (15)$$

$$E_{cR} = V_{cr}(cos\, \delta_{cR} + j\, sin\, \delta_{cR}) \quad (16)$$

Where: V_{vr} and δ_{vR} are the controllable magnitude and phase angle of the voltage source representing the shunt converter respectively (equation (17),(18)).

$$V_{vR\, min} \leq V_{vR} \leq V_{vR\, max} \quad (17)$$

$$0 \leq \delta_{vR} \leq 2\pi \quad (18)$$

V_{cr} and δ_{cR} are the controllable magnitude and phase angle of the voltage source representing the series converter respectively (equation (19),(20)).

$$V_{cR\, min} \leq V_{cR} \leq V_{cR\, max} \quad (19)$$

$$0 \leq \delta_{cR} \leq 2\pi \quad (20)$$

The équation of the active power and reactive power at bus *k* and *m* are:

$$P_k = V_k^2 G_{kk} + V_k V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] + V_k V_{cR} [G_{km} \cos(\theta_k - \delta_{cR}) + B_{km} \sin(\theta_k - \delta_{cR})] + V_k V_{vR} [G_{vR} \cos(\theta_k - \delta_{vR}) + B_{vR} \sin(\theta_k - \delta_{vR})] \quad (21)$$

$$Q_k = -V_k^2 G_{kk} + V_k V_m [G_{km} \cos(\theta_k - \theta_m) - B_{km} \sin(\theta_k - \theta_m)] + V_k V_{cR} [G_{km} \cos(\theta_k - \delta_{cR}) - B_{km} \sin(\theta_k - \delta_{cR})] + V_k V_{vR} [G_{vR} \cos(\theta_k - \delta_{vR}) - B_{vR} \sin(\theta_k - \delta_{vR})] \quad (22)$$

$$P_m = V_m^2 G_{mm} + V_m V_k [G_{mk} \cos(\theta_m - \theta_k) + B_{mk} \sin(\theta_m - \theta_k)] + V_m V_{cR} [G_{mm} \cos(\theta_m - \delta_{cR}) + B_{mm} \sin(\theta_m - \delta_{cR})] \quad (23)$$

$$Q_m = -V_m^2 B_{mm} + V_m V_k [G_{mk} \cos(\theta_m - \theta_k) - B_{mk} \sin(\theta_m - \theta_k)] + V_m V_{cR} [G_{mm} \cos(\theta_m - \delta_{cR}) - B_{mm} \sin(\theta_m - \delta_{cR})] \quad (24)$$

The active and the reactive power of the series converter :

$$P_{cR} = V_{cR}^2 G_{mm} + V_{cR} V_k [G_{mk} \cos(\delta_{cR} - \theta_k) + B_{mk} \sin(\delta_{cR} - \theta_k)] + V_{cR} V_m [G_{mm} \cos(\delta_{cR} - \theta_m) + B_{mm} \sin(\delta_{cR} - \theta_m)] \quad (25)$$

$$Q_{cR} = -V_{cR}^2 B_{mm} + V_{cR} V_k [G_{mk} \cos(\delta_{cR} - \theta_k) - B_{mk} \sin(\delta_{cR} - \theta_k)] + V_{cR} V_m [G_{mm} \cos(\delta_{cR} - \theta_m) - B_{mm} \sin(\delta_{cR} - \theta_m)] \quad (26)$$

The active and the reactive power of the shunt converter :

$$P_{vR} = -V_{vR}^2 G_{vR} + V_{vR} V_k [G_{vR} \cos(\delta_{vR} - \theta_k) + B_{vk} \sin(\delta_{vR} - \theta_k)] \quad (27)$$

$$Q_{vR} = V_{vR}^2 B_{vR} + V_{vR} V_k [G_{vR} \sin(\delta_{vR} - \theta_k) - B_{vk} \cos(\delta_{vR} - \theta_k)] \quad (28)$$

4. Differential evolution

DE was first proposed by Storn and price at Berkely in 1995[6]. It is heuristic, population based search algorithm [16]. It is based on the concept of a population of individuals that evolve and improve their fitness through probabilistic operators like mutation. DE technique is a technically simple, population based evolutionary algorithm such as genetic algorithms using similar operators; crossover, mutation and selection, It using for minimizing non-linear and multi-modal objective functions.

- **Initialization**

The population is initialized by randomly generating individuals between the given constraints limits (equation (29))[17].

$$X_{ij}^0 = X_j^{min} + rand * \left(X_j^{max} - X_j^{min}\right) \qquad i = 1,2, \dots Np \quad \& \quad j = 1,2, \dots D \quad (29)$$

Where ; the *j*th variable of the given problem has its lower X_j^{min} and upper X_j^{max} bound. Np is the size of the population and D is the number of decision variables.

- **Mutation**

The mutation operator of DE creates new parameters into the population. This operation creates mutant vectors $X_i^{'k}$ by perturbing a randomly selected three vectors, X_a^k, X_b^k and X_c^k such that the indices i, r1, r2 and r3 are distinct at the th iteration equatuion (30) [16]

$$X_i'^k = X_a^k + Fx * \left(X_b^k - X_c^k\right) \qquad i = 1,2, \dots Np \quad (30)$$

Where ; the scaling factor is F, it used to control the amount of perturbation in the process (F∊ [0 2]) ,

- **Crossover**

Based on the mutant vector, the parent vector is mixed with the mutated vector to create a trial vector, Then to get the trail vector the crossing operation is used and the trail vector ui is given by:

$$X_{ij}^{''k} = \begin{cases} X_{ij}^{'k} & if\ rand\ j < Cr\ or\ j = randn \\ X_{ij}^k & otherwise, \end{cases} \quad (31)$$

Where, i = 1, 2, 3 , . . ., Np; j = 1, . . ., ,D. $X_{ij}^k, X_{ij}^{'k}$ *and* $X_{ij}^{''k}$ are jth individual of ith target vector, mutant vector, and trial vector at kth iteration respectively. Cr e [0, 1] is the Crossover constant [18].

- **Selection**

Selection process is used among the set of trial vector and the updated target vector to choose the best. At last the fitness of the vector X_i^k and X_i^{k+1} is compared, and the best is chosen to generate offspring through greedy selection, that is:

$$X_i^{k+1} = \begin{cases} X_i^{''k} & iff(X_i^{''k}) \le f(X_i^{''k}) \\ X_i^k & otherwise, \end{cases} \quad (32)$$

i=1,2,… Np

The selection operator is repeated for both pair of target/trial vector until the new population is completed. The pseudo-code of the DE algorithm is shown as [16] :

The Pseudo-code of the DE algorithm

```
Generate the initial population P
Evaluate the fitness for each individual in P
while The termination criterion is not satisfied
    for i = 1 to NP
        Select uniform randomly r1 ≠ r2 ≠ r3 ≠ i
        j_rand = randint(1, D)
        for j = 1 to D do
            if randj (0, 1) > CR or j == j_rand
                U_i(j) = X_r1(j) + F (X_r2(j) − X_r3(j))
            else
                U_i(j) = X_i(j)
            end
        end
    end
for i = 1 to NP
    Evaluate the offspring U_i
    if U_i is better than P_i
        P_i = U_i
    end
    end
end
```

- **Algorithm of DE applied to OPF**

In the optimal power flow problem each vector in the Differential evolution population represent a candidate solution. The vector of that solution consist the output power generated by a generation unit.

Evaluation function: DE algorithm searches for the optimal solution by the maximization of the fitness function, and for that reason an evaluation function which provides a determine of the quality of the problem solution must be given. The objective is to minimize the total cost of generation while satisfying the equality constraints by running the algorithm of Newton Raphson power flow and the inequality constraints by adding a quadratic penalty function.(Fig.2)

5. Application Study

The OPF with UPFC device using Differential evolution approach has been developed and implemented by the use of Matlab 9. The applicability and validity of this method (DE) have been tested on IEEE 30-bus system. This system consist of 6 generators (n°:1, 2, 5, 8, 11 and 13), 41 transmission lines and 4 transformers (Figure 3).

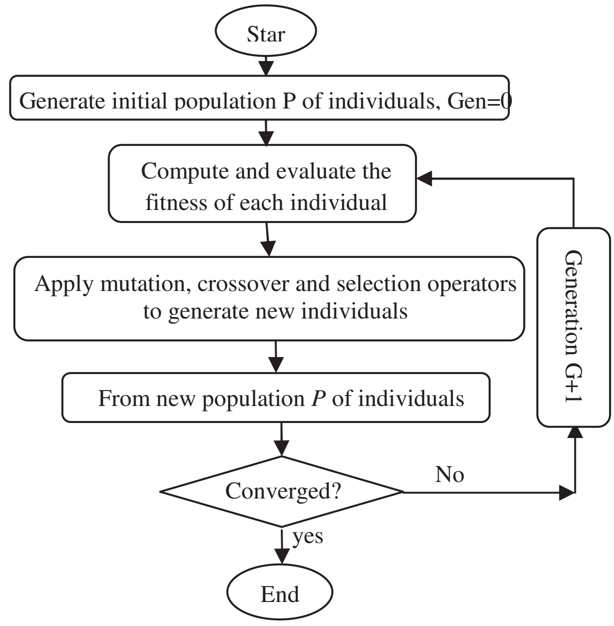

Fig.2 optimal power flow using DE

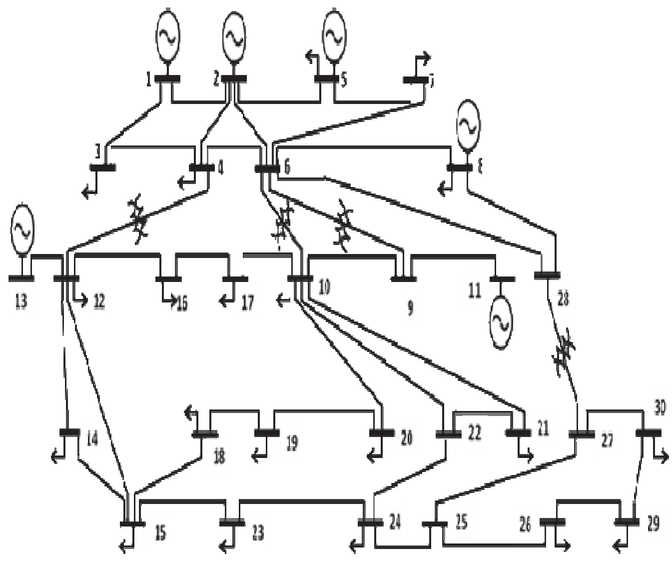

Fig. 3. Topology of the IEEE 30-Bus electrical network

To demonstrate the effectiveness of the proposed approach two cases to be discussed:

Case 1: represent the solution of optimal power flow using DE without UPFC device installed.
In this case the vector of control variables include only the generated active power (P_{gi}).

Case 2: represent the solution of optimal power flow using DE with one UPFC device is installed.
The vector of control variables include the generated active power (P_{gi}), the shunt and the series voltage source of UPFC (V_{cR}, V_{vR})

$$x = [P_{g_2}, P_{g_5}, P_{g_8}, P_{g_{11}}, P_{g_{13}}, V_{vr}, V_{cr}] \qquad (33)$$

The upper and lower bounds on the the shunt and the series voltage source of UPFC are set as

$0.95 \leq V_{vr} \leq 1.10$ pu. & $0.95 \leq V_{cr} \leq 1.10$ pu.

The control parameters of UPFC are showed in Table 1.

TABLE 1.
Parameters of UPFC

	NUPFC	Xcr (pu)	Xvr (pu)	Qmax (MVar)	Qmin (MVar)
UPFC	1	0.1	0.1	35	-35

The active power generating limits and the unit costs of all generators of the IEEE 30-bus test system are presented in Table 2 [19].

The total active load in the system was 283.4 MW, the upper and lower bounds on the of generator buses and load buses are set between 0,9 & 1,1 pu and The phase angles are set between -14 °& 0°.

TABLE 2
Power generation limits and cost coefficients for IEEE 30-bus system

Pg_i (MW)	$Pg_{i(min)}$ (MW)	$Pg_{i(max)}$ (MW)	A_i ($/h)	$B_i.10^{-2}$ ($/MWh)	$C_i.10^{-4}$ ($/MW²h)
Pg1	50	200	0.00	200	37.5
Pg2	20	80	0.00	175	175.0
Pg5	15	50	0.00	100	625.0
Pg8	10	35	0.00	325	83.0
Pg11	10	30	0.00	300	250.0
Pg13	12	40	0.00	300	250.0

The DE parameters are set as:

TABLE 3
DE parameters

DE parameters	
Population size NP	20
Maximum number of generations Gmax :	100
Crossover constant CR	0.5
Weighting factor F	0.9

The proposed approach with FACTS devices is applied on IEEE 30 Bus system. UPFC devices are placed at the bus 24 (between bus 24 and bus 25) system performance is observed with and without UPFC device. Table 4 shows the results of the total cost of generation, active power and total losses obtained by DE with and without installation of UPFC. Fig. 4 present the typical convergence characteristics for the best solutions of the minimum fuel cost obtained for each generation with and without UPFC. The results of the voltage magnitude of all buses are shown in fig.5

From TABLE 4 we observe that power loss as well as operational cost reduced significantly in all cases of loading with UPFC device as compared to without such device (9.2504 MW compared to 9.6073 MW) and (801.5290$/hr compared to 802.8047$/hr). Fig. 4 show that the convergence of fuel cost with UPFC is fast and better than without UPFC. From fig.5 the voltage profile before and after UPFC is clearly identified that all voltage magnitude profiles are within the constraint limit and are ameliorated.

TABLE 4
Comparison of results obtained by DE-OPF with/without UPFC

	Min	OPF-DE Without UPFC	OPF-DE With UPFC	Max
Pg1 (MW)	50	176.3184	176.3803	200
Pg2 (MW)	20	48.8386	48.6344	80
Pg5 (MW)	15	21.5388	21.1435	50
Pg8 (MW)	10	22.0872	22.5892	35
Pg11 (MW)	10	12.2363	12.5488	30
Pg13 (MW)	12	12.0000	11.3543	40
Cr		0.5100	0.5100	
F		1.9627	1.9627	
Vcr		-	0.1115	
Vvr		-	0.9197	
Generation cost ($/h)	-	802.8047	**801.5290**	-
Active power losses (MW)	-	9.6073	**9.2504**	-
UPFC	-		**24-25**	-

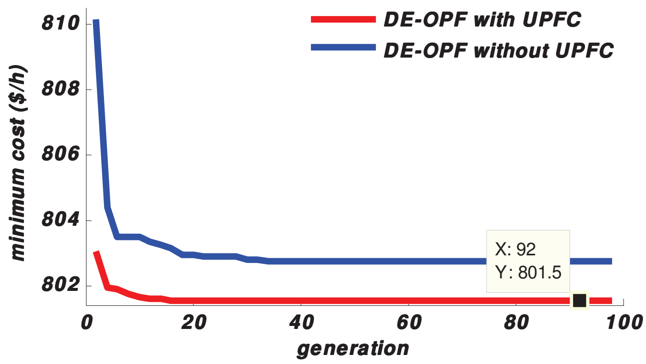

Fig. 4. The convergence profile of DE-OPF with/without UPFC for IEEE 30-bus system

6. Conclusion

In this paper, a new stochastic optimization algorithm is a differential evolution has been presented to solve the optimal power flow problem with consideration of FACTS device (UPFC). The effectiveness of DE method was demonstrated and tested with IEEE 30 bus system. It is clearly evident from the results that effective installation of UPFC devices by using suitable globally acceptable optimization technique like DE can significantly improve system performance.

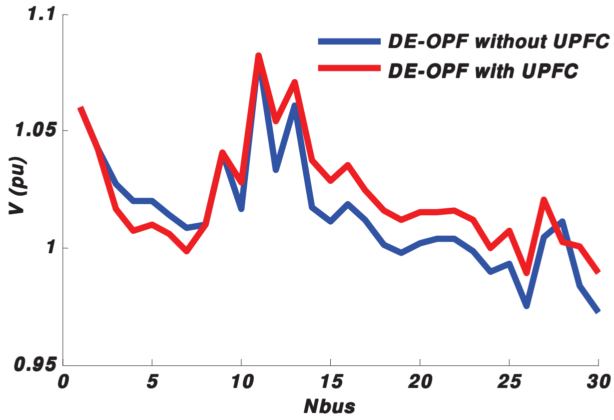

Fig.5. Voltage profile of all buses for IEEE 30-Bus with & without UPFC

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