Abstract_ This paper present the solution of optimal power flow problems using a Differential Evolution algorithm (DE) with consideration of unified power flow controller (UPFC). Differential Evolution method is one of the best evolutionary algorithms for global optimization, it using three basic operations, namely: mutation, crossover, and selection operators. The objective of this paper is to minimize the total fuel cost of generation, total real power loss and also maintain an acceptable system performance in terms of limits on generator real power, bus voltages. In this work, the proposed method (DE) has been applied to solve the optimal power flow problems with and without installation of UPFC device on the standard IEEE 30-bus system.

Index Terms_ Optimal power flow, Differential evolution (DE), FACTS, Unified Power Flow Controller (UPFC).

1. Introduction

The optimal power flow problem (OPF) has been one of the most usually studied subjects in the power system community [1]. He was first discussed by Carpenter in 1962 [2]. OPF problem has been generally considered as the minimization of an objective function representing the generation cost and the transmission loss. The constraints concerned are the physical laws governing the power generation-transmission systems and the operating limitations of the equipment.

The unified power flow controller (UPFC) is an advanced devices member of the group of Flexible Alternating Current Transmission Systems (FACTS) [3]. This device can control many parameters, so it is the combination of the properties of two FACTS: static synchronous compensator STATCOM and a static synchronous series compensator SSSC [4]. It is able to control all the parameters affecting power flow in the transmission line: voltage, impedance, and phase angle [5].

Differential evolution (DE) is a population-based algorithm with the generation-and-test feature for global optimization problems using real-valued parameters, It was invented by Price and Storn in 1995 [6]. Differential evolution improves a population of candidate solutions over several generations using three basic operations in order to reach an optimal solution, the operations are, mutation, crossover, and selection operators [7].

In this paper voltage source model of unified power flow controller is integrated in Newton Raphson algorithm in order to investigate the control of power flow using DE algorithm [8] the simulations has been implemented by the use of MATLAB software and have been tested on the IEEE 30-bus power system.

The rest of this paper is planned as follows. Section 2 describes the mathematical formulation of optimal power flow problem. Sections 3 present the mathematical equivalent model of UPFC. The DE algorithm is represented in section 4. Section 5 present results and simulation. The last section, Sect. 6, is devoted to conclusions and future work.

2. Problem Formulation

The mathematical formulation of optimal power flow problem can be written in the following from :

\[
\begin{align*}
\text{Minimize } & F(x) \\
\text{Subject to } & g(x) = 0 \\
& h(x) \leq 0
\end{align*}
\]

Where: \( F(x) \) is the objective function, \( g(x) \) is the equality constraints, \( h(x) \) is the inequality constraints and \( x \) is the vector of control variables, It can be generated active and reactive power \( P_g, Q_g \), generation bus magnitudes \( V_g \), and transformers tap setting \( T \), … etc.

\[
x = [p_g, q_g, v_g, t, \ldots ]
\]

2.1 Objective function

In this paper, The OPF problem can be expressed as minimizing the cost of production of the real power which is given by a quadratic function of generator power output as [9, 10].

\[
F(x) = \sum_{i=1}^{n_g} (A_i + B_i P_{gi} + C_i P_{gi}^2)
\]

Where:

\( F(x) \) is the fuel cost function. 
\( A_i, B_i, C_i \) are the fuel cost coefficients.
where $i$ represent the corresponding generator $(1, 2, ..., n_g)$. $P_{g_i}$ is the generated active power at bus $i$. $n_g$ is number of generators including the slack bus.

2.2 Equality and Inequality constraints

OPF constraint can be split into two parts: The equality and inequality constraint:

The equality constraints reflect the physics of the power system, equality constraints $g(x)$ are the power flow equations describing bus injected active and reactive powers of the $i^{th}$ bus, expressed as

$$P_{g_i} - P_{d_i} = V_i \sum_{j=1}^{N} V_j (g_{ij} \cos \delta_{ij} + \sin \delta_{ij})$$

$$Q_{g_i} - Q_{d_i} = V_i \sum_{j=1}^{N} V_j (g_{ij} \sin \delta_{ij} + \cos \delta_{ij})$$

Where $P_{g_i}, Q_{g_i}$ are the active and reactive power generation at bus $i$.

$P_{d_i}, Q_{d_i}$ are the real and reactive power demand at bus $i$.

$V_i, V_j$ the voltage magnitude at bus $i,j$ respectively.

$g_{ij}, z_{ij}$ are the real and imaginary part of the admittance ($Y_{ij}$).

$\delta_{ij}$ is the phase angle difference between buses $i$ and $j$ respectively.

$N$ is the total number of buses.

The inequality constraints reflect the limits on physical devices in the power system as well as the limits created to ensure system security that they are presented in the following inequalities:

- Limits on active power at generator buses:
  $$P_{g_i} \text{ min} \leq P_{g_i} \leq P_{g_i} \text{ max}$$

- Limits on reactive power at generator buses:
  $$Q_{g_i} \text{ min} \leq Q_{g_i} \leq Q_{g_i} \text{ max}$$

- Limits on voltage magnitude of at the all buses:
  $$V_i \text{ min} \leq V_i \leq V_i \text{ max}$$

- Limits on tap positions of a transformer:
  $$T \text{ min} \leq T \leq T \text{ max}$$

- Limits on the bus voltage phase angles:
  $$\theta_i \text{ min} \leq \theta_i \leq \theta_i \text{ max}$$

- Limits on transmission lines loading:
  $$S_{ij} \text{ min} \leq S_{ij} \leq S_{ij} \text{ max}$$

- Limits on FACTS controllers:
  $$X_{\text{min}} \leq X_{\text{FACTS}} \leq X_{\text{max}}$$

3. Modeling of UPFC

The Unified Power Flow Controller (UPFC), which was proposed by L. Gyugyi in 1991 [11-13], is one of the most complex FACTS devices in a power system today. It is a multipurpose FACTS’s device which allows simultaneous control of active and reactive power flow and the voltage magnitude at the UPFC terminals [14].

UPFC is modeled as a series reactance together with the dependent loads injected at each end of the series reactance. The model is simple and helpful in understanding the UPFC impact on the power system. A simpler schematic representation of UPFC is shown in figure 1 with its equivalent circuit [15].

![UPFC equivalent circuit](image)

The equivalent circuit of UPFC shown in Figure 1(b) consists of a shunt-connected voltage source, a series-connected voltage source, and an active power constraint equation, which links the two voltage sources. The two voltage sources are connected to the AC system through inductive reactance representing the VSC transformers. The UPFC voltage sources $E_{vR}$ and $E_{vC}$ are [18]:

$$E_{vR} = V_{vR} (\cos \delta_{vR} + j \sin \delta_{vR})$$

$$E_{vC} = V_{vC} (\cos \delta_{vC} + j \sin \delta_{vC})$$

Where: $V_{vR}$ and $\delta_{vR}$ are the controllable magnitude and phase angle of the voltage source representing the shunt converter respectively (equation (17),(18)).

$$V_{vR} \text{ min} \leq V_{vR} \leq V_{vR} \text{ max}$$

$$0 \leq \delta_{vR} \leq 2\pi$$

$V_{vC}$ and $\delta_{vC}$ are the controllable magnitude and phase angle of the voltage source representing the series converter respectively (equation (19),(20)).

$$V_{vC} \text{ min} \leq V_{vC} \leq V_{vC} \text{ max}$$

$$0 \leq \delta_{vC} \leq 2\pi$$
The equation of the active power and reactive power at bus $k$ and $m$ are:

$$
P_k = V_k^2 G_{kk} + V_k V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] + V_k V_m [G_{km} \cos(\theta_k - \delta_{CR}) + B_{km} \sin(\theta_k - \delta_{CR})] + V_k V_m [G_{GR} \cos(\theta_k - \delta_{GR}) + B_{GR} \sin(\theta_k - \delta_{GR})]
$$

(21)

$$
Q_k = -V_k^2 G_{kk} + V_k V_m [G_{km} \cos(\theta_k - \theta_m) - B_{km} \sin(\theta_k - \theta_m)] + V_k V_m [G_{km} \cos(\theta_k - \delta_{CR}) - B_{km} \sin(\theta_k - \delta_{CR})] + V_k V_m [G_{GR} \cos(\theta_k - \delta_{GR}) - B_{GR} \sin(\theta_k - \delta_{GR})]
$$

(22)

$$
P_m = V_m^2 G_{mm} + V_m V_k [G_{mk} \cos(\theta_m - \theta_k) + B_{mk} \sin(\theta_m - \theta_k)] + V_m V_k [G_{mm} \cos(\theta_m - \delta_{CR}) + B_{mm} \sin(\theta_m - \delta_{CR})] + V_m V_k [G_{mm} \cos(\theta_m - \delta_{CR}) - B_{mm} \sin(\theta_m - \delta_{CR})]
$$

(23)

$$
Q_m = -V_m^2 B_{mm} + V_m V_k [G_{mk} \cos(\theta_m - \theta_k) - B_{mk} \sin(\theta_m - \theta_k)] + V_m V_k [G_{mm} \cos(\theta_m - \delta_{CR}) - B_{mm} \sin(\theta_m - \delta_{CR})] + V_m V_k [G_{mm} \cos(\theta_m - \delta_{CR}) - B_{mm} \sin(\theta_m - \delta_{CR})]
$$

(24)

The active and the reactive power of the series converter:

$$
P_{cr} = V_c^2 G_{mm} + V_c V_k [G_{mk} \cos(\delta_{CR} - \theta_k) + B_{mk} \sin(\delta_{CR} - \theta_k)] + V_c V_k [G_{mm} \cos(\delta_{CR} - \theta_m) + B_{mm} \sin(\delta_{CR} - \theta_m)] + V_c V_k [G_{mm} \cos(\delta_{CR} - \theta_m) - B_{mm} \sin(\delta_{CR} - \theta_m)]
$$

(25)

$$
Q_{cr} = -V_c^2 B_{mm} + V_c V_k [G_{mk} \cos(\delta_{CR} - \theta_k) - B_{mk} \sin(\delta_{CR} - \theta_k)] + V_c V_k [G_{mm} \cos(\delta_{CR} - \theta_m) - B_{mm} \sin(\delta_{CR} - \theta_m)] + V_c V_k [G_{mm} \cos(\delta_{CR} - \theta_m) - B_{mm} \sin(\delta_{CR} - \theta_m)]
$$

(26)

The active and the reactive power of the shunt converter:

$$
P_{vr} = -V_v^2 G_{VR} + V_v V_k [G_{VR} \cos(\delta_{VR} - \theta_k) + B_{VR} \sin(\delta_{VR} - \theta_k)] + V_v V_k [G_{VR} \cos(\delta_{VR} - \theta_k) - B_{VR} \sin(\delta_{VR} - \theta_k)]
$$

(27)

$$
Q_{vr} = V_v^2 B_{VR} + V_v V_k [G_{VR} \sin(\delta_{VR} - \theta_k)] + V_v V_k [G_{VR} \sin(\delta_{VR} - \theta_k)]
$$

(28)

4. Differential evolution

DE was first proposed by Storn and price at Berkely in 1995[6]. It is heuristic, population based search algorithm [16]. It is based on the concept of a population of individuals that evolve and improve their fitness through probabilistic operators like mutation. DE technique is a technically simple, population based evolutionary algorithm such as genetic algorithms using similar operators; crossover, mutation and selection, It using for minimizing non-linear and multimodal objective functions.

- **Initialization**

The population is initialized by randomly generating individuals between the given constraints limits (equation (29) [17].

$$
X_{ij}^0 = X_{ij}^{min} + rand \ast (X_{ij}^{max} - X_{ij}^{min})
$$

$i = 1,2, \ldots Np$ \& $j = 1,2, \ldots D$

(29)

Where $r_{ij}$ the $j$th variable of the given problem has its lower $X_{ij}^{min}$ and upper $X_{ij}^{max}$ bound. Np is the size of the population and D is the number of decision variables.

- **Mutation**

The mutation operator of DE creates new parameters into the population. This operation creates mutant vectors $X_{ij}^k$ by perturbing a randomly selected three vectors, $X_{ij}^r$, $X_{ij}^k$ and $X_{ij}^v$ such that the indices $i$, $r1$, $r2$ and $r3$ are distinct at the $th$ iteration equatuion (30) [16]

$$
X_{ij}^k = X_{ij}^r + F \ast (X_{ij}^v - X_{ij}^k)
$$

$i = 1,2, \ldots Np$

(30)

Where $F$ the scaling factor is, it used to control the amount of perturbation in the process (Fc [0 2]),

- **Crossover**

Based on the mutant vector, the parent vector is mixed with the mutated vector to create a trial vector. Then to get the trail vector the crossing operation is used and the trail vector $ui$ is given by:

$$
X_{ij}^k = \begin{cases} 
X_{ij}^k \text{if } rand < Cr \text{ or } j = randn \\
X_{ij}^r \text{otherwise}
\end{cases}
$$

(31)

Where, $i = 1, 2, 3, \ldots, Np$; $j = 1, \ldots, D$. $X_{ij}^k$, $X_{ij}^v$ and $X_{ij}^r$ are $j$th individual of $i$th target vector, mutant vector, and trial vector at $k$th iteration respectively, $Cr \in [0, 1]$ is the Crossover constant [18].

- **Selection**

Selection process is used among the set of trial vector and the updated target vector to choose the best. At last the fitness of the vector $X_{ij}^k$ and $X_{ij}^{k+1}$ is compared, and the best is chosen to generate offspring through greedy selection, that is:

$$
X_{ij}^{k+1} = \begin{cases} 
X_{ij}^k \text{if } f(X_{ij}^k) \leq f(X_{ij}^v) \\
X_{ij}^v \text{otherwise}
\end{cases}
$$

(32)

$i = 1, 2, \ldots, Np$

The selection operator is repeated for both pair of target/trial vector until the new population is completed. The pseudo-code of the DE algorithm is shown as [16]:

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**References**


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(309) **Editors:** Tarek Bouktir & Rafik Neji
The Pseudo-code of the DE algorithm

1: Generate the initial population P
2: Evaluate the fitness for each individual in P
3: while The termination criterion is not satisfied
   4: for i = 1 to NP
      5: Select uniform randomly \( r_1 \neq r_2 \neq r_3 \neq i \)
      6: \( j_{rand} = \text{randint}(1, D) \)
      7: for j = 1 to D do
         8: if rand(0, 1) > CR or j == \( j_{rand} \)
            9: \( U_i(j) = X_i(j) + F (X_{r_1}(j) - X_{r_3}(j)) \)
         10: else
            11: \( U_i(j) = X_i(j) \)
      12: end
      13: end
   14: end
5: for i = 1 to NP
6: Evaluate the offspring \( U_i \)
7: if \( U_i \) is better than \( P_i \)
8: \( P_i = U_i \)
9: end
10: end
11: end

• Algorithm of DE applied to OPF

In the optimal power flow problem each vector in the Differential evolution population represent a candidate solution. The vector of that solution consist the output power generated by a generation unit.

Evaluation function: DE algorithm searches for the optimal solution by the maximization of the fitness function, and for that reason an evaluation function which provides a determine of the quality of the problem solution must be given. The objective is to minimize the total cost of generation while satisfying the equality constraints by running the algorithm of Newton Raphson power flow and the inequality constraints by adding a quadratic penalty function (Fig.2).

5. Application Study

The OPF with UPFC device using Differential evolution approach has been developed and implemented by the use of Matlab 9. The applicability and validity of this method (DE) have been tested on IEEE 30-bus system. This system consist of 6 generators (\( n^*: 1, 2, 5, 8, 11 \) and 13), 41 transmission lines and 4 transformers (Figure 3).

To demonstrate the effectiveness of the proposed approach two cases to be discussed:

Case 1: represent the solution of optimal power flow using DE without UPFC device installed.
In this case the vector of control variables include only the generated active power \( (P_{gi}) \).

Case 2: represent the solution of optimal power flow using DE with one UPFC device is installed.
The vector of control variables include the generated active power \( (P_{gi}) \), the shunt and the series voltage source of UPFC \( (V_{cr}, V_{sr}) \).
\[ x = [P_{g2}, P_{g5}, P_{g6}, P_{g11}, P_{g13}, V_{tr}, V_{cr}] \]  

The upper and lower bounds on the shunt and the series voltage source of UPFC are set as

\[ 0.95 \leq V_{tr} \leq 1.10 \text{ pu.} \quad \& \quad 0.95 \leq V_{cr} \leq 1.10 \text{ pu.} \]

The control parameters of UPFC are showed in Table 1.

<table>
<thead>
<tr>
<th>Parameters of UPFC</th>
<th>NUPFC</th>
<th>Xcr (pu)</th>
<th>Xvr (pu)</th>
<th>Qmax (MVar)</th>
<th>Qmin (MVar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UPFC</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>35</td>
<td>-35</td>
</tr>
</tbody>
</table>

The active power generating limits and the unit costs of all generators of the IEEE 30-bus test system are presented in Table 2 [19].

The total active load in the system was 283.4 MW, the upper and lower bounds on the of generator buses and load buses are set between 0.9 & 1.1 pu and The phase angles are set between -14° & 0°.

<table>
<thead>
<tr>
<th>Parameters of UPFC</th>
<th>NUPFC</th>
<th>Xcr (pu)</th>
<th>Xvr (pu)</th>
<th>Qmax (MVar)</th>
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<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>35</td>
<td>-35</td>
</tr>
</tbody>
</table>

The DE parameters are set as:

<table>
<thead>
<tr>
<th>DE parameters</th>
<th>Population size NP</th>
<th>Maximum number of generations Gmax :</th>
<th>Crossover constant CR</th>
<th>Weighting factor F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>100</td>
<td>0.5</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The proposed approach with FACTS devices is applied on IEEE 30 Bus system. UPFC devices are placed at the bus 24 (between bus 24 and bus 25) system performance is observed with and without UPFC device. Table 4 shows the results of the total cost of generation, active power and total losses obtained by DE with and without installation of UPFC. Fig. 4 present the typical convergence characteristics for the best solutions of the minimum fuel cost obtained for each generation with and without UPFC. The results of the voltage magnitude of all buses are shown in fig.5.

From Table 4 we observe that power loss as well as operational cost reduced significantly in all cases of loading with UPFC device as compared to without such device (9.2504 MW compared to 9.6073 MW) and (801.5290$/hr compared to 802.8047$/hr). Fig. 4 show that the convergence of fuel cost with UPFC is fast and better than without UPFC. From fig.5 the voltage profile before and after UPFC is clearly identified that all voltage magnitude profiles are within the constraint limit and are ameliorated.

<table>
<thead>
<tr>
<th>Comparison of results obtained by DE-OPF with/without UPFC</th>
<th>Min OPF-DE Without UPFC</th>
<th>OPF-DE With UPFC</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pg1 (MW)</td>
<td>176.3184</td>
<td>176.3803</td>
<td>200</td>
</tr>
<tr>
<td>Pg2 (MW)</td>
<td>48.8386</td>
<td>48.6344</td>
<td>80</td>
</tr>
<tr>
<td>Pg5 (MW)</td>
<td>21.5388</td>
<td>21.1435</td>
<td>50</td>
</tr>
<tr>
<td>Pg8 (MW)</td>
<td>22.0872</td>
<td>22.5892</td>
<td>35</td>
</tr>
<tr>
<td>Pg11 (MW)</td>
<td>12.2363</td>
<td>12.5488</td>
<td>30</td>
</tr>
<tr>
<td>Pg13 (MW)</td>
<td>12.0000</td>
<td>11.3543</td>
<td>40</td>
</tr>
<tr>
<td>Cr</td>
<td>0.5100</td>
<td>0.5100</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>1.9627</td>
<td>1.9627</td>
<td></td>
</tr>
<tr>
<td>Vcr</td>
<td>-</td>
<td>0.1115</td>
<td></td>
</tr>
<tr>
<td>Vvr</td>
<td>-</td>
<td>0.9197</td>
<td></td>
</tr>
<tr>
<td>Generation cost ($/h)</td>
<td>802.8047</td>
<td>801.5290</td>
<td></td>
</tr>
<tr>
<td>Active power losses (MW)</td>
<td>9.6073</td>
<td>9.2504</td>
<td></td>
</tr>
<tr>
<td>UPFC</td>
<td>24-25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Conclusion

In this paper, a new stochastic optimization algorithm is a differential evolution has been presented to solve the optimal power flow problem with consideration of FACTS device (UPFC). The effectiveness of DE method was demonstrated and tested with IEEE 30 bus system. It is clearly evident from the results that effective installation of UPFC devices by using suitable globally acceptable optimization technique like DE can significantly improve system performance.
7. References


