Modeling and Simulation of the Lightning Return Stroke Current Using Electromagnetic Models and the 3D-FDTD Method

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Abstract-**In this paper we present a comparison between lightning return stoke current distributions calculated in a vertical lightning channel. The latter is excited at its bottom by a lumped current source above a flat perfectly conducting ground. The computation is based on the use of the three dimensions finite difference time domain method (3D-FDTD) associated to uniaxial perfectly matched layer (UPML) boundary conditions. The calculating approach which is based on the Taflove formulation of the 3D-FDTD method combined to the UPML boundary conditions is implemented on Matlab environment in order to develop calculation codes to obtain lightning return stroke current distributions. Otherwise, due to electromagnetic model representation, the lightning channel is modelled alternately by : a vertical conducting wire in air loaded by additional distributed series inductance and distributed series resistance, a vertical perfectly wire embedded in a dielectric of relative permittivity great to the air, a vertical perfectly conducting wire embedded in parallelepiped dielectric having the same relative permittivity that the second representation surrounded by air, and a vertical perfectly wire coated by a material having a high relative permittivity and permeability in the air. Finally, the obtained lightning return stroke space and time distributions are compared with others taken from specialised literature for validation needs.**

Index terms—**Electromagnetic models, , 3D-FDTD method, Taflove formulation.**

I. Introduction

In Electromagnetic Compatibility studies, relative to lightning and its effects on various systems, it is necessary to determine the space-time lightning return stroke distribution along the lightning channel to identify the electromagnetic field and its effects on electrical system. So, several lightning return stroke distribution models exist. They are currently classified [1] into four types of models namely the physical models, the electromagnetic models, the RLC models, and the engineering models [1]. These models are currently used in lightning return stroke analysis, in lightning electromagnetic environment characterization and in lightning induced effects studies.

Furthermore electromagnetic models which are based on Maxwell's equations are relatively new but effective to analyze the coupling lightning electromagnetic field effects on various systems [5- 6-7].

So several works relative to lightning return stroke electromagnetic models have been performed by the implementation of numerical methods allowing the lightning electromagnetic field calculation. These methods solve Maxwell's equations in order to have the lightning channel current space-time distribution Among these numerical methods we quote the time domain moment method (MoM) (Van Baricum and Mailler [2]), the frequency domain moment method (MoM) (Harrington [3]) and the finite difference time domain method (FDTD) (Yee [4]). Indeed, the MoM method, in time domain, was used by Moini *et al*. [5] to have a numerical solution of the electric field integral equation; this can give the lightning channel current stroke distribution. Shoory *et al*. [6] employed the frequency domain MoM method with a vertical resistive wire excited by a lumped current source to analyze the lightning current. Finally the FDTD method was used by Baba and Rakov [7-8-9-10] for the specification of the current time-space distribution and the associate electromagnetic field components. The latter employs a simple way to discretize the entire work space into small cubic or rectangular parallelepiped cells for computing the electromagnetic field components and the return stroke current distribution along the lightning channel. This task needs a special lightning channel representation taking into account the current wave propagation velocity knowledge and the excitation sources representation.

The aim of this paper is to contribute in the characterization of lighting by the use of electromagnetic models and the 3D FDTD method. The paper is structured as follow. In the first section, we review some theoretical aspects concerning the Taflove formulation related to the 3D-FDTD method using uniaxial perfectly matched layer (UPML) absorbing boundary conditions [11]. These conditions are employed to solve Maxwell's equation to obtain the current distribution along the lightning channel. At the end of this section we give two formulas linking the space and the time increments which allow us to obtain the computational stability. A review on the lightning channel representations used in electromagnetic models and FDTD calculation is given in the second section ; we also present in this section the channel representations adopted in our analyze. The third section, of this paper, is devoted to the presentation of the simulation results. We also present, in this section, the calculation methodology used in lightning return stroke current analysis and a comparison between four lightning channel representations. For validation reasons, return stroke currents calculated are compared with measurements results taken from literature [12].

Moreover, for obtaining of the lightning current space-time distribution waveforms, it was necessary for us to develop on Matlab environment a 3D computation code. The latter, is based on the FDTD method including Taflove formulation. Uniaxial perfectly matched layer (UPML) absorbing boundary conditions were implemented in this computation code. Finally conclusions, related to the lightning analysis based on electromagnetic models and the 3D-FDTD method, are given in the last section.

II. 3D-FDTD FORMULATIONS AND BOUNDARY CONDITIONS

The FDTD method based on Yee algorithm [4] was employed in 2D and 3D by Baba and Rakov associated with the perfectly matched layer (PML) conditions based on Berenger's formulation [13] and the second order Lioas boundary conditions in various works such us [7], [8], [9] and [10] to analyze the lightning return stroke current and to compute the associated electromagnetic field components using different lightning channel representations and different excitation sources methods above a perfectly conducting ground and a lossy ground and also to study the influence of the strike tall grounded object on lightning electromagnetic field.

The Taflove formulation [11] of the 3D-FDTD method used to analyze lightning current and electromagnetic field in this work has the advantage that it can be used to calculate the electromagnetic components in the entire work space and set a uniaxial perfectly matched layer (UPML) absorbing boundary conditions using the same equations of electric and magnetic field associated to their current densities, employing a simple changing into mediums parameters (working space medium and UPML regions).

Equations representing this formulation can be written as follow:

$$D_x^{n+1}\left(i+\frac{1}{2},j,k\right)=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right).D_x^{n}\left(i+\frac{1}{2},j,k\right)+\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right).\left[\frac{H_z^{n+1/2}\left(i+\frac{1}{2},j+\frac{1}{2},k\right)-H_z^{n+1/2}\left(i+\frac{1}{2},j-\frac{1}{2},k\right)}{\Delta y}-\frac{H_y^{n+1/2}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)-H_y^{n+1/2}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}{\Delta z}\right] \quad (1\text{-a})$$

$$E_x^{n+1}\left(i+\frac{1}{2},j,k\right)=\left(\frac{2\varepsilon k_z-\sigma_z\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right).E_x^{n}\left(i+\frac{1}{2},j,k\right)+\left[\frac{1}{(2\varepsilon k_z+\sigma_z\Delta t)\varepsilon}\right].\left[(2\varepsilon k_x+\sigma_x\Delta t).D_x^{n+1}\left(i+\frac{1}{2},j,k\right)-(2\varepsilon k_x-\sigma_x\Delta t).D_x^{n}\left(i+\frac{1}{2},j,k\right)\right] \quad (1\text{-b})$$

$$B_x^{n+3/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right).B_x^{n+1/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)+\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right).\left[\frac{E_z^{n+1}\left(i,j+1,k+\frac{1}{2}\right)-E_z^{n+1}\left(i,j,k+\frac{1}{2}\right)}{\Delta y}-\frac{E_y^{n+1}\left(i,j+\frac{1}{2},k+1\right)-E_y^{n+1}\left(i,j+\frac{1}{2},k\right)}{\Delta z}\right] \quad (1\text{-c})$$

$$H_x^{n+3/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)=\left(\frac{2\varepsilon k_z-\sigma_z\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right).H_x^{n+1/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)+\left[\frac{1}{(2\varepsilon k_z+\sigma_z\Delta t)\mu}\right].\left[(2\varepsilon k_x+\sigma_x\Delta t).B_x^{n+3/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)-(2\varepsilon k_x-\sigma_x\Delta t).B_x^{n+1/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)\right] \quad (1\text{-d})$$

Similar expressions can be derived for the remaining electric and magnetic fields components.

With:

$$\sigma_x(x)=\left(\frac{x}{d}\right)^m.\sigma_{max} \quad (2)$$

$$k_x(x)=1+\left(k_{x,max}-1\right).\left(\frac{x}{d}\right)^m \quad (3)$$

$$\sigma_{max}=-\frac{(m+1)ln(R(0))}{2\,\eta\,d} \quad (4)$$

$$R(\theta)=e^{-2\,\eta\,cos\theta\int_0^d\sigma(x)\,dx} \text{ (Reflection error)} \quad (5)$$

$$\eta=\sqrt{\frac{\mu}{\varepsilon}} \quad (6)$$

d: The PML area thickness.
x: A positive integer number corresponding to the layer's number ($0<x<d$).

Defining the multiplying coefficients of electric and magnetic fields and current densities permits a unified treatment of both the interior working volume and the UPML area. The parameters σ and k values, in the interior of the working volume, depend on the medium nature. For example in a free space they worth $\sigma=0$ and $k=1$. However, in the

UPML area the parameters σ and k are assumed to have a polynomial-graded profile given by equations (3) and (4).
For computational stability, it is necessary to satisfy a relation between the space increment Δs and the time increment Δt namely [7]:

$$\frac{\Delta t}{\sqrt{\mu\varepsilon}} \leq \frac{\Delta s}{\sqrt{3}} \quad (8)$$

In the same context Noda and Yokoyama [14] proposed the following formula to determine the time step Δt is:

$$\Delta t = \Delta s \sqrt{\frac{\mu\varepsilon}{3}}(1-\alpha) \quad (9)$$

α is a small positive value specified by user in order to prevent instability of the numerical integration.

III. STUDY AND RESULT'S PRESENTATION

a. Lightning channel representation

In references [7],[8],[9], and [10], electromagnetic models of lightning return stroke are classified into seven several types depending on lightning return stroke channel representations. These types are:

1) a perfectly conducting/resistive wire in air above ground;
2) a wire loaded by a additional distributed series inductance in air above ground;
3) a wire surrounded by a dielectric medium (other than air) that occupies the half space above ground;
4) a wire coated by a dielectric material in air above ground;
5) a wire coated by a fictitious material having high relive permittivity and high relative permeability in air above ground;
6) two parallel wires having additional distributed shunt capacitance in air); and
7) a phase current source array in air above ground.

Among these seven electromagnetic models we considered, in this work, four models (figure 1) it is:

Model 1: A vertical wire loaded by an additional distributed series inductance and a distributed series resistance (L= 6.57 µH/m, R=0,13 Ω/m).

Model 2: lightning channel considered as a vertical wire of 0.2 m radius surrounded by a dielectric medium of permittivity $\varepsilon_r = 4.12$ greater than 1 occupying the entire half space above a flat ground.

Model 3: a vertical wire embedded in a parallelepiped of $\varepsilon_r = 4.12$.

Model 4: a vertical wire embedded in a parallelepiped of $\varepsilon_r = 4.12$ and $\mu_r = 4.12$.

Note that this types are used by Baba and Rakov [7], [8], [9], and authors of reference [10] in their FDTD computation and analysis of lightning current distribution along the channel and by Moini *et al*. [5] in their lightning electromagnetic field calculation based using the method of moment (MoM) in time domain.

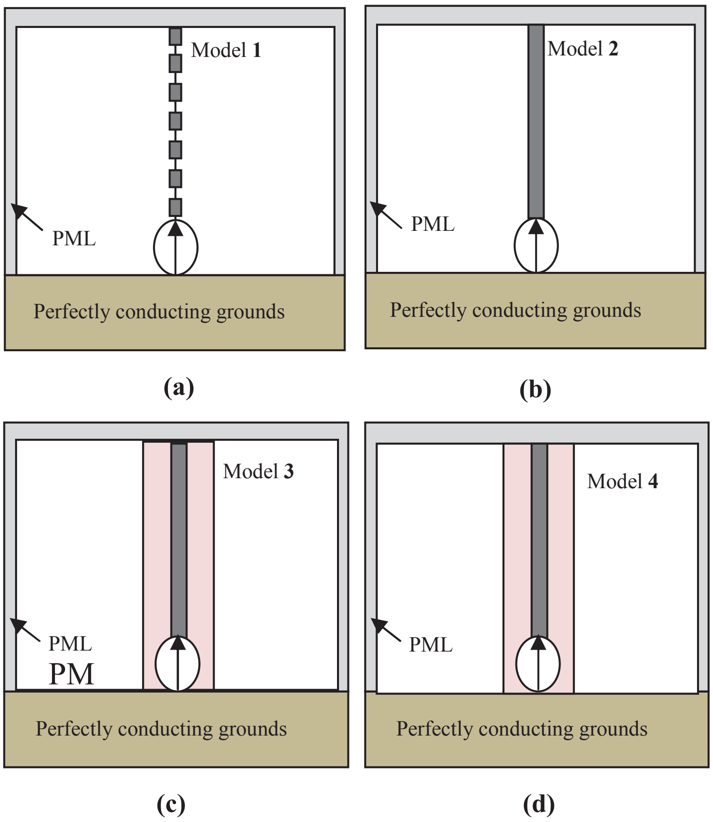

Fig. 1. Two lightning return stroke channel representations with a perfectly conducting ground excited at its bottom by a current source:
(a) Model **1** (L= 6.57 µH/m, R=0,13 Ω/m),
(b) Model **2** ($\varepsilon_r = 4.12$).
(c) Model **3** (parallelepiped of $\varepsilon_r = 4.12$).
(d) Model 4 (parallelepiped of $\varepsilon_r = 4.12$ and $\mu_r = 4.12$).

b. simulation principal

In this study the lightning return stroke channel is represented by a vertical wire placed at the center of a horizontal perfectly conducting plane and excited at its bottom by a current source. The later is represented by four kinds of electromagnetic models (Figure 1).

In this part of study we analyze the return stroke current space-time distribution along the lightning channel using the electromagnetic model, illustrated in Figure (1) and based on 3D-FDTD formulations presented before.

The wire representing the lightning channel has a length of 4 km; this length value is close enough to reality since the lightning channel length must not exceed 7.5 km. This wire is placed in a working volume of 90 m x 90 m x 4500 m, which is divided into rectangular parallelepiped cells of 1.5 m $\times$ 1.5 m $\times$ 10 m. In this way a vertical wire has an equivalent radius of 0.2 m ($r_{eq} = 0.135 \times \Delta x$, according to Taniguchi *et al* [27]). The time increment was fixed to 2 ns.

Finally, in the second part of this section, we compare our results, consisting of the lightning channel distribution current, with those computed by Izadi *et al* [12].

c. Current time- space distribution along the lightning channel

The vertical wire shown in Figure (1) is excited at its bottom by a lumped current source, this latter produce a current waveform having a peak of 16 kA and a rise time of 0.7 µs. Note that the channel-base current waveform shown at Figure (1), at 0 m, is calculated using Heidler function (equation 10) applied to a subsequent return stroke and using parameters illustrated in table 1 (taken from reference [12] for comparison and validation needs).

$$i(0,t) = \frac{i_{01}}{\eta_1}\frac{\left(t/\tau_{11}\right)^{n_1}}{1+\left(t/\tau_{11}\right)^{n_1}} exp\left(\frac{-t}{\tau_{12}}\right) + \frac{i_{02}}{\eta_2}\frac{\left(t/\tau_{21}\right)^{n_2}}{1+\left(t/\tau_{21}\right)^{n_2}} exp\left(\frac{-t}{\tau_{22}}\right) \quad (10)$$

Where:

i_{01}, i_{02} are current amplitudes,
τ_{11}, τ_{12} are the front time constants,
τ_{21}, τ_{22} are the decay time constants,
n_1, n_2 are the exponents

$$\eta_1 = \left[-\left(\frac{\tau_{11}}{\tau_{12}}\right)\left(n_1.\frac{\tau_{12}}{\tau_{11}}\right)^{\frac{1}{n_1}}\right], \quad \eta_2 = \left[-\left(\frac{\tau_{21}}{\tau_{22}}\right)\left(n_2.\frac{\tau_{22}}{\tau_{21}}\right)^{\frac{1}{n_2}}\right]$$

Table 1
Lightning channel base current parameters.

i_{01} (kA)	τ_{11} (µs)	τ_{12} (µs)	i_{01} (kA)	τ_{21} (µs)	τ_{22} (µs)	n_1	n_1
14.8	0.244	2.77	6.86	4.18	40.66	2	2

The speed of lightning return stroke current wave is equal to 1.4752.10^8 m/s [12] (lower to the light velocity 3.10^8 m/s) for this reason we take L = 6.57 µH/m, and $\varepsilon_r = 4.12$.

In figure (2) we present a comparison between our results, obtained by the achievement of the 3D-FDTD method and EM models, and those obtained by Izadi and *al* [12] using measured electromagnetic fields. These results consist in current waveforms which are plotted at different lightning channel heights namely: 0 m, 250 m, 500 m, and 750 m. Thus, current waveforms plotted in figure (2 a) are those obtained by Izadi *et al* [12] and curves plotted in figure (2 b-c-d-e) are our results.

Through this comparison, we can see the good agreement between the two results. As seen in figure. (2 a), the current wave suffers both attenuation and dispersion as it propagates along the lightning channel. In all the four electromagnetic models results we can remark that the current waveforms have attenuation in there magnitude along the lightning channel and there is also a difference between current speeds obtained for each model. So, in figure (2 b-c) the lightning current speed is equal to 1.47. 10^8 m/s, in figure (2 d) this latter equal to 2.38. 10^8 m/s and the speed values taken from figure (2 e) is equal to 1.7857.108 m/s. from this values of lighting return stroke current speeds it is clearly remarked that all the four electromagnetic models implemented in this calculation give a speed lower than that of light velocity, the difference between these values is due to the series inductance value (model 1) and the size of artificial medium

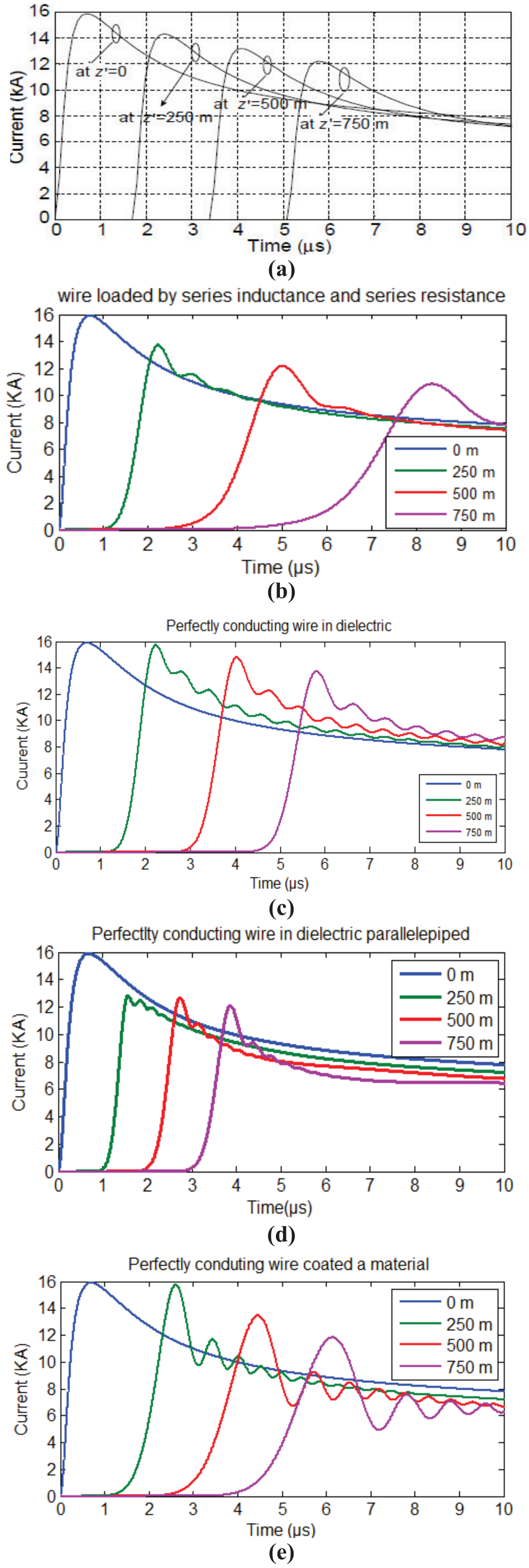

Fig. 2. Current waveforms calculated at different heights using the 3D-FDTD method and EM models:
(a) Results taken from [12],
(b), **(c)**, **(d)** **(e)**: Our results.

and to it's relative permittivity and permeability (model 2,3 and 4) we can notice also that there is a difference in the rate of attenuation of current waveforms along the lightning channel of each model.

Finally we can notice that the use of four electromagnetic models and the 3D- FDTD method based on Taflove formulation has given good results validating thus our computation code developed on Matlab environment.

IV. CONCLUSION

In this paper, we implemented electromagnetic models in order to calculate the lightning return stroke current distribution, along a vertical channel and above a flat perfectly conducting ground. In this computation we also implemented Taflove formulation in the 3D-FDTD method associated to the UPML absorbing boundary conditions. To that effect, we developed two 3D calculation codes based on this numerical method. Finally, this computation approach gives a good agreement between the obtained results (current distribution along the lightning channel) and results of the same type taken from literature allowing us to validate the proposed approach based on the use of electromagnetic models and Taflove formulation in the 3D FDTD calculating method. At the same time we also validated the developed codes.

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