Comparison between rate equations model and traveling wave model in large signal transient response of Fabry-Pérot Laser diodes

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Abstract—In this paper we compare and analysis the large signal transient response of Fabry-Pérot semiconductor laser using two different theoretical models: rate equations model and traveling wave model. The results are obtained by numerical solution of the two models and show an excellent agreement.

Keywords—Rate equations model; Traveling wave model; Fabry-Pérot Laser diodes, Transient response.

I. INTRODUCTION

Laser diodes are key components of the coherent high capacity optical communication systems due to their attractive properties such as: emission wavelength, compact structure, low threshold, high-speed modulation [1]. With the enhancement of computer hardware, computer science and numerical methods; computer aided model plays an important role in semiconductor lasers design and optimize [2].

Two well-known theoretical frameworks are used for semiconductor lasers modeling and analysis: rate equations model, and traveling wave model TWM. The rate equations model is a classic approach in laser modeling and it used since 1960 [3], it describe the rate of injection and recombinaison of electronic carriers, and the rate of emission and loss of photons. Mathematically it consists of a set of ordinary differential equations for the electric field and electron density within the laser cavity, where the carrier density and the optical field are assumed to be uniformly distributed along the laser cavity.

Comparing with rate equations model, traveling wave model is a recent approach in semiconductor laser modeling; where the basics are putted by many authors [5-6-7]. It provides information about the longitudinal distribution of the carrier density and the optical power where the optical field is split into forward and backward fields. Mathematically it is described by partial differential equations of the time-dependent coupled-mode equation for electric waves traveling in the opposite direction along longitudinal axis of the laser; these equations are coupled to the ordinary differential carrier rate equation in the active layer. The boundary conditions of the model are defined by the reflectivity conditions at both facets of the laser.

The purpose of this paper is to provide quantitative comparison of large signal transient response of Fabry-Pérot laser diodes, using the rate equations model and the traveling wave model. A comparison between these models is desirable to demonstrate the efficiency and the accuracy and highlights the differences in approach to various problems

The transient response is an important characteristic for application of semiconductor lasers in optical communications, the high-speed optical signals are generated by modulating the injection currents into the laser, where the modulation characteristics are determined by the intrinsic character of the semiconductor laser itself: turn on delay time, and the relaxation oscillation, this two main characteristics of the semiconductor laser will be discussed in detail in section III.

The paper is organized as follows. Section II will give a brief presentation of rate equations model and the traveling wave model TWM for Fabry-Pérot semiconductor laser. In Section III, we give the simulation results and physically interpret them. We finally give conclusion in Section IV.

II. DESCRIPTION OF THE MODELS

A. Rate equations model:

The Fabry-Pérot laser diodes dynamic is governed by single mode rate equations for the time variation of the optical intensity $S$ and the carrier density $N$ on the active layer as follows [4]:

$$\frac{dS}{dt} = G(N)S - \frac{S}{\tau_p} + \beta R_p$$

(1)

$$\frac{dN}{dt} = \frac{J(t)}{qv} - \frac{N}{\tau_e} - G(N)S$$

(2)

Where $G(N)$ is the optical gain in the active region of the device and is given by:
\[ G(N) = \frac{\Gamma G_{\alpha} (N - N_0)}{1 + \varepsilon S} \quad (3) \]

\( \varepsilon \) is the gain compression factor, when the saturation effect is very small, we can approximate the gain as:

\[ G(N) = \Gamma G_{\alpha} (N - N_0) (1 - \varepsilon S) \quad (4) \]

\( \tau_p \) is the photon lifetime, \( \beta \) is the spontaneous emission factor, \( R \) total spontaneous emission, \( J(t) \) is the injection current, \( \tau_c \) is the carrier lifetime dependant to the carrier density it is given by :

\[ \frac{1}{\tau_c} = A + BN + CN^2 \quad (5) \]

Where coefficient \( A \) describes non-radiative processes, \( B \) is responsible for spontaneous recombinaton and \( C \) describes non-radiative Auger recombinaton.

**B. Traveling wave model:**

Starting from the time dependent coupled wave equation of the DFB laser \([8-9-10]\), and putting the grating period and the coupling coefficient to zero for the F-P lasers case, a partial differential equation, called an advection wave equation is obtained for Fabry-Pérot Laser, and it is used to describe the time and spatial development of the field in the laser cavity. For both forward and backward waves, we have :

\[ \frac{1}{C_G} \frac{\partial E^+}{\partial t} + \frac{\partial E^+}{\partial z} = \frac{(\Gamma gm - am)}{2} E^+(z,t) + \xi^+(z,t) \quad (6) \]

\[ \frac{1}{C_G} \frac{\partial E^-}{\partial t} - \frac{\partial E^-}{\partial z} = \frac{(\Gamma gm - am)}{2} E^-(z,t) + \xi^-(z,t) \quad (7) \]

\( E^+(z,t), E^-(z,t) \) are the forward and backward slowly varying amplitude of the traveling optical fields. \( C_G \) is known as the group velocity, \( gm, am \) are respectively the material amplitude gain and loss. In some text, the term \( (\Gamma gm - am) \) is replaced by the net gain \( G \) given by \([8]\) :

\[ G = \Gamma G_{\alpha} \left\{ N(z,t) - N_0 \right\} \frac{\alpha m}{2(1 + \varepsilon S)} \quad (8) \]

Where \( G_{\alpha} \) is the differential gain, \( N(z,t) \) is the carrier density, \( \varepsilon \) is the gain compression factor, \( N_0 \) is the carrier density at transparency, \( S \) is the photon density and it is function of optical field and is given by :

\[ S = \left| E^+ \right|^2 + \left| E^- \right|^2 \quad (9) \]

\( \xi^+(z,t) \) and \( \xi^-(z,t) \) are the spontaneous noises associated with the forward and backward traveling waves and they are a function of the carrier density, they appear in the laser output as noise but they are also an important mechanism for the operation of the laser, when the laser turn on, spontaneous emission initiates the photon distribution inside the cavity, where it is supposed to have a zero mean :

\[ \langle \xi(z,t), \xi(z,t) \rangle = 0 \]

and their correlation function is given by:

\[ \langle \xi(z,t), \xi(z',t') \rangle = K \omega \delta(t - t') \delta(z - z') \quad (10) \]

\( K \) is the Peterman coefficient and \( \Psi \) is the bimolecular recombination per unit length contributed to spontaneous emission. Using the boundary conditions at right and left facets of the laser:

\[ E^+(z = 0, t) = r_1 E^-(z = 0, t) \]

\[ E^-(z = L, t) = r_2 E^+(z = L, t) \]

\( r_1, r_2 \) are the amplitude reflectivity of the left and right facets respectively. The rate equation for carrier density is given by:

\[ \frac{\partial N}{\partial t} = \frac{J(t)}{ed} - \frac{N(z,t)}{\tau_c} - AN(z,t) - BN(z,t)^2 - CN(z,t)^3 - GC_{\alpha} P \quad (12) \]

\( J(t) \) is the injection current, \( e \) is the modulus of the electron charge, \( d \) is the volume of the active layer, \( \tau_c \) is the carrier life time, \( B \) is the radiative spontaneous emission coefficient, \( C \) is the Auger recombination coefficient.

**TABLE I**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>2</td>
<td>Non-radiative recombination coefficient</td>
</tr>
<tr>
<td>( B )</td>
<td>1</td>
<td>Bimolecular coefficient</td>
</tr>
<tr>
<td>( C )</td>
<td>1</td>
<td>Auger coefficient</td>
</tr>
<tr>
<td>( G_{N} )</td>
<td>2.5</td>
<td>Differential gain</td>
</tr>
<tr>
<td>( \alpha_m )</td>
<td>11.2</td>
<td>Waveguide loss</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>1.5</td>
<td>Transparency carrier density</td>
</tr>
<tr>
<td>( L )</td>
<td>300</td>
<td>Section cavity length</td>
</tr>
<tr>
<td>( w )</td>
<td>0.8</td>
<td>Active layer width</td>
</tr>
<tr>
<td>( d )</td>
<td>0.14</td>
<td>Active layer thickness</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>0.25</td>
<td>Confinement factor</td>
</tr>
<tr>
<td>( r_2, r_1 )</td>
<td>0.32, 0.32</td>
<td>Right and left facet amplitude reflectivity</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1310</td>
<td>Wavelength lasing</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>3</td>
<td>Gain compression factor</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 5 \times 10^5 )</td>
<td>Spontaneous emission coefficient</td>
</tr>
</tbody>
</table>
III. RESULTS AND DISCUSSION

Using the parameters listed in Table I and the models mentioned in section 2, we simulate the large transient response of Fabry-Pérot laser by solving numerically the two models. To solve the rate equations we have used fourth Runge-Kutta method, while for TWM we have used finite difference time domain method FDTD. Because the traveling wave model is a stochastic model we ignore the effect of spontaneous noise in the simulation of the TWM.

The current density $J(t)$ is given by:

$$ J(t) = J_b u(t) \quad (13) $$

$$ u(t) = \begin{cases} 0(t < 0) \\ 1(t \geq 0) \end{cases} \quad (14) $$

$J_b$ is the biased current.

We plot in Fig.1 and Fig.2 the output optical power as a function of time of a Fabry-Pérot semiconductor laser biased close threshold at 5mA using TWM and rate equations model respectively. Note that the pulse width of the injection current is much larger the carrier life time $\tau_c$. The results in both models show that the laser takes less than 4 ns to reach a stable output, this time is called the turn on delay time and is associated with the fact that laser takes time to inject enough electrons to bring the laser to start the lasing process, and the peak in the two graphs and correspond to shorter wavelength which is suppressed by lasing mode after the laser reaches the steady state, then we obtain the quasi single mode oscillation. On other word, the relaxation oscillation appear in laser output because the carrier cannot follow the photon decay rate. Above the relaxation oscillation the modulation efficiency is greatly degraded and the intensity modulation through the injection current becomes difficult.

The large signal transient response biased above threshold at 15mA are shown in Fig. 3 and Fig. 4 using TWM and rate equations model respectively. The device take around 1ns to reach the steady state in both models, we also notice that as the biasing current increase, the electrons density increase that lead the photon density, this gives rise to significant increase photon density overshoot and relaxation oscillations appear rapidly and become dominant at the laser output.

Our calculation demonstrates that the turn on delay time and damping rate of relaxation oscillation are same for both models, and damping oscillation is more prominent for lower drive current near to threshold.

In terms of difference in the simulation time between the two models the traveling wave model take more longue time than the rate equations because it needs multiple computational run.
Fig. 4. Large signal transient response of F-P laser using rate equations model above threshold (15mA)

On the other hand, traveling wave model is still more efficient compared to rate equations model in the large signal dynamic analysis when the spontaneous emission noise is taken into account [7], [11].

IV. CONCLUSION

In this paper we have compared the large signal transient response of Fabry-Pérot semiconductor laser using two different theoretical models. Simulation results are based on InGaAsP 1310nm wavelength and they show clearly a good agreement between the traveling wave model and the rate equations model in the simulation of the large signal transient response. This agreement demonstrates also that the two models can be used as robust simulation tools for designing and optimizing of semiconductor lasers.

In The future work we will compare other dynamical proprieties of semiconductor lasers using the two models.

REFERENCES