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Virtual Reference Model and Disturbance Rejection based Fuzzy Tracking Control for a PMSM

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***Abstract*—This paper develops a new fuzzy tracking control method using Takagi-Sugeno (T-S) fuzzy models and virtual reference model integrating disturbance estimation, with an application to permanent magnet synchronous machine (PMSM). The major contribution of the proposed method is to consider the load disturbance as a part of the reference model to compensate completely its effect on the PMSM. The design procedure can be summarized in three stages: i) construct the T-S fuzzy controller and calculate its matrix gains by solving a set of linear matrix inequalities (LMIs). ii) construct the nonlinear tracking controller and reference model based on the concept of virtual desired variables (VDVs). The reference model is designed according to the desired speed and load torque which is considered to be an unknown disturbance. iii) construct the disturbance observer using fuzzy sliding mode observer (FSMO), and calculate its matrix gains by solving a set of LMIs. To show the validity and effectiveness of the proposed fuzzy tracking control method, some simulation results are provided.**

***Index Terms*—permanent magnet synchronous motor; T-S fuzzy models; sliding mode observer.**

I. Introduction

Permanent Magnet Synchronous Motor (PMSM) drives are widely used in many complex processes, such as generators and motors, due to a number of advantages: simple structure, high torque-to-current radio, high power density, low noise, and friendly maintenance [1], [2]. However, the control of a PMSM is not easy and still a challenging task because such a system exhibits nonlinear behavior with inherent uncertainties and load disturbance variation. Thus, a conventional control method is already widely used for controlling the speed of PMSM drives. This method involves the use of two proportional-integral (PI) controllers for loop speed control and outer inner loop current control [3], [4]. However, the main drawbacks of this control stratify are the sensitivity of performances to the system parameter variations, inadequate rejection of load disturbance. In order to overcome these drawbacks, many algorithms and control strategies have been proposed, eg., adaptive control [5], [6], [7], predictive control [8]-[10], sliding mode control [11], [12], backstepping control [13], [14], neural network control [15], [16], finite-time control [17], [18] and fuzzy control [19], [20], etc. Furthermore, many attempts have been made to develop disturbance rejection control strategies, as an effective way to suppress the unfavorable influence of load disturbance; these strategies using the feedforward compensation principle with some kind of disturbance estimation [21]-[24].

On the other hand, the control of PMSM drives using Takagi-Sugeno (T-S) fuzzy models is recently considered by many researchers [25]-[30], as an alternative to the traditional heuristics-based fuzzy control. Using T-S fuzzy model-based, a complex dynamic system can be represented by means of an averaged sum of local linear subsystems, thus providing a systematic framework to cope with modeling, analysis, and control of nonlinear systems [31]. In [25], [27], [29], [30], control strategies combine the T-S fuzzy models and a concept called virtual desired variables (VDVs) have been proposed to deal with the control of PMSM drives. This concept has been introduced by [32], to facilitate the design of the control law and reference model, and ensure a fast dynamic response with good tracking performance. However, these control strategies have been discussed the control of PMSM drives without taking into account the variations of the load torque disturbance. Hence, a method combines the advantages of VDVs and T-S fuzzy control with integral action has been proposed by [33], to minimize the effect of load disturbance on the PMSM drives, where the stability analysis has been treated using Lyapunov's method and formulated into Linear Matrix Inequalities (LMIs).

In this paper, we propose a new T-S fuzzy tracking control method for a PMSM using the concept of VDVs and load disturbance observer. First, we use the TS fuzzy models to describe the dynamic behaviour of PMSM. And then, we design a T-S fuzzy controller using parallel distributed compensation (PDC) technique [34]. Next, we derive a virtual reference model and nonlinear tracking controller based on the concept of VDVs. By using this concept, the reference model can be designed according to the desired states and the load disturbance. In this case, instead of minimizing the effect of the load disturbance on the system using integral action, it becomes a part of the virtual reference model to compensate completely its effect on the PMSM, but the problem here lies in the fact that the load disturbance is usually unknown or difficult to measure. To overcome this problem, we propose a fuzzy sliding mode observer based on the method developed in [35], to estimated effectively the variations of the load torque. The main idea behind it is to extend the traditional sliding mode observers to dynamical systems described by a T-S fuzzy models. Finally, simulation results are presented to demonstrate the effectiveness and the good performance of the proposed method.

II. PMSM Model in a Rotating $d-q$ Frame

The mathematical model of the PMSM can be described by the following equations [36]:

$$u_d = Ri_d + L_d \frac{di_d}{dt} - pL_q \omega i_q \tag{1}$$

$$u_q = Ri_q + L_q \frac{di_q}{dt} + p\omega(L_d i_d + \lambda) \tag{2}$$

where ω is the rotor speed, (u_d, u_q) are the stator voltage components in the d-q axis, (i_d, i_q) are the current components in the d-q axis, (L_d, L_q) are the stator inductors in the d-q axis. R, λ and p represent the the stator winding resistance, flux linkage of the permanent magnets and number of poles, respectively. The electromagnetic torque of the PMSM can be expressed, in the $d-q$ reference frame, as follows:

$$T_e = \frac{3}{2} p \left(\lambda - (L_q - L_d) i_d\right) i_q \tag{3}$$

The electromagnetic torque depends to the speed by following mechanical equation:

$$T_e - T_L = J \frac{d\omega}{dt} + f\omega \tag{4}$$

where T_L is the load torque, J is the moment of inertia of the rotor and f is the friction coefficient relating to the rotor speed. The position can be written as:

$$p\frac{d\theta}{dt} = \omega \tag{5}$$

In our work, the smooth-air-gap of the synchronous machine systems are considered, i.e., $L_q = L_d = L$. Using the equations (1) to (4), the mathematical model of the PMSM can be rewritten as:

$$\begin{cases} \dot{x}(t) = f(x(t)) + gu(t) + \upsilon T_L(t) \\ y(t) = \varphi(x(t)) \end{cases} \tag{6}$$

where

$$x = \begin{bmatrix} \omega \\ i_q \\ i_d \end{bmatrix}, f = \begin{bmatrix} -\frac{f}{J}\omega + \frac{3p\lambda}{2J} i_q \\ -\frac{p\lambda}{L}\omega - \frac{R}{L} i_q - p\omega i_d \\ p\omega i_q - \frac{R}{L} i_d \end{bmatrix},$$

$$g = \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}, u = \begin{bmatrix} u_q \\ u_d \end{bmatrix}, \upsilon = \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \end{bmatrix}, \varphi = \omega(t).$$

III. Proposed Control Design

The goal is to design a fuzzy tracking control scheme allows to drive the states of the PMSM system $x = \begin{bmatrix} \omega & i_q & i_d \end{bmatrix}^T$ to track a specified desired trajectory $x_d = \begin{bmatrix} \omega_d & i_{qd} & i_{dd} \end{bmatrix}^T$ and to reject completely the effect of disturbance (load torque variation) on the system. In the first step, we develop the T-S fuzzy controller (FC) based on the mathematical model of the PMSM. Next, we use the concept of VDVs to design a virtual reference model (VRM) and nonlinear tracking controller (NLC). Finally, We design a fuzzy sliding mode observer (FSMO) to estimate the load torque disturbance. Thus, a control scheme with FC, VRM, NTC and FMSO is proposed, as shown in Fig 1.

A. T-S fuzzy controller

In order to develop the fuzzy controller, the PMSM model (6) is transformed into T-S fuzzy models using the measurable speed as a decision variable. This leads to the following nonlinear state space form:

$$\begin{cases} \dot{x}(t) = A(\omega(t))x(t) + Bu(t) + ET_L(t) \\ y(t) = Cx(t) \end{cases} \tag{7}$$

where:

$$A(\omega(t)) = \begin{bmatrix} -\frac{f}{J} & \frac{3p\lambda}{2J} & 0 \\ -\frac{p\lambda}{L} & -\frac{R}{L} & -p\omega(t) \\ 0 & p\omega(t) & -\frac{R}{L} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix},$$

$$E = \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

Assuming that the speed is bounded as: $\underline{\omega} \leq \omega(t) \leq \overline{\omega}$ and using the well-known sector nonlinearity transformation [37], the nonlinear system (7) can be described by a T-S fuzzy models with $r = 2^1$ fuzzy If-Then rules, as follows:

Rule i : If $z(t)$ is F_{1i} Then

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_i T_L(t) \\ y(t) = C_i x(t) \end{cases} \quad i = 1, ..., r$$

where $z = \omega$ is the premise variable, F_{11} and F_{12} are the membership functions which can be defined by the following equations:

$$F_{11}(\omega) = \frac{\omega(t) - \underline{\omega}}{\overline{\omega} - \underline{\omega}}, \quad F_{12}(\omega) = 1 - F_{11}(\omega) \tag{8}$$

The matrices of the local models can be defined as:

$$A_1 = \begin{bmatrix} -\frac{f}{J} & \frac{3p\lambda}{2J} & 0 \\ -\frac{p\lambda}{L} & -\frac{R}{L} & -p\overline{\omega} \\ 0 & p\overline{\omega} & -\frac{R}{L} \end{bmatrix}, A_2 = \begin{bmatrix} -\frac{f}{J} & \frac{3p\lambda}{2J} & 0 \\ -\frac{p\lambda}{L} & -\frac{R}{L} & -p\underline{\omega} \\ 0 & p\underline{\omega} & -\frac{R}{L} \end{bmatrix},$$

$$E_1 = E_2 = \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

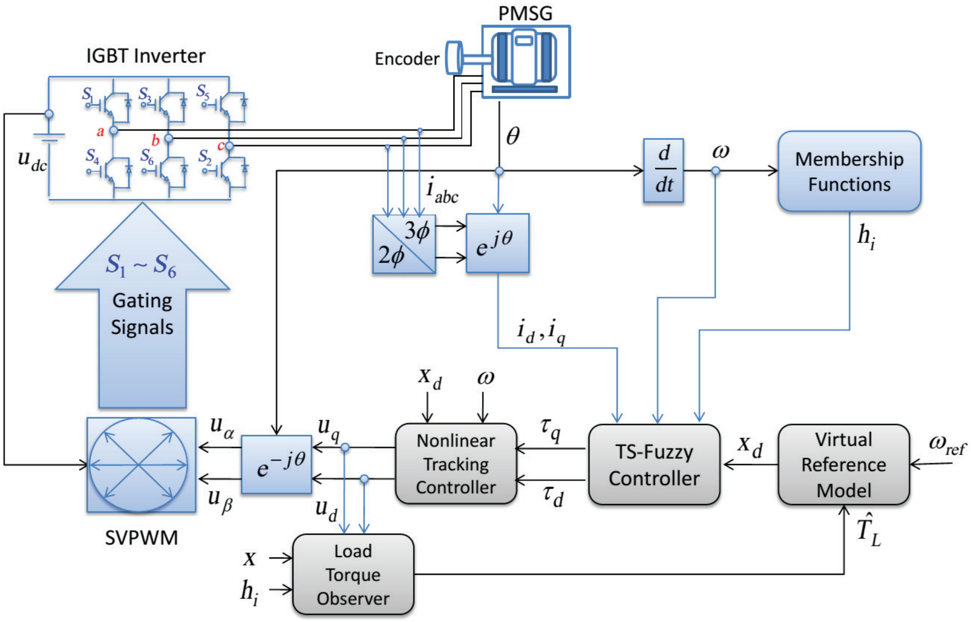


Fig. 1: Overall block diagram of the proposed fuzzy tracking control

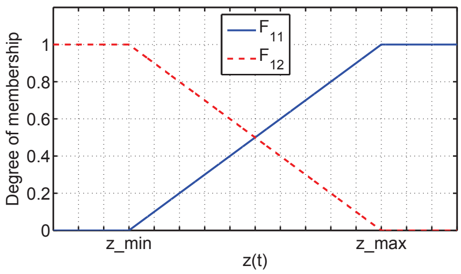


Fig. 2: Membership functions of the decision variable

Using the product-inference rule, singleton fuzzifier, and the center of gravity defuzzifier, the overall output of the fuzzy rule-based system:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t) + E_i T_L(t)) \quad (9)$$

where

$$h_i(z(t)) = \frac{F_{1i}(z(t))}{\sum_{j=1}^{r} F_{1j}((z(t))} \quad (10)$$

for all $t > 0$, $h_i(z(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(z(t)) = 1$.

The objective is to design a fuzzy controller capable capable of driving the state of the PMSM system $x(t)$ to track a specified set of VDVs $x_d(t)$. Then, the feedback tracking control is required to satisfy:

$$x(t) - x_d(t) \to 0 \quad as \quad t \to \infty \quad (11)$$

According to $y(t) = \varphi(x(t))$, it is natural to require $y_d(t) = \varphi(x_d(t))$, which denotes the desired output state. Now, let $\tilde{x}(t) = x(t) - x_d(t)$ be defined as the tracking error and its time derivative is given by:

$$\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{x}_d(t) \quad (12)$$

Replacing the equation (9) by its value in (12) and adding the term $\sum_{i=1}^{r} h_i(z(t))A_i(x_d(t) - x_d(t))$, the equation (12) becomes:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} h_i(z)\left(A_i\tilde{x}(t) + B_i u(t) + E_i T_L(t) + A_i x_d(t)\right) - \dot{x}_d(t) \quad (13)$$

By introducing a new control variable $\tau(t)$ that satisfy the following relation:

$$\sum_{i=1}^{r} h_i(z(t))B_i\tau(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x_d(t) + B_i u(t) + E_i T_L(t)) - \dot{x}_d(t) \quad (14)$$

Using the equation (14), the tracking error system (13) can be rewritten as follows:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i\tilde{x}(t) + B_i\tau(t)) \quad (15)$$

The new local state feedback controllers are designed to deal with the tracking control problem as:

Rule i: If z(t) is F_{1i} Then $\tau(t) = -K_i\tilde{x}(t)$

The final output of the fuzzy controller is determined by the following summation:

$$\tau(t) = -\sum_{i=1}^{r} h_i(z(t))K_i\tilde{x}(t) \quad (16)$$

Applying control law (16) to model (15), the closed-loop system takes the following form:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))(A_i - B_iK_j)\tilde{x}(t) \quad (17)$$

By letting $G_{ij} = (A_i - B_iK_j)$, the equation (17) can be written as follows:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))G_{ij}\tilde{x}(t) \quad (18)$$

Stability Analysis

In order to determine the gains K_i of the fuzzy control law (16), the following theorem is considered:

Theorem 1. [32] *The equilibrium of the closed-loop continuous fuzzy system* (18) *is asymptotically stable via the controller* (16) *if there exist a symmetric matrix* $P > 0$*, a diagonal matrix* D *and matrices* Q_{ij} *with:* $Q_{ii} = Q_{ii}^T$ *and* $Q_{ji} = Q_{ij}^T$ *for* $i \neq j$*, such that:*

$$G_{ii}^T P + PG_{ii} + Q_{ii} + DPD < 0, i = 1, ..., r \quad (19)$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) + Q_{ij} \le 0, i < j \le r \quad (20)$$

$$\begin{bmatrix} Q_{11} & \dots & Q_{1r} \\ \vdots & \ddots & \vdots \\ Q_{1r} & \dots & Q_{rr} \end{bmatrix} \equiv \tilde{Q} > 0 \quad (21)$$

for $i,\ j = 1, ..., r$*, s.t. the pairs* $(i,\ j)$ *such that* $h_i(z(t))h_j(z(t)) = 0$*,* $\forall\ t$*.*

Remark 1: The conditions of Theorem 1 can be transformed into an equivalent problem in the form of LMIs. The transformation corresponds to simple objective changes of variables $X = P^{-1}$, $K_i = M_iX^{-1}$ and the use of a congruence in inequalities (19), (20), (21). The following LMI expressions in variables X and M_i can be obtained:

$\exists\ X = X^T > 0, \exists\ Y_{ii} = Y_{ii}^T, \exists\ Y_{ij} = Y_{ji}^T, \exists\ M_i$

$$\begin{bmatrix} XA_i^T + A_iX - B_iM_i - M_i^TB_i^T + Y_{ii} & XD^T \\ DX & -X \end{bmatrix} < 0, \quad (22)$$

$$XA_i^T + A_iX + XA_j^T + A_jX - B_iM_j - M_j^TB_i^T - B_jM_i - M_i^TB_j^T + 2Y_{ij} \le 0, i < i \le r \quad (23)$$

$$\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1r} \\ Y_{12} & Y_{22} & \dots & Y_{2r} \\ \vdots & \ddots & \vdots & \\ Y_{1r} & Y_{2r} & \dots & Y_{rr} \end{bmatrix} \equiv \tilde{Y} > 0 \quad (24)$$

B. Nonlinear Tracking Controller and Virtual Reference Model

The virtual desired variables $x_d(t)$ and control law $u(t)$ can be obtained using Eq. (14) which is rewritten as:

$$\sum_{i=1}^{r} h_i(z(t))B_i(u(t) - \tau(t)) = -\sum_{i=1}^{r} h_i(z(t))A_ix_d(t) - \sum_{i=1}^{r} h_i(z(t))E_iT_L(t) + \dot{x}_d(t) \quad (25)$$

Noting that:

$$A = \sum_{i=1}^{r} h_i(z)A_i, \quad g = \sum_{i=1}^{r} h_i(z)B_i, \quad \upsilon = \sum_{i=1}^{r} h_i(z)E_i \quad (26)$$

Then, Eq. (26) can be rewritten as:

$$g(u(t) - \tau(t)) = -A(x(t))x_d(t) - \upsilon T_L(t) + \dot{x}_d(t) \quad (27)$$

Applying the equation (28) to the PMSM model, we obtain the following matrix form

$$\begin{bmatrix} 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_q - \tau_q \\ u_d - \tau_d \end{bmatrix} = -\begin{bmatrix} -\frac{f}{J} & \frac{3p\lambda}{2J} & 0 \\ -\frac{p\lambda}{L} & -\frac{R}{L} & -p\omega \\ 0 & p\omega & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \omega_d \\ i_{qd} \\ i_{dd} \end{bmatrix} - \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \end{bmatrix} T_L + \frac{d}{dt}\begin{bmatrix} \omega_d \\ i_{qd} \\ i_{dd} \end{bmatrix} \quad (28)$$

From the first equation of (28), it remains that:

$$i_{qd} = (\dot{\omega}_d + \frac{f}{J}\omega_d + \frac{1}{J}T_L)\frac{2J}{3p\lambda}. \quad (29)$$

According to field oriented vector control, the best choice for the desired d-axis current is $i_{dd} = 0$. Then, the vector of VDVs becomes:

$$x_d(\omega_d, T_L) = \begin{bmatrix} \omega_d \\ (\dot{\omega}_d + \frac{B}{J}\omega_d + \frac{1}{J}T_L)\frac{2J}{3p\lambda} \\ 0 \end{bmatrix} \quad (30)$$

From the second and the third equation of (28), we obtain the following nonlinear tracking control inputs:

$$\begin{cases} u_q = p\lambda\omega_d + Ri_{qd} + L\dot{i}_{qd} + \tau_q \\ u_d = -pL\omega i_{qd} + \tau_d \end{cases} \quad (31)$$

C. Load torque observer

The reference model obtained in (30) needs the load torque, which is usually an inaccessible state variable or difficult to measure. Then, it is important to look for its estimated value. In our case the states are available but we are interested in estimating an unknown input (load torque). The problem considered here consists of the reconstruction of the unknown load torque disturbance by using the information provided by the input voltages and PMSM outputs. The proposed observer is a linear combination of local observers, each of them having the structure proposed in [35]. In this context, we consider that the load torque is bounded, such as $\|T_L\| \le \eta$, where η

is a scalar and $\|.\|$ represents the Euclidean norm, It is also assumed that there exists matrices L_i such as $\tilde{A}_i = A_i - L_iC$ have stable eigenvalues, and there exist a symmetric matrix $\tilde{P}$, a matrix $\tilde{Q}$, matrices F_i respecting the following constraints:

$$\begin{cases} \tilde{A}_i^T\tilde{P} + \tilde{P}\tilde{A}_i = -\tilde{Q} \\ F_iC = E_i^T\tilde{P} \end{cases} \tag{32}$$

The following structure of fuzzy sliding mode observer is proposed:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z)\left(A_i\hat{x}(t) + B_iu(t) + L_ir(t) + E_iv_i(t)\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{33}$$

The matrices L_i and the control variables v_i must be determined in order to guarantee the asymptotic convergence of $\hat{x}(t)$ towards $x(t)$. The terms v_i are designed in order to compensate the error due to the load torque. $r(t)$ is the output error which is defined as:

$$r(t) = y(t) - \hat{y}(t) = Ce(t) \tag{34}$$

where $e(t)$ is the state estimation error which is defined as:

$$e(t) = x(t) - \hat{x}(t) \tag{35}$$

and its time derivative can be written as:

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(z)\left((A_i - L_iC)e + B_iu + E_iT_L - E_iv_i(t)\right) \tag{36}$$

The state estimation error (36) converges toward zero, if the following theorem is verified:

Theorem 2*. The state estimation error* (36) *converges toward zero, if v_i satisfied the following conditions:*

$$\begin{cases} v_i(t) = \eta\frac{F_ir}{\|F_ir\|} \quad if \quad r \neq 0 \\ v_i(t) = 0 \quad if \quad r = 0 \end{cases} \tag{37}$$

and if there exist a symmetric matrix $\tilde{P}$, such that:

$$\tilde{A}_i^T\tilde{P} + \tilde{P}\tilde{A}_i < 0 \tag{38}$$

Remark 2. In order to calculate the control variables $v_i(t)$, the following LMI expressions in variables $\tilde{P}$, $\tilde{Q}$ and F_i can be obtained:

$\exists\ \tilde{P} = \tilde{P}^T > 0$, $\exists\ \tilde{Q}$, $\exists\ F_i$:

$$\begin{cases} (A_i - L_iC)_i^T\tilde{P} + \tilde{P}(A_i - L_iC) = -\tilde{Q} \\ F_iC = E_i^T\tilde{P} \end{cases} \tag{39}$$

where L_i can be calculated using pole placement technique. The estimated load torque is derived, as follows:

$$\hat{T}_L \approx \left(\sum_{i=1}^{r} h_iE_i \right)^+ (\hat{x} - \bar{x}) - \left(\sum_{i=1}^{r} h_iE_i \right)^+ (x - \hat{x}) \tag{40}$$

where $(.)^+$ denotes the pseudo-inverse and $\bar{x}$ denotes the state resulting from the fuzzy model without disturbance which is defined as:

$$\dot{\bar{x}}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i\bar{x}(t) + B_iu(t)) \tag{41}$$

IV. Simulation Results

In order to verify the effectiveness and the validity of the proposed method, a numerical simulation was carried out by using MATLAB/Simulink with the parameters of PMSM listed in Table I. The following fuzzy controller gains are calculated by solving the LMIs:

$$K_1 = \begin{bmatrix} 8.1338 & 18.8361 & 0.0758 \\ -0.0765 & 0.0780 & 18.8743 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 12.4762 & 16.8344 & -0.3105 \\ -0.1569 & -0.2428 & 17.9380 \end{bmatrix}.$$

The FSMO matrix gains F_i are calculated by solving the LMIs (39), as follows:

$$F_1 = F_2 = -4.2745 \times 10^3$$

where the matrix gains L_i are calculated using the pole placement technique, as follows

$$L_1 = 10^4.\begin{bmatrix} 0.1353 & 3.1135 & 0.7554 \end{bmatrix}$$

$$L_2 = 10^4.\begin{bmatrix} 0.1353 & 3.1135 & -0.7554 \end{bmatrix}$$

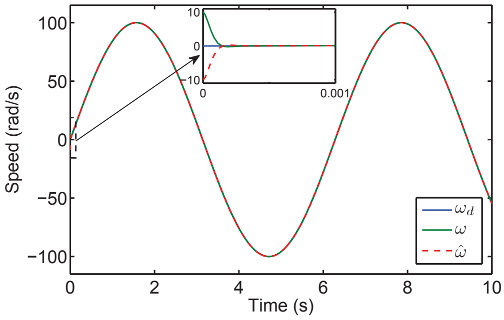
The proposed fuzzy tracking control has been verified for the following cases:

A. Speed tracking

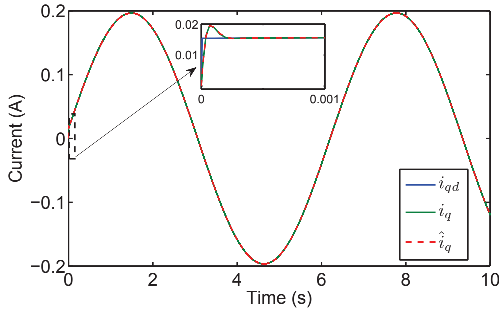
The aim of the first simulation is to force the output speed of the PMSM to track sinusoidal reference speed $\omega_d(t) = 100sin(t)\ rad/s$, where the initial states are assumed to be: $x(0) = \begin{bmatrix} 10 & 0 & 0 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} -10 & 0 & 0 \end{bmatrix}^T$. The responses of different steady states are shown in Figs 3(a), 3(c), 3(b) and 3(d). The simulation results indicate that the time response of the tracking is very low. It is clear that the less speed, current and voltage tracking errors, the better the tracking performance. Indeed, the speed and d-q axis currents track well the reference trajectory with good reliability over the whole speed range. From Figs 3(c) and 3(d), it is clear that the current and voltage responses are in the expected ranges.

B. Speed regulation with applied load torque disturbance

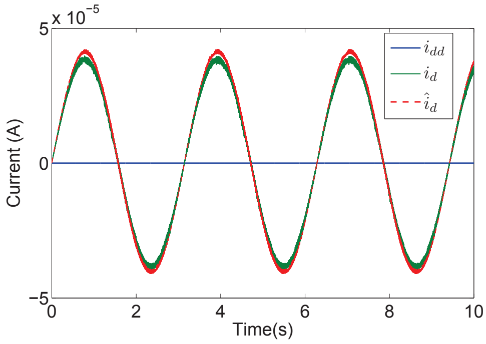
The aim of the second simulation is to force the output speed of the PMSM to track the step-reference speed $\omega_d(t) = 100\ rad/s$, where the load torque disturbance increases from $0\ N.m$ to $5.5\ N.m$ at $t = 2\ s$ and decreases from $5.5\ N.m$ to $0\ N.m$ at $t = 4\ s$, as shown in Fig. 4(d). The initial states are set to be $x(0) = \hat{x}(0) = 0$. The dynamic responses of PMSM system are shown in Figs. 4(a), 4(b) and 4(c). It can be seen from these figures that the time response of the regulation control is very low, also, the tracking error is very small when the load disturbance changes from $0\ N.m$ to 5.5 or vice-versa. This means that the robustness of the fuzzy tracking control is improved with the disturbance compensation. From Fig 4(d), it is clear that the load torque variation can be successfully identified via the proposed fuzzy sliding mode observer.



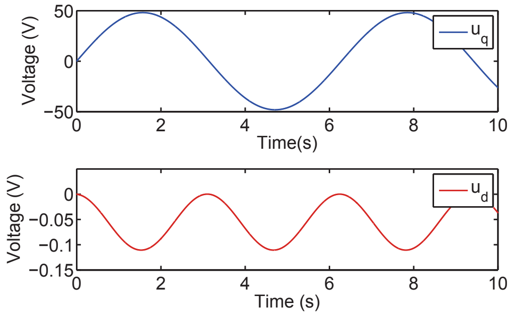
(a) Reference, actual and estimated speed



(b) Reference, actual and estimated q-axis current

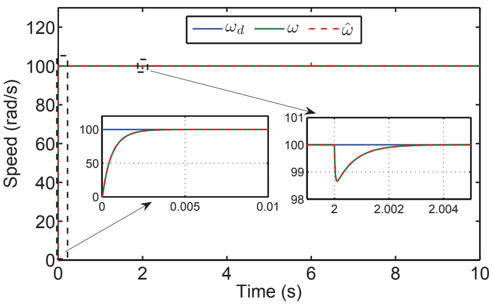


(c) Reference, actual and estimated d-axis current

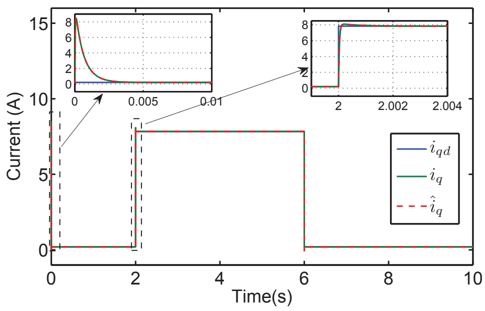


(d) Control voltages.

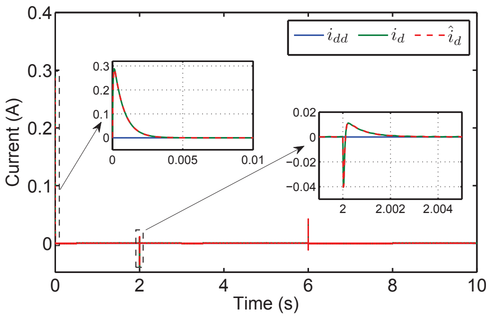
Fig. 3: Closed loop control with $\omega_d(t) = 100sin(t)\ rad/s$.



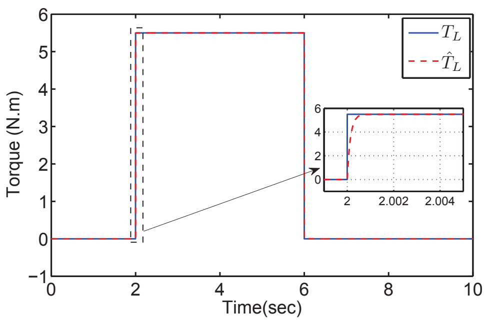
(a) Reference, actual and estimated speed



(b) Reference, actual and estimated q-axis current



(c) Reference, actual and estimated d-axis current



(d) Actual and estimated load torque

Fig. 4: Closed loop control with $\omega_d(t) = 100\ rad/s$ and load disturbance applied at $2s$ with magnitude $5\ N.m$.

TABLE I: Parameters of the PMSM.

Parameter	Symbol	Value
Number of poles pairs	p	4
Stator Resistance	R	$2.875\ \Omega$
d-axis Inductance	L_d	$8.5\ mH$
q-axis Inductance	L_q	$8.5\ mH$
Rotor inertia	J	$0.0008\ Kg.m^2$
Friction coefficient	f	$0.001\ N.m/rad/s$
Magnet flux linkage	λ	$0.175\ wb$

V. Conclusion

This paper presents a fuzzy tracking control method for a PMSM based on the concept of VDVs and sliding mode observer. The concept of VDVs is used to simplify the design of the control law and the construction of the reference model according to the desired speed and the load torque disturbance. A fuzzy sliding mode observer is proposed to estimated the variation of the load torque disturbance. Sufficient conditions for stability are derived from Lyapunov's method and solved by convex programming techniques. The simulation results show the effectiveness of the proposed fuzzy tracking control method and the robustness of the designed controller.

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