

# Sensorless Speed Control of Salient Pole PMSM According to the Backstepping Observer

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**Abstract**--In this work, a novel sensorless speed control scheme is developed for the design of salient pole permanent magnet synchronous motors (PMSM). This design strategy consists of nonlinear backstepping controllers and an angular velocity observer to achieve the purpose of speed tracking. The PMSM design, in general, requires angular velocity measurement to achieve control goals, but most PMSM systems do not have speed measurement devices. Therefore, a backstepping observer is used to estimate the rotor speed, and then the nonlinear controllers based on backstepping design algorithm coupled with introduction of integral actions are developed for the PMSM system. The proposed control scheme is not only to stabilize the PMSM system, but also to drive the speed tracking error to converge to zero asymptotically. Furthermore, the complete system model is simulated by using MATLAB/Simulink software, performance of the proposed controller is investigated extensively at different dynamic operating conditions such as sudden load change, command speed change and parameter variation.

**Index Terms**--Backstepping control, Lyapunov theorem, integral action, salient pole permanent magnet synchronous motor (PMSM), backstepping angular velocity observer, observer.

## 1. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are widely used in high performance servo applications due to their high efficiency, high power density, and large torque to inertia ratio [1]. The PMSM is very similar to the standard wound rotor synchronous machine except that the PMSM has no damper windings and the field winding of the rotor is replaced by a permanent magnet [2]. Hence, the mathematical model of the PMSM is identical to that of the classical synchronous machine with the equations of the damper windings and field current dynamics removed. However, the dynamic model of a PMSM is strongly nonlinear and coupled, because of the coupling between the motor speed and the electrical quantities, such as the d-q axis currents [3].

Recently, various nonlinear control methodologies have been applied to the control design of PMSM systems in order to construct the controllers directly by considering the nonlinear PMSM dynamics. Backstepping control is a new type recursive and systematic design methodology for the feedback control of uncertain nonlinear system,

particularly for the system with matched uncertainties. The most appealing point of it is to use the virtual control variable to make the original high order system simple, thus the final outputs can be derived step by step systematically through suitable Lyapunov functions. The influence of the change some parameters and of the perturbation of charge can be greatly reduced by introducing integral action in each step, in order to ensure high accuracy speed control [4]-[6].

In addition, the main reason why the industrial application of PMSM drives has grown is that the algorithm driving PMSM without speed sensors has been steadily improved. The design of PMSM systems, in general, the information of rotor position and speed are necessary to achieve a high-performance drive, but lots of PMSM systems do not have speed measurement devices. Eliminating speed sensors simplifies the structure of the driver system and widens the application fields. Therefore, various sensorless control strategies have been investigated [7]-[10].

In this paper, with the proposed backstepping angular velocity observer, a nonlinear backstepping control design scheme is developed for the speed tracking control of PMSM that has exact model knowledge. The asymptotic stability of the resulting closed-loop system is guaranteed according to Lyapunov stability theorem. As a result, the proposed nonlinear backstepping control design without using speed sensor is not only to stabilize the PMSM system, but also to force the speed tracking error to converge to zero asymptotically.

The paper is organized as follows: in Section 2, the dynamic model of a PMSM is introduced with some important system properties. The sensorless backstepping control scheme with integral action consisting of an angular velocity observer and the voltage input controllers is developed for the purpose of speed tracking in Section 3. The simulation results are illustrated in Section 4, and some concluding remarks are given in Section 5.

## 2. MATHEMATICAL MODEL

The dynamic model of a PMSM can be described in the well-known d-q frame through the Park transformation [3], [11] as follows:

$$\begin{aligned} \frac{di_d}{dt} &= -\frac{R}{L_d}i_d + p\frac{L_q}{L_d}\omega i_q + \frac{1}{L_d}v_d \\ \frac{di_q}{dt} &= -\frac{R}{L_q}i_q - p\frac{L_d}{L_q}\omega i_d - p\frac{1}{L_q}\phi_f\omega + \frac{1}{L_q}v_q \\ \frac{d\omega}{dt} &= \frac{p}{J}(L_d - L_q)i_d i_q - \frac{F}{J}\omega + \frac{P}{J}\phi_f i_q - \frac{1}{J}T_L \end{aligned} \quad (1)$$

where  $i_d$  Stator's d-axis current.

$i_q$  Stator's q-axis current.

$v_d$  Stator's d-axis voltage.

$v_q$  Stator's q-axis voltage.

$\omega$  Motor speed.

$R$  Stator resistance.

$L_d$  d-axis stator inductance.

$L_q$  q-axis stator inductance.

$p$  Number of pole pairs.

$\phi_f$  Permanent magnet flux.

$J$  Rotor moment of inertia.

$F$  Viscous friction coefficient.

$T_L$  Load torque.

$\theta$  Rotor angular position.

Here, the available measurements are assumed to be the rotor position  $\theta$  and the d-q axis currents  $i_d$  and  $i_q$ . If rotor position can be obtained, then the relationship between the d-q frame currents and the actual three-phase current quantities  $i_a$ ,  $i_b$  and  $i_c$  is defined by the following change of coordinates:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = AB \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

where

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & c \cos\theta \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

### 3. SENSORLESS BACKSTEPPING CONTROL WITH INTEGRAL ACTION

This part of the article studies the strategy of speed control by the sensorless Backstepping technique, it can be effectively used for linearizing a nonlinear system in the presence of uncertainties [4], [6].

The control technique is a nonlinear Backstepping control having properties of strength. The pursuit of speed takes place with a high yield by the control voltage  $v_q$  as long as the current  $i_q$  is kept equal to zero.

The basic idea of the type Backstepping control is to make curly equivalent subsystems of order 1 systems in cascade stable within the meaning of Lyapunov, which gives them the qualities of robustness and a global asymptotic stability [5], [11]. In other words, this is a multi-step method. At each step of the process, a virtual control is also generated to ensure the convergence of system to its equilibrium state. This can be reached from Lyapunov functions to ensure step by step the stabilization of each synthesis step. The idea is to compute a control law to ensure that the Lyapunov function is positive definite and its derivative is always negative.

The calculation of Lyapunov function is performed in a recursive way. It is based on the previous system state. A new Control Lyapunov Function (CLF) is constructed by the increase of CLF of the previous step [6]. This procedure calculates allow us to ensure overall system stability. The robustness improvement of this technique by incorporation of introduction in terms integrations of the control design of PMSM thereafter. The corresponding block diagram of a PMSM system with coordinate transformation is shown in Fig. 1.

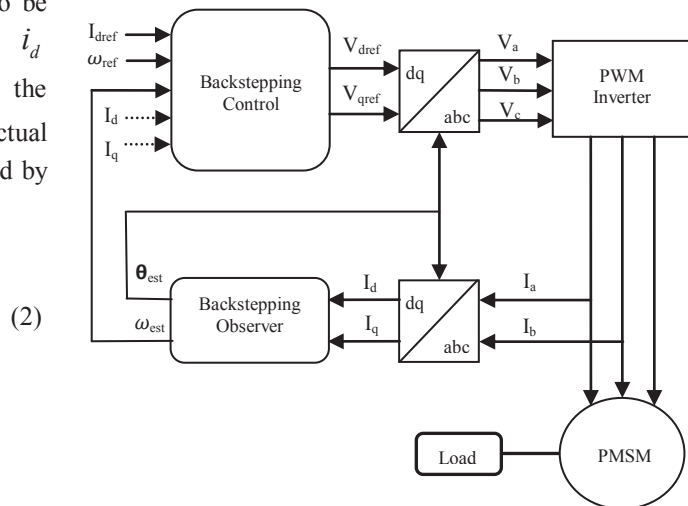


Fig. 1. The block diagram of speed tracking design scheme.

Since the speed measurement of the PMSM is usually unavailable, an angular velocity observer must be employed to estimate the actual motor speed. So, the speed estimation error is defined by

$$\tilde{\omega} = \omega - \hat{\omega} \quad (3)$$

where  $\hat{\omega}$  speed estimation

$\tilde{\omega}$  speed estimation error

$k_1, k'_1, k_2, k'_2, k_3$  and  $k'_3$  positive design

constants.

Now, we employ backstepping schemes to design the nonlinear controllers with the angular velocity observer, the backstepping design procedure consists of three steps:

#### A. Current Loop $i_d$

First of all, since the direct axis current  $i_d$  must be forced to be zero, let us define the following tracking error:

$$e_1 = i_{dref} - i_d + e'_1 \quad (4)$$

where  $e'_1 = k'_1 \int_0^t (i_{dref} - i_d) dt$  is an integral action.

Defining the following candidate Lyapunov function:

$$V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} e_1'^2 \quad (5)$$

the time derivative is computed as:

$$\dot{V}_1 = e_1 \left\{ -\frac{di_d}{dt} - k'_1 i_d \right\} + e'_1 \{ -k'_1 i_d \} \quad (6)$$

Since  $i_{dref} = 0$ , then replacing  $i_d = e'_1 - e_1$  in the above equation, we obtain:

$$\dot{V}_1 = e_1 \left\{ \frac{R}{L_d} i_d - p \frac{L_q}{L_d} \hat{\omega} i_q - p \frac{L_q}{L_d} \tilde{\omega} i_q - \frac{1}{L_d} v_d \right\} + k'_1 \{ e_1 + e'_1 \} \{ e_1 - e'_1 \} \quad (7)$$

#### B. Speed Loop

To solve speed tracking problem, define the following speed tracking error as:

$$e_2 = \omega_{ref} - \hat{\omega} + k'_2 \int_0^t (\omega_{ref} - \hat{\omega}) dt \quad (8)$$

where  $k'_2 \int_0^t (\omega_{ref} - \hat{\omega}) dt$  is an integral action term added

to the control in Backstepping to ensure the convergence of the tracking error to zero in spite of the uncertainties of such piecewise constant at each step of the algorithm.

The virtual control input  $i_{qref}$  used to ensure the stability of the speed loop. The error dynamics of speed from (8) is given by:

$$\dot{e}_2 = \dot{\omega}_{ref} - \frac{p}{J} (L_d - L_q) i_d i_{qref} + \frac{F}{J} \hat{\omega} - \frac{p}{J} \phi_f i_{qref} + \frac{1}{T_L} + k'_2 (\omega_{ref} - \hat{\omega}) \quad (9)$$

Consider the following Lyapunov candidate function

$V_2 = \frac{1}{2} e_2^2$  its derivative is given by the following equation:

$$\dot{V}_2 = e_2 \left\{ \dot{\omega}_{ref} - \frac{p}{J} (L_d - L_q) i_d i_{qref} + \frac{F}{J} \hat{\omega} - \frac{p}{J} \phi_f i_{qref} + \frac{1}{J} T_L + k'_2 (\omega_{ref} - \hat{\omega}) \right\} \quad (10)$$

#### C. Current Loop $i_q$

Now to design the control input  $v_q$  we define the tracking error in the current as follows:

$$e_3 = i_{qref} - i_q + e'_3 \quad (11)$$

where  $e'_3 = k'_3 \int_0^t (i_{qref} - i_q) dt$  is an integral action.

Consider the following Lyapunov candidate function:

$$V_3 = V_2 + \frac{1}{2} e_3^2 + \frac{1}{2} e_3'^2 \quad (12)$$

By taking the time derivative of  $V_3$  is given by:

$$\dot{V}_3 = -k_2 e_2^2 + e_3 \left\{ \frac{di_{qref}}{dt} - \frac{di_q}{dt} + k'_3 (i_{qref} - i_q) \right\} + e_3' k'_3 (i_{qref} - i_q) \quad (13)$$

Replacing by:

$$\frac{di_q}{dt} = -\frac{R}{L_q} i_q - p \frac{L_d}{L_q} \hat{\omega} i_d - p \frac{1}{L_q} \phi_f \hat{\omega} + \frac{1}{L_q} v_q, \quad \text{then} \quad (13)$$

becomes:

$$\dot{V}_3 = -k_2 e_2^2 + e_3 \left\{ \frac{di_{qref}}{dt} + \frac{R}{L_q} i_q + p \frac{L_d}{L_q} \hat{\omega} i_d + p \frac{L_d}{L_q} \tilde{\omega} i_d + p \frac{1}{L_q} \phi_f \hat{\omega} + p \frac{1}{L_q} \phi_f \tilde{\omega} - \frac{1}{L_q} v_q + k'_3 (i_{qref} - i_q) \right\} + e_3' k'_3 (i_{qref} - i_q) \quad (14)$$

Therefore, by following the choice of (5) and (12), the complete Lyapunov function candidate is selected as

$$V = V_1 + V_2 + \frac{1}{2} e_3^2 + \frac{1}{2} e_3'^2 + \frac{1}{2\gamma} \tilde{\omega}^2 \quad (15)$$

where  $\gamma$  is the positive adaptation gain. From (7), (10) and (14), the derivative of (15) is computed as follows:

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + e_3 \dot{e}_3 + e_3' \dot{e}_3' + \frac{1}{\gamma} \tilde{\omega} \dot{\tilde{\omega}} \quad (16)$$

$$\begin{aligned} \dot{V} = e_1 \left\{ \frac{R}{L_d} i_d - p \frac{L_q}{L_d} \hat{\omega} i_q - p \frac{L_q}{L_d} \tilde{\omega} i_q - \frac{1}{L_d} v_d \right\} \\ + k_1' \{ e_1 + e_1' \} \{ e_1 - e_1' \} + e_2 \left\{ \dot{\omega}_{ref} - \frac{p}{J} (L_d - L_q) i_d i_{qref} \right. \\ \left. + \frac{F}{J} \hat{\omega} - \frac{p}{J} \phi_f i_{qref} + \frac{1}{J} T_L + k_2' (\omega_{ref} - \hat{\omega}) \right\} \\ + e_3 \left\{ \frac{di_{qref}}{dt} + \frac{R}{L_q} i_q + p \frac{L_d}{L_q} \hat{\omega} i_d + p \frac{L_d}{L_q} \tilde{\omega} i_d \right. \\ \left. + p \frac{1}{L_q} \phi_f \hat{\omega} + p \frac{1}{L_q} \phi_f \tilde{\omega} - \frac{1}{L_q} v_q \right\} \\ + (e_3 + e_3') k_3' (i_{qref} - i_q) + \frac{1}{\gamma} \tilde{\omega} \left\{ \frac{p}{J} (L_d - L_q) i_d i_{qref} \right. \\ \left. - \frac{F}{J} \hat{\omega} - \frac{F}{J} \tilde{\omega} + \frac{p}{J} \phi_f i_{qref} - \frac{1}{J} T_L - \dot{\tilde{\omega}} \right\} \quad (17) \end{aligned}$$

At last, we are able to obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - \tilde{\omega} \frac{F}{J} \frac{1}{\gamma} \quad (18)$$

Clearly,  $\dot{V}$  in (18) is negative definite, so it implies that the resulting closed-loop system is asymptotically stable and, hence, all the error variables  $e_1$ ,  $e_2$ ,  $e_3$  and the estimation error  $\tilde{\omega}$  will converge to zero asymptotically. Therefore, the d-axis current  $i_d$  will converge to zero and the angular velocity will converge to the reference speed. In addition, because  $\tilde{\omega}$  can converge to zero, the estimation speed will converge to the actual speed eventually. As a result, the desired control objective of sensorless speed tracking for the PMSM system is indeed achieved by the proposed non linear backstepping control scheme.

#### 4. SIMULATION RESULTS

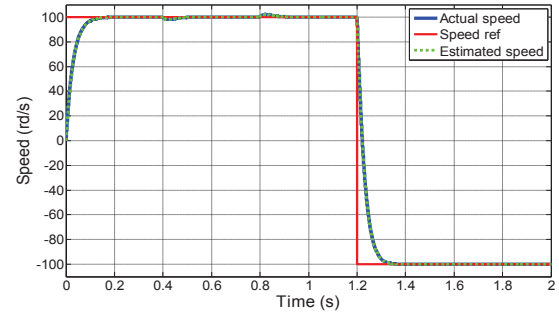
To illustrate the mathematical analysis and to investigate the performance of the proposed PMSM control of Backstepping with integral action according to the Backstepping observer, simulations are carried out following the overall block diagram of the control shown in Fig. 1. Table I gives the nominal parameters of the PMSM which is used in the simulation tests. The simulation has been carried out using Matlab/ Simulink software. The machine started on the vacuum, then a load torque is applied of 5 (N.m) at time 0.4 s, and then eliminated where the moment was 0.8 s. At time  $t = 1.2$  s, the direction of rotation is reversed at 100 (rd/s) to -100 (rd/s).

TABLE I

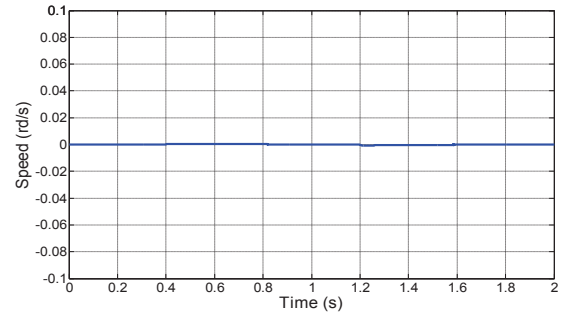
The Parameters of the PMSM.

$\omega$	$R$	$L_d$	$L_q$	$\phi_f$	$p$	$J$	$F$
(rd/s)	( $\Omega$ )	(mH)	(mH)	(Wb)		(kg.m <sup>2</sup> )	(N.m/rd/s)
314	6.2	2.5025	4.01	0.305	3	0.0036	0.0011

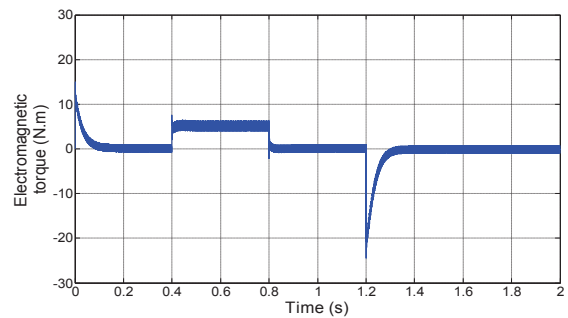
Speed of rotation (rd/s)



Speed estimation error (rd/s)



Electromagnetic torque (N.m)



Current  $i_d, i_q$  (A)

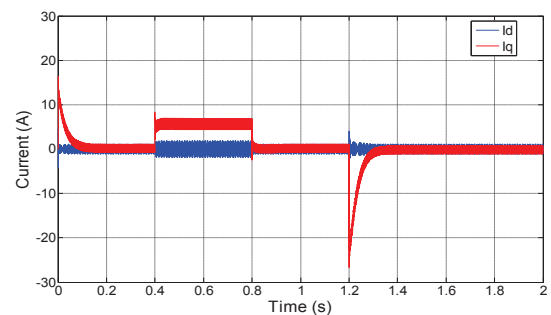


Fig. 2. Simulation results during the variation of stator resistance.

Figure 2 show error of +100 % on the nominal value of the stator resistance, it is noted that according to the shape the speed follows perfectly its reference which is achieved very rapidly, with a very fast response, we can find that the estimation speed error can converge to zero asymptotically, we can see that the effect of the load perturbation is rapidly disappearing. The electromagnetic torque stabilize at the load torque value and the response of the two current components  $i_d$  and  $i_q$  show good decoupling introduced by PMSM control (The direct axis current  $i_d$  is always forced to zero in order to orient all the linkage flux in the d-axis and achieve maximum torque per ampere), we also can see that the current  $i_q$  is the couple image. From the speed tracking simulation result we can find that the sensorless backstepping controllers have good performance.

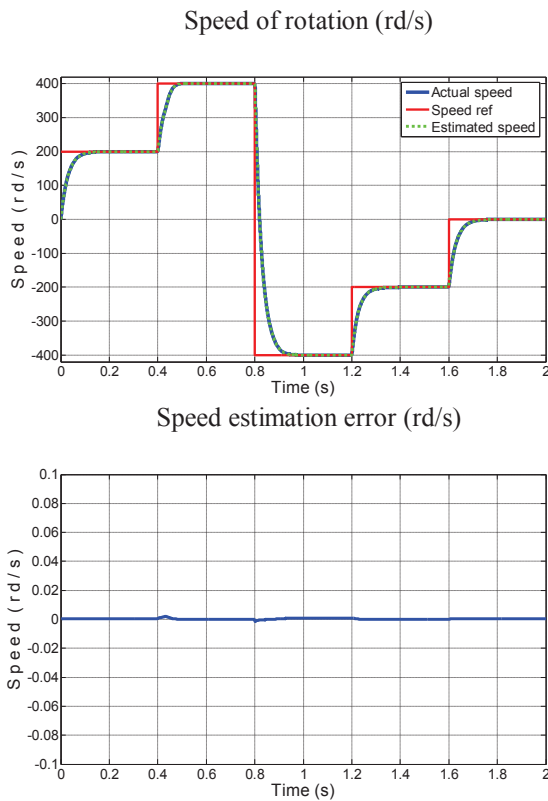


Fig. 3. Speed behaviour of the PMSM regulated by backstepping.

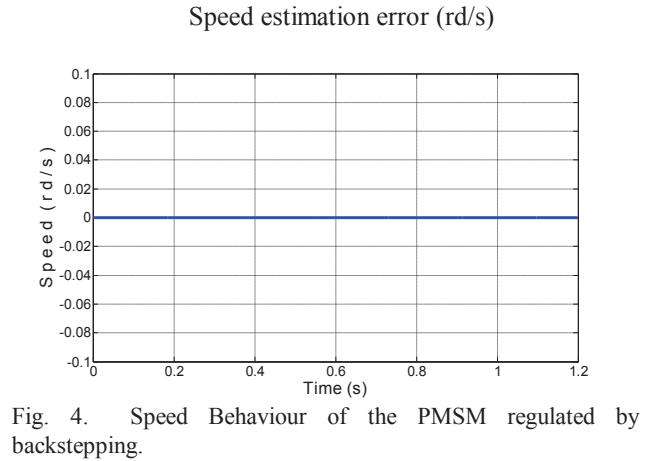
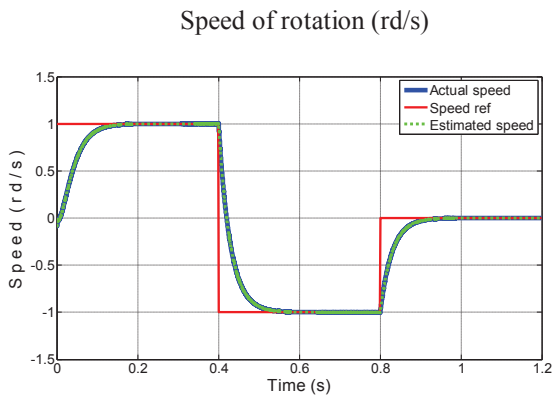


Fig. 4. Speed Behaviour of the PMSM regulated by backstepping.

Figures 3 and 4 illustrate the dynamics behaviours and speed tracking error of PMSM according Backstpping observer at a servo-control of speed, load torque is applied at 5 (N.m) from starting. In the first figure and at time  $t = 0.4$  the speed of rotation is increased at 200 (rd/s) to 400 (rd/s), at time  $t = 0.8$  the direction of rotation is reversed to -400 (rd/s) then decreased to -200 (rd/s) then a stop 0 (rd/s). In the last figure and at time  $t = 0.4$  the direction of rotation is reversed at 1 (rd/s) to -1 (rd/s), then a stop 0 (rd/s). The simulation results show the robustness of the controller used in high and low speeds, it is also shown that this estimated speed follows the actual speed with precision (speed estimation error converge to zero) on permanent regime and the transitional regime.

### 5. CONCLUSION

This paper based on Backstepping control with an integral action for salient pole permanent magnet synchronous motor according to the Backstepping observer, which is addressed in a hand, as a tool for a new nonlinear control speed PMSM, and in the other hand as a tool for studying dynamic stability.

The performance of the proposed controller has been investigated in simulation. The different results show its effectiveness and fast response without overshoot at tracking a reference speed under critical situation of benchmark commands which rapidly change, robust performance to parameter variations and disturbances throughout the system. Moreover, to overcome the problem of control without mechanical sensors, we used a method based on Backstepping technique, for estimating speed from the measured currents. The speed elimination sensor reduces the constraints and gives more flexibility to control the machine.

In the light of the simulation results, we can conclude that the objective of this study is achieved. Thus, the Backstepping technique to the integral action combined with vector control offers high control performance. The intrinsic robustness Backstepping is strengthened through full terms added to it.

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